G14CAM 2017–2018	**Computational Applied Mathematics** Coursework 1 Part A
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Existing codes	Names of approved existing codes that you used

LaTeX instruction: This TeX-file template can be compiled using PDFLaTeX.

Problem 1

Problem 1.A

Your solution to problem 1.A could be typed in LaTeX. You can use LaTeX commands for typing symbols (e.g.: $M\alpha\tau\hbar$) and equations:

$$y = y_h + \mathcal{O}(h^p) \tag{1}$$

See the LaTeX manual (https://www.ctan.org/tex-archive/info/lshort/english/?lang=en) for more help.
Alternatively, you can include scans of hand written material:

G14CAM Computational Applied Mathematics

ELLA TAYLOR

Coursework 1 Part A

Problem 1

Use the definition of O to show that if $y = y_h + O(h^p)$, then $hy = hy_h + O(h^{p+1})$

If $y = y_h + O(h^p)$ then $|y - y_h| \leq Ch^p$ h small

and $\lim_{h \to 0} \left| \frac{y - y_h}{h^p} \right| = C$ $C < \infty$ $C > 0$

Multiplying top and bottom by h

$$\lim_{h \to 0} \left| \frac{y - y_h}{h^p} \right| = \lim_{h \to 0} \left| \frac{hy - hy_h}{h^{p+1}} \right| = C < \infty$$

From definition of O this implies

$hy = hy_h + O(h^{p+1})$

Problem 2

Problem 2.A

```
#include <iostream>

//Writing a code that implements the tridiagonal matrix algorithm for
solving an nxn matrix

void tridiagonal_matrix_solver(int n, double* x, double* lower, double* diag,
double* upper, double* f)
    {
    //Elimination stage

    //f[0] = f[0] and d[0] = d[0] no change
    for (int i = 1; i<n; i++)
        {
        diag[i] = diag[i] - ((upper[i-1]*lower[i-1])/diag[i-1]);
        f[i]    = f[i] - ((f[i-1]*lower[i-1])/diag[i-1]);
        }

    //Backsolving
    //Bottom row is a special case
    x[n-1] = f[n-1]/diag[n-1];

    for (int i = n-2; i >= 0; i--)
        {
        x[i] = (f[i] - upper[i]*x[i+1])/diag[i];
        }

    }
```

Problem 2.B

```
#include <iostream>
#include <ctime>

void tridiagonal_matrix_solver(int n, double*x, double* lower, double* diag,
double* upper, double* f);

int main()
{
int n;
std::cout<< "Enter␣an␣integer␣n␣\n";
std::cin>>n;

double* lower;
lower = new double[n-1];

double* diag;
diag = new double[n];

double* upper;
upper = new double[n-1];
```

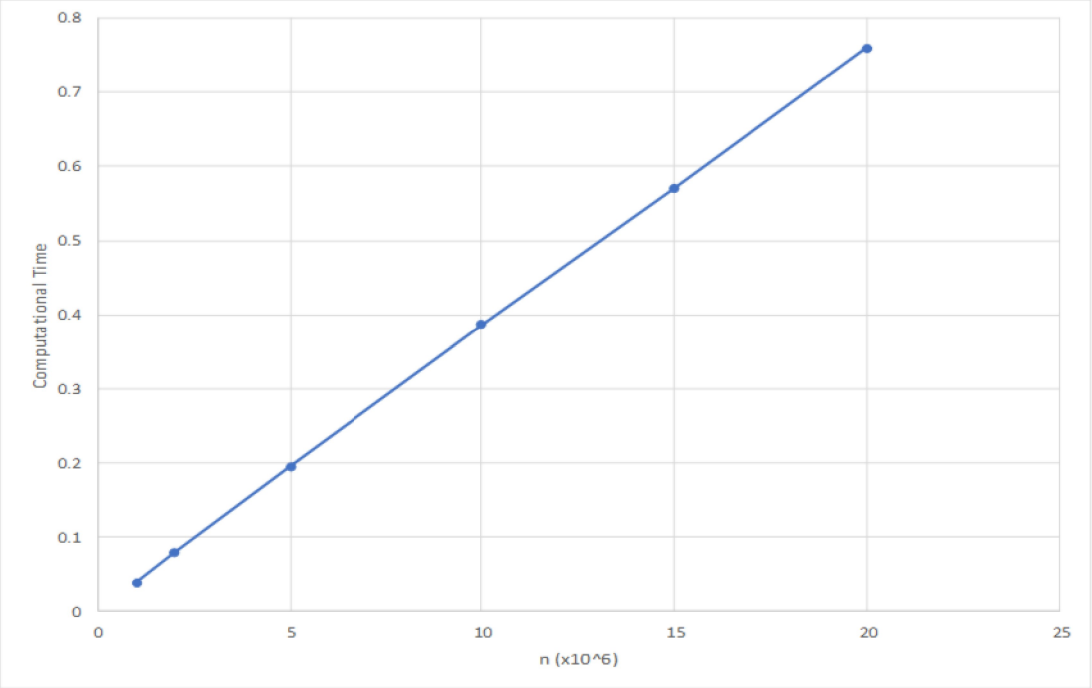
```
double* f;
f = new double[n];

double* x;
x = new double[n];

x[0] = 0;
diag[0] = 4;
upper[0] = 2;
f[0] = 1;
lower[n-2] = 2;
diag[n-1] = 4;
f[n-1] = (double)(n);
x[n-1] = 0;

for (int i=1; i<n-1; i++)
    {
    x[i] = 0;
    lower[i-1] = 1;
    diag[i] = 4;
    upper[i] = 1;
    f[i] = (double)(i)+1;
    }

clock_t start_s = clock();
tridiagonal_matrix_solver(n,x,lower,diag,upper,f);
clock_t stop_s = clock();
std::cout<<"Time = "<<(stop_s - start_s)/((double)(CLOCKS_PER_SEC))<<"\n";

std::cout<<"x[1] = "<<x[0]<<"\n";
std::cout<<"x[n] = "<<x[n-1]<<"\n";

delete[] lower;
delete[] upper;
delete[] diag;
delete[] f;
delete[] x;

return 0;
}
```

n	2	3	4	8	16	1,000	1,000,000
x_1	0	0.0833333	0.0666667	0.0704225	0.0704416	0.0704416	0.0704416
x_n	0.5	0.583333	0.766667	1.42958	2.76289	166.763	166,667

Problem 2.C

n	10^6	$2x10^6$	$5x10^6$	10^7	$1.5x10^7$	$2x10^7$
$Time$	$3.8x10^-2$	7.9x1-$^-2$	$1.95x10^-1$	$3.87x10^-1$	$5.71x10^-1$	$7.6x10^-1$

Plotting a graph of these results it is clear that the relationship between n and computational effort is linear (alpha = 1)



On Wikipedia it says that the standard Gaussian Elimination Algorithm has arithmetic complexity O(n^3) so has significantly more computations than the tridiagonal matrix algorithm as n increases