G14CAM 2017–2018	**Computational Applied Mathematics** Coursework 1 Part C
Name **Student ID** **Date**	Ella Taylor 429052 March 1, 2018
Existing codes	Names of approved existing codes that you used

Problem 1

Problem 1.a

Type your solution here, or upload hand written material

Problem 1.b

```
#include <iostream>
#include <cmath>
#include <cassert>

double f1(double x)
{
const double pi = 3.14159265358979323846;
return (1.0/18.0)*pow(x,4) - sin(pi*x/6.0);
}

double GQ1(double a, double b)
{
assert(a < b);
//Mapping Xi on to x using x = (b-a)/2 * Xi + (a+b)/2
//Integral approx equal to 2*f(0) for integrating between -1,1.
// Xi = 0 the x = (a+b)/2
//GQ1 = 2(b-a)/2 *f((a+b)/2)= (b-a)*f((a+b)/2)

return (b-a)*f1((a+b)/2);
}

int main()
{
const double a = 0.0;
const double b = 3.0;

double GQ_1;

GQ_1 = GQ1(a,b);

std::cout<<"GQ1 = "<<GQ_1<<"\n";

return 0;
}
```

Problem 1.c

```
#include <cmath>
#include <cassert>
#include <iostream>

double f1(double x)
{
const double pi = 3.14159265358979323846;
return (1.0/18.0)*pow(x,4) - sin(pi*x/6.0);
}

double GQ3(double a, double b)
{
assert(a < b);
//Mapping Xi on to x using x = (b-a)/2 * Xi + (a+b)/2
//GQ3 = 5/9 * f(-sqrt(3/5)) + 8/9 * f(0) + 5/9  * f(sqrt(3/5))
//f(x) = 1/18 * pow(x,4) - sin(pi x /6)
//sin functions cancel as odd
double x[3];
x[0] = -sqrt(3.0/5.0);
x[1] = 0;
x[2] = sqrt(3.0/5.0);

double Xi[3];
for (int i=0; i<3; i++)
    {
    Xi[i] = x[i]*(b-a)/2.0 + (a+b)/2.0;
    }

return ((b-a)/2.0) * ( (5.0/9.0) * f1(Xi[0]) +(8.0/9.0) * f1(Xi[1]) +
(5.0/9.0) * f1(Xi[2]) );
}

int main()
{
const double a = 0.0;
const double b = 3.0;

double GQ_3;

GQ_3 = GQ3(a,b);

std::cout<<"GQ3 = "<<GQ_3<<"\n";

return 0;
}
```

Problem 2

Problem 2.a

```
#include <cmath>
#include <iostream>
#include <iomanip>
```

```
double f2(double x)
{
return tanh(100.0*(x-2.0)) * ((3.0-x)/10.0) * pow(pow(x,2)-x*pow(2,0.5)+
0.5005,0.5);
}


double CGQ1(int n, double a, double b)
{
double h = (b-a)/(double)(n);
double sum = 0;
double Xi;

//f = (tanh(100(x-2)))((3-x)/10)sqrt(x^2 - x sqrt(2)+ 0.5005)

for(double i = a; i<b; i = i+h)
    {
    Xi = (2.0 * i + h)/2.0;
    sum = sum + h * f2(Xi);
    }

return sum;
}

double CGQ3(int n, double a, double b)
{
double sum = 0.0;
double h = (b-a)/((double)(n));

double x[3];
x[0] = -sqrt(3.0/5.0);
x[1] = 0;
x[2] = sqrt(3.0/5.0);

double Xi[3];

//f = (tanh(100(x-2)))((3-x)/10)sqrt(x^2 - x sqrt(2)+ 0.5005)

for (double i = a; i<b; i = i+h)
    {

    for (int j=0; j<3; j++)
    {
    //mapping points on to interval [-1,1]
    Xi[j] = x[j]*h/2.0 + (2.0*i+h)/2.0;
    }

    sum = sum + (h/2.0)*( (5.0/9.0)*f2(Xi[0]) + (8.0/9.0)*f2(Xi[1]) +
    (5.0/9.0)*f2(Xi[2]));
    }

return sum;
}

int main()
{
```

```
int n; //n is number of intervals
double h;
const double a = 0;
const double b = 3;

std::cout<< "N"<<std::setw(20)<<"h"<<std::setw(30)<<"CGQ1"<<
std::setw(35)<<"CGQ3"<<std::endl;
std::cout<<"--------------------------------------------
---------------------------------------------"<<std::endl;

for (int i=0; i<=7; i++)
{
n = pow(2,i);
h = (b-a)/n;
std::cout<<n<<std::setw(20)<<h<<std::setw(30)<<
CGQ1(n,a,b)<<std::setw(35)<<CGQ3(n,a,b)<<std::endl;
}

return 0;
}
```

N	h	CGQ_1	CGQ_3
1	3	-0.356944	-0.185565
2	$\frac{3}{2}$	0.157268	-0.0754361
4	$\frac{3}{4}$	-0.168996	-0.11855
8	$\frac{3}{8}$	-0.0737691	-0.102471
16	$\frac{3}{16}$	-0.123251	-0.110649
32	$\frac{3}{32}$	-0.100556	-0.106934
64	$\frac{3}{64}$	-0.109829	-0.108087
128	$\frac{3}{128}$	-0.107711	-0.10793

Problem 2.b

```
#include <iostream>
#include <cmath>
#include <iomanip>

double CGQ1(int n, double a, double b);
double CGQ3(int n, double a, double b);

double adapt_int(double a, double b, double tau)
{
int n = 1; //number of  intervals
int n_new;
int N = 1000; //max number of mesh points
int counter = 0;
double Est= 10;
double Gauss_quad1 = 0.0;
double Gauss_quad3 = 0.0;


double* tau_k;
double* Est_k;
double* x;
```

```
x = new double[N];
tau_k = new double[N];
Est_k = new double[N];

//Setting end points for single interval
x[0] = a;
x[1] = b;

std::cout<< "␣i␣"<<std::setw(10)<<"␣N␣"<<std::setw(20)<<"␣CGQ1␣"<<std::setw(30)
<<"␣CGQ3␣"<<std::setw(40)<<"␣Sum␣Est_k␣"<<std::endl;

while (Est > tau)
    {
    counter = counter + 1;
    //Est =/= 0 for while loop to begin. Now recalculating Est
    //Resetting Gauss_Quad calculations to zero
    Est = 0.0;
    Gauss_quad1 = 0.0;
    Gauss_quad3 = 0.0;

    for (int k = 0; k<n; k++)
        {
        //Storing each tolerance
        tau_k[k] = tau*(x[k+1] - x[k])/(b-a);
        //Storing each error
        Est_k[k] = fabs(CGQ3(1,x[k],x[k+1]) - CGQ1(1,x[k],x[k+1]));

        Est = Est + Est_k[k];

        Gauss_quad1 = Gauss_quad1 + CGQ1(1, x[k],x[k+1]);
        Gauss_quad3 = Gauss_quad3 + CGQ3(1, x[k],x[k+1]);
        }

    std::cout << counter  <<std::setw(10)<<n<<std::setw(20)<< Gauss_quad1
    <<std::setw(30)<<Gauss_quad3 <<std::setw(40)<<Est<<std::endl;

    n_new = n;

    for(int k = 0; k<n_new; k++)
        {
        if(Est_k[k] > tau_k[k])
            {
            n_new= n_new+1;

            //Shifting entries of x along to make space for new mesh point
            for(int i=n_new; i>k+1; i--)
                {
                x[i] = x[i-1];
                }
            for(int i=n_new-1; i>k+1 ; i--)
                {
                tau_k[i] = tau_k[i-1];
                Est_k[i] = Est_k[i-1];
                }
            //New mesh point
            x[k+1] = 0.5*(x[k]+x[k+1]);
```

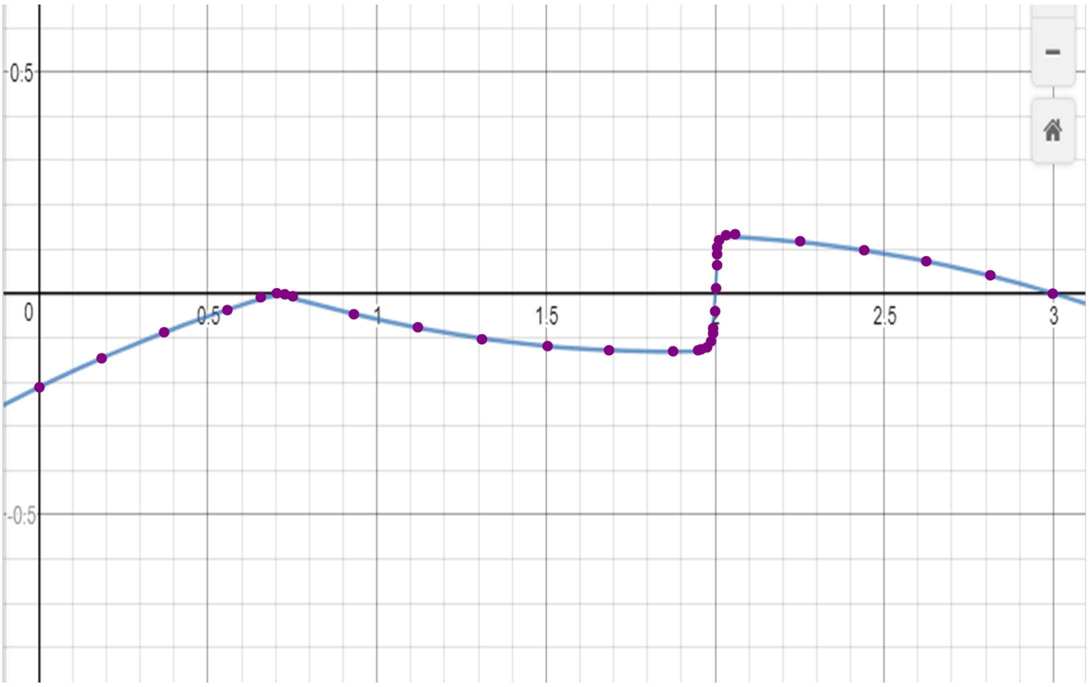
```
                std::cout<<"x["<<k+1<<"] = "<<x[k+1]<<"\n";
                k=k+1;
                }
            }

        n=n_new;
        }

delete[] x;
delete[] tau_k;
delete[] Est_k;

return Gauss_quad1;
}

int main()
{
const double a = 0.0;
const double b = 3.0;
const double tau = pow(10,-3);

std::cout<<"Gauss Quad 1 = " << adapt_int(a,b,tau)<<"\n";

return 0;
}
```

Problem 2.c

i	N	$\sum_k \mathrm{GQ}_1^{(k)}$	$\sum_k \mathrm{GQ}_3^{(k)}$	$\sum_k \mathrm{Est}_k$
1	1	-0.356944	-0.185565	0.171379
2	2	0.157268	-0.0754361	0.232704
3	4	-0.168996	-0.11855	0.06494
4	8	-0.0737691	-0.102471	0.0312698
5	16	-0.123251	-0.110649	0.0146713
6	18	-0.10046	-0.106934	0.00711414
7	20	-0.109705	-0.108087	0.00268892
8	23	-0.10757	-0.107931	0.0012512
9	26	-0.107787	-0.107937	0.000927684

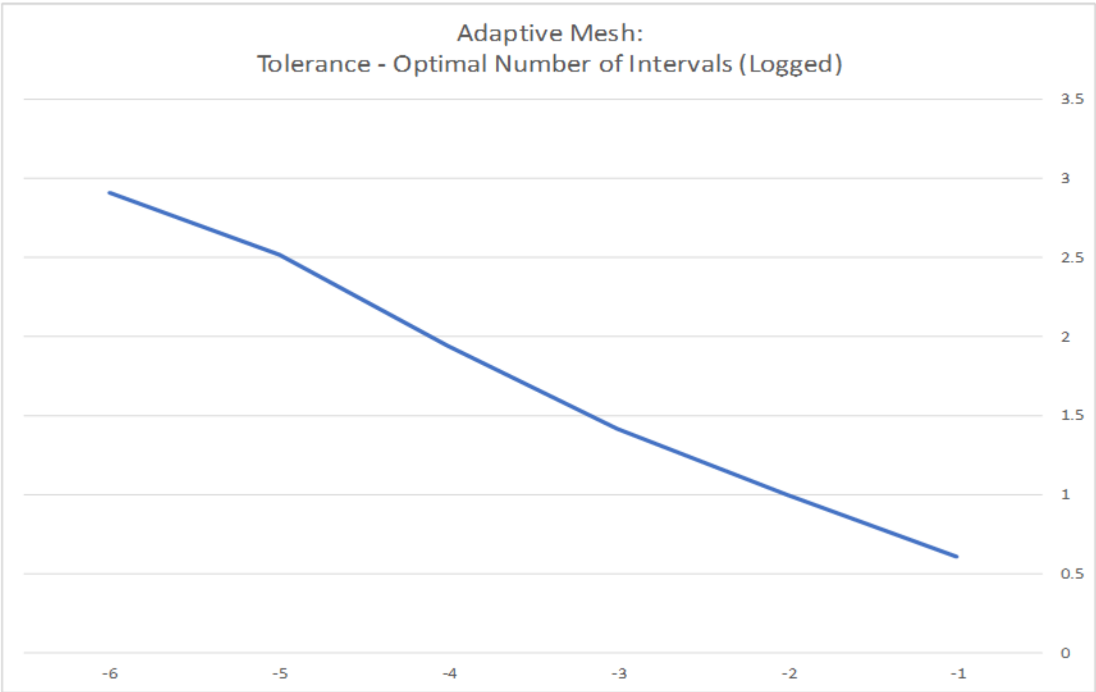
```
Gauss Quad 1 = -0.107787
```

Visualisation of optimally-found grid: ...

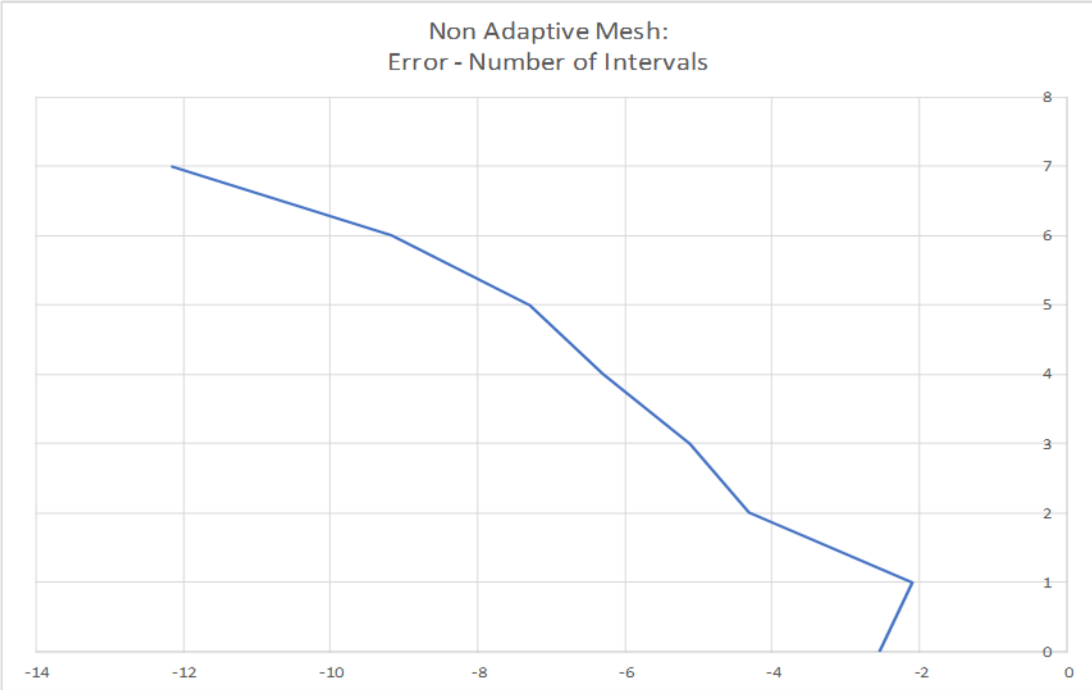


0.5
0
0.5
1
1.5
2.5
3
-0.5

Problem 2.d



Adaptive Mesh:
Tolerance - Optimal Number of Intervals (Logged)
3.5
3
2.5
2
1.5
1
0.5
0
-6
-5
-4
-3
-2
-1



Taking logs (base 10) of both the number of intervals and the tolerance of the adaptive mesh method I found that when I plotted the graph the gradient was around -2. When I took logs (base 2) of the number of intervals and the error with CGQ3 I found that the gradient was around -1/2