G14CAM 2017–2018	**Computational Applied Mathematics** Coursework 1 Part B
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Existing codes	Names of approved existing codes that you used

Problem 1

$$\begin{pmatrix} 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$c_{-1} = 2c_0 - c_1$$
$$c_3 = 2c_2 - c_1$$

$$c_{-1} + -2c_0 + c_1 = 0$$
$$c_1 - 2c_2 + c_3 = 0$$

$$\begin{pmatrix} 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2c_0 - c_1 \\ c_0 \\ c_1 \\ c_2 \\ 2c_2 - c_1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

$$c_0 = 1/2, \quad c_2 = 1/3$$

$$c_{-1} = \frac{35}{24}, \quad c_1 = \frac{-11}{24}, \quad c_3 = 9/8$$

Problem 2

Problem 2.A

```
#include <iostream>
#include <cmath>
#include <iomanip>

void tridiagonal_matrix_solver(int n, double* c, double* lower, double* diag,
double* upper, double* f);

int main()
{
double h; //stepsize
std::cout<<"Enter␣stepsize␣h␣\n";
std::cin>>h;
double L; //Length of  interval
std::cout<<"Enter␣length␣of␣interval␣(0,L)␣\n";
std::cin>>L;
int n; //n is the number of nodes
n = int((L/h) +1.0);

double x[n];
double pi = 3.14159265358979323846;

double* lower;
lower = new double[n-1];

double* diag;
diag = new double[n];

double* upper;
upper = new double[n-1];

double* f;
f = new double[n];

for (int i=0; i<n; i++)
    {
    x[i] = 0.0 + h*(double)(i); //Interval (0,1)
    f[i] = exp(x[i])*sin((5.0/4.0)*pi*x[i]); //f(x) = exp(x)*sin(5pi/4)x)
    }


f[0]   = f[0] + (h/3.0)*exp(x[0])*(sin(5.0*pi*x[0]/4.0)+
(5.0*pi/4.0)*cos(5.0*pi*x[0]/4.0));
f[n-1] = f[n-1] - (h/3.0)*exp(x[n-1])*(sin(5.0*pi*x[n-1]/4.0)+
(5.0*pi/4.0)*cos(5.0*pi*x[n-1]/4.0));

//f dash evaluated at end points
double f_diff[2];
f_diff[0] = exp(x[0])*(sin(5.0*pi*x[0]/4.0)+(5.0*pi/4.0)*cos(5.0*pi*x[0]/4.0));
f_diff[1] = exp(x[n-1])*(sin(5.0*pi*x[n-1]/4.0)+
(5.0*pi/4.0)*cos(5.0*pi*x[n-1]/4.0));
```

```
double* c;
c = new double[n]; //c not including c[-1] and c[n+1]

c[0] = 0.0;
diag[0] = 4.0;
upper[0] = 2.0;
lower[n-2] = 2.0;
diag[n-1] = 4.0;
c[n-1] = 0.0;

for (int i=1; i<n-1; i++)
    {
    c[i] = 0.0;
    lower[i-1] = 1.0;
    diag[i] = 4.0;
    upper[i] = 1.0;
    }

tridiagonal_matrix_solver(n,c,lower,diag,upper,f);

//Printing out entries of c
double c_end[n+2]; //c including end values

c_end[0] = c[2] - (1.0/3.0)*h*f_diff[0];
c_end[n+1] = c[n-2]+(1.0/3.0)*h*f_diff[1];

std::cout<<"c[-1] = "<<c[0]<<"\n";
for( int i = 0; i<n; i++)
{
c_end[i+1] = c[i];
std::cout<<"c["<<i<<"] = "<<c_end[i+1]<<"\n";
}
std::cout<<"c["<<n<<"] = "<<c_end[n+1]<<"\n";

delete[] lower;
delete[] upper;
delete[] diag;
delete[] f;
delete[] c;

return 0;
}
```

Problem 2.B

i	c_i
-1	-0.132442
0	-0.0155912
1	0.194807
2	0.303991
3	0.112447
4	-0.340775
5	-0.671465
6	-0.396644
7	0.542973
8	1.42184
9	1.15873

Problem 2.C

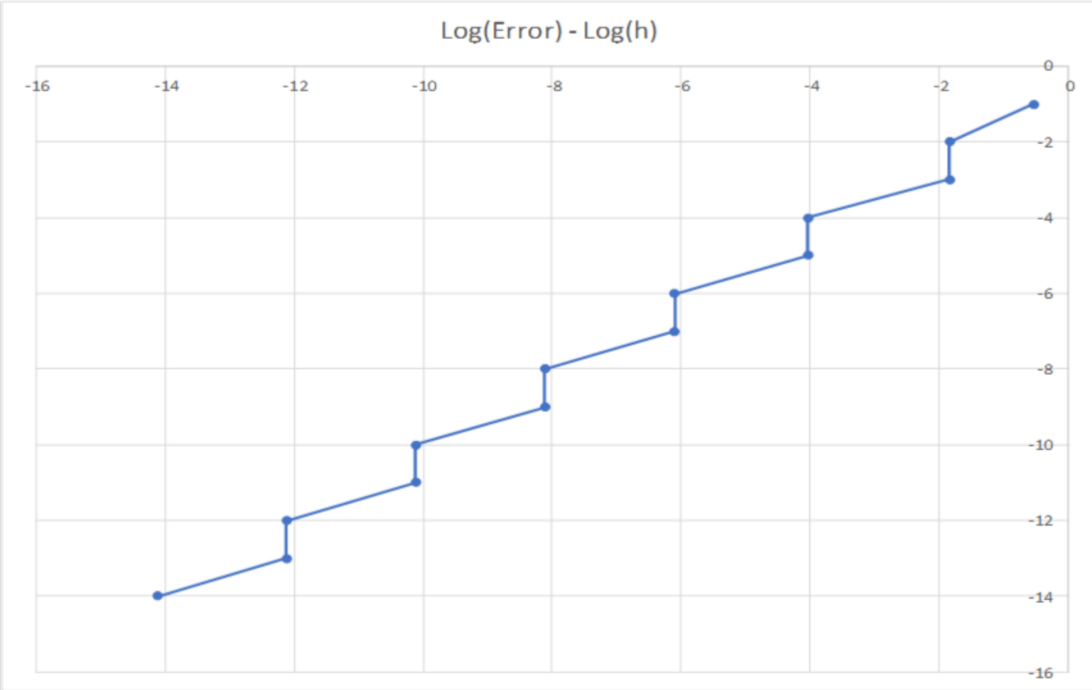
h	$q_h(\frac{1}{3})$	$\left\lvert f(\frac{1}{3}) - q_h(\frac{1}{3})\right\rvert$
$1/2$	0.654498	0.69356
$1/4$	1.06763	0.28043
$1/8$	1.06763	0.28043
$1/16$	1.28694	0.0611199
$1/32$	1.28694	0.0611199
$1/64$	1.28694	0.0611199
$1/128$	1.33343	0.0146324

```
#include <iomanip>
#include <iostream>
#include <cmath>
#include <cassert>

double evaluate_qh(int n, double h, double x_val, double x0, double* c)
{
double qh = 0;

//For any x in [xk-1,xk] bi is only locally non zero
//therefore q(x) = ck-2Bk-2(x) + ck-1Bk-1(x) + ckBk(x) + ck+1Bk+1(x)
int k = int((x_val - x0)/h +1.0);

assert(k>=0);

if (k<2)
    {
    qh = 4.0*c[k]+ 2.0*c[k+1];
    }
else if ((k>1) && (k<=n))
    {
    qh = c[k-1] +4.0*c[k]+c[k+1];
    }
else
    {
```

```
    qh = 2.0*c[k-1]+4.0*c[k];
    }
return qh;
}


void tridiagonal_matrix_solver(int n, double* x, double* lower, double* diag,
 double* upper, double* f);
double evaluate_qh(int n, double h, double x_val, double x0, double* c);

int main()
{
double h; //stepsize
double L = 2; //Length of  interval
int n; //n is the number of nodes
double pi = 3.14159265358979323846;

double x_val;
std::cout<<"Enter␣x␣\n";
std::cin>>x_val;
double q_h;
double Err = 0;

std::cout << "-------------------------------------------
-------------------------------"
            << std::endl;

std::cout << std::setw(10) << "h"
              << std::setw(20) << "q_h"
              <<std::setw(40) << "Err"
              << std::endl
              << "-------------------------------------------
␣␣␣␣␣␣␣␣␣␣␣␣-------------------------------"
            << std::endl;
double* x;
double* lower;
double* diag;
double* upper;
double* f;
double* c;
double* c_end;

double f_diff[2];
f_diff[0] = exp(x[0])*(sin(5.0*pi*x[0]/4.0)+(5.0*pi/4.0)*cos(5.0*pi*x[0]/4.0));

for (int j = 1; j<15; j++)
    {

    h = pow(0.5,j);
    n = int((L/h) +1.0);
    x = new double[n];
    lower = new double[n-1];
    diag = new double[n];
    upper = new double[n-1];
    f = new double[n];
    c = new double[n];
```

```
    c_end = new double[n+2]; //c including end values

    for (int i=0; i<n; i++)
        {
        x[i] = 0.0 + h*(double)(i); //Interval (0,1)
        f[i] = exp(x[i])*sin((5.0/4.0)*pi*x[i]); //f(x) = exp(x)*sin(5pi/4)x)
        }
    f[0]   = f[0] + (h/3.0)*exp(x[0])*(sin(5.0*pi*x[0]/4.0)+
    (5.0*pi/4.0)*cos(5.0*pi*x[0]/4.0));
    f[n-1] = f[n-1] - (h/3.0)*exp(x[n-1])*(sin(5.0*pi*x[n-1]/4.0)+
    (5.0*pi/4.0)*cos(5.0*pi*x[n-1]/4.0));

    c[0] = 0.0;
    diag[0] = 4.0;
    upper[0] = 2.0;
    lower[n-2] = 2.0;
    diag[n-1] = 4.0;
    c[n-1] = 0.0;

    for (int i=1; i<n-1; i++)
        {
        c[i] = 0.0;
        lower[i-1] = 1.0;
        diag[i] = 4.0;
        upper[i] = 1.0;
        }

    tridiagonal_matrix_solver(n,c,lower,diag,upper,f);

    f_diff[1] = exp(x[n-1])*(sin(5.0*pi*x[n-1]/4.0)+
    (5.0*pi/4.0)*cos(5.0*pi*x[n-1]/4.0));

    c_end[0] = c[2] - (1.0/3.0)*h*f_diff[0];
    c_end[n+1] = c[n-2]+(1.0/3.0)*h*f_diff[1];


    for( int i = 0; i<n; i++)
    {
    c_end[i+1] = c[i];
    }

    q_h = evaluate_qh(n,h,x_val,x[0],c_end);

    Err = fabs(exp(1.0/3.0)*sin(pi*5.0/12.0) - q_h);


    std::cout<< std::setw(10)<<h
    <<std::setw(20)<<q_h
    <<std::setw(40)<<Err<<std::endl;

    }

delete[] lower;
delete[] upper;
delete[] diag;
delete[] f;
```

```
delete[] c;
delete[] x;
delete[] c_end;
}
```



Gradient is roughly one, so step size is related to convergence by order one.

Problem 3

Coursework 1 | Part B

Problem 3

$$q(x,t) = \sum_{i=-1}^{3} \sum_{j=-1}^{3} c_{ij} B_i(x) B_j(t)$$

9 data points for q

$$\begin{pmatrix} q_{00} & q_{02} & q_{04} \\ q_{10} & q_{12} & q_{14} \\ q_{20} & q_{22} & q_{24} \end{pmatrix} = \begin{pmatrix} 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} c_{-1-1} & c_{-10} & c_{11} & c_{-12} & c_{-13} \\ c_{0-1} & c_{00} & c_{01} & c_{02} & c_{03} \\ c_{1-1} & c_{-10} & c_{-11} & c_{-12} & c_{-13} \\ c_{2-1} & c_{20} & c_{21} & c_{22} & c_{23} \\ c_{3-1} & c_{30} & c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

Matices must go this way, otherwise do not commute.
Here ~~[illegible]~~ we have 9 equations for 25 unknowns.

Now impose boundary conditions

i) Natural:

In 1D impose $q''(x)=0$ at end points

Could impose

$q_{xx}(x,t) = 0$ when $x = 0, 1, 2$ and $t = 0, 2, 4$

$q_{tt}(x,t) = 0$ when $x = 0, 1, 2$ and $t = 0, 2, 4$

Giving $8 + 8 = 16$ equations, totalling 25 equations

2) Complete:

In 1D impose $q'(x) = f'(x)$ at end points

Could impose

$q_x(x,t) = f_x(x,t)$ when $x = 0, 1, 2$ and $t = 0, 2, 4$

$q_t(x,t) = f_t(x,t)$ when $x = 0, 1, 2$ and $t = 0, 2, 4$

Totalling 25 equations