Criar um TAD para manipular matrizes de *double*. Crie um programa que utiliza todas as funções do TAD. A alocação interna da matriz deverá ser feita na forma de um vetor (array) unidimensional. Por exemplo, para armazenar uma matriz de tamanho 3x4, deve-se usar um array de tamanho 12.

Operações:

- Criar/destruir o TAD
- Escrever elemento i,j
- Acessar elemento i,j
- Preencher com números aleatórios entre um valor mínimo e máximo
- Somar duas matrizes
- Multiplicar duas matrizes
- Multiplicar uma matriz por um valor escalar
- Calcular o Traço da matriz
- Retornar um vetor com a soma das linhas
- Retornar um vetor com a soma das colunas

Observações:

- Em caso de sucesso, a operação deverá retornar 0 para indicar o sucesso. -1 em caso de erro ou algum outro código que você definir.
- Não colocar comandos de entrada/saída na TAD. Toda mensagem (de sucesso ou erro) deverá ser apresentada no programa principal

Basic Matrix Operations

A *matrix* is a rectangular or square grid of numbers arranged into rows and columns. Each number in the matrix is called an *element*, and they are arranged in what is called an *array*. The plural of "matrix" is "matrices". Matrices are often used in algebra to solve for unknown values in linear equations, and in geometry when solving for vectors and vector operations.

Example 1) Matrix M $\quad M = \begin{bmatrix} 4 & 1 & 5 \\ 3 & 6 & 2 \end{bmatrix}$

- There are 2 rows and 3 columns in matrix M. M would be called a 2 x 3 (i.e. "2 by 3") matrix.

PART A - Matrix Addition

We can add matrices together as long as their dimensions are the same, i.e. both matrices have the same number of rows and columns. To add two matrices, we add the numbers of each matrix that are in the same element position.

Example 2) $\begin{bmatrix} \mathbf{8} & 4 & 2 \\ 6 & 1 & 5 \end{bmatrix} + \begin{bmatrix} \mathbf{3} & 10 & 4 \\ 5 & 6 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{11} & 14 & 6 \\ 11 & 7 & 6 \end{bmatrix}$

- For the elements in bold: **8 + 3 = 11**

PART B - Matrix Subtraction

We can subtract matrices in a similar way to addition. Both matrices need to have the same dimensions, and we subtract the numbers of the second matrix from the first that are in the same element position.

Example 3) $\begin{bmatrix} \mathbf{8} & 4 & 2 \\ 6 & 1 & 5 \end{bmatrix} - \begin{bmatrix} \mathbf{3} & 10 & 4 \\ 5 & 6 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{5} & -6 & -2 \\ 1 & -5 & 4 \end{bmatrix}$

- For the elements in bold: **8 - 3 = 5**

PART C - Multiplying a Matrix by a Constant

We can multiply a matrix by some value by multiplying each element with that value. The value can be positive or negative.

Example 4) $2 \times \begin{bmatrix} 4 & 1 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 6 & 12 \end{bmatrix}$

Example 5) $-1 \times \begin{bmatrix} -4 & 1 \\ -3 & 5 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 3 & -5 \\ -6 & 2 \end{bmatrix}$

- We can get the negative of a matrix by using the above multiplication method:

Example 6) $-\begin{bmatrix} -4 & 1 \\ -3 & 5 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 3 & -5 \\ -6 & 2 \end{bmatrix}$

PART D - Multiplying Matrices

We can multiply a matrix (A) by another matrix (B) if the number of columns in A is equal to the number of rows in B (in bold). Multiplication of A by B is typically written as A(B) or (A)B.

Example 7) A = $\begin{bmatrix} 4 & -1 & 5 \\ 3 & 6 & -2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 6 \\ -2 \end{bmatrix}$

2 x **3** **3** x 1

- A has 2 rows and **3 columns** and B has **3 rows** and 1 column so we can multiply A by B. Notice that we can't multiply B by A in this particular case because the number of columns in B is not equal to the number of rows in A (in bold).

Example 8) B = $\begin{bmatrix} 3 \\ 6 \\ -2 \end{bmatrix}$ A = $\begin{bmatrix} 4 & -1 & 5 \\ 3 & 6 & -2 \end{bmatrix}$

3 x **1** **2** x 3

- This means that multiplying matrices is not commutative: A(B) ≠ B(A)

To multiply matrices, there's a convention that is followed.

Let matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and B = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$$A(B) = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ c & d \end{bmatrix} \begin{bmatrix} \mathbf{e} & f \\ \mathbf{g} & h \end{bmatrix} = \begin{bmatrix} \mathbf{ae + bg} & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

- Let's look at the first row of A and the first column of B. Element **a** is multiplied by element **e**. Element **b** is multiplied by element **g**. The value of the element in the first row and first column of A(B) is the sum of the products (**ae + bg**).

Example 9) A = $\begin{bmatrix} 5 & 1 & 6 \\ 0 & 8 & -2 \end{bmatrix}$ B = $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

2 x **3** **3** x 1

Basic Matrix Operations

A(B) = $\begin{bmatrix} 5(2)+1(3)+6(4) \\ 0(2)+8(3)+(-2)(4) \end{bmatrix} = \begin{bmatrix} 37 \\ 16 \end{bmatrix}$

- Notice A(B) is now a 2 x 1 matrix.

PART E - Transposing a Matrix

To transpose a matrix, we swap the rows for the columns. To indicate that we are transposing a matrix, we add a “T” to the top right-hand corner of the matrix.

Example 10) $\begin{bmatrix} 4 & 1 & 5 \\ 3 & 6 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 3 \\ 1 & 6 \\ 5 & 2 \end{bmatrix}$

Practice Questions

Given the following matrices, please solve the questions below and if you can’t solve the problem, explain why:

A = $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 2 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix}$ C = $\begin{bmatrix} 2 & 4 \\ 3 & 6 \\ -1 & 2 \end{bmatrix}$ D = $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ E = $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$

F = $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 7 & -2 \end{bmatrix}$

1) A + F
2) E – D
3) C + B
4) C(D)
5) A(F)
6) C^T
7) $F^T(E)$

Basic Matrix Operations

Solutions

1) A + F = $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \\ 5 & 7 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 8 \\ 11 & 9 & -2 \end{bmatrix}$

2) E – D = $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ -2 & 3 \end{bmatrix}$

3) C + B = $\begin{bmatrix} 2 & 4 \\ 3 & 6 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix}$

This problem cannot be solved because the matrices have different dimensions.

4) C(D) = $\begin{bmatrix} 2 & 4 \\ 3 & 6 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 22 & 8 \\ 33 & 12 \\ 1 & 0 \end{bmatrix}$

3 x **2** **2** x 2 3 x 2

5) A(F) = $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 5 & 7 & -2 \end{bmatrix}$

2 x **3** **2** x 3

This problem cannot be solved because the number of columns in A does not equal to the number of rows in F.

6) C^T = $\begin{bmatrix} 2 & 4 \\ 3 & 6 \\ -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 6 & 2 \end{bmatrix}$

7) F^T(E) = $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 7 & -2 \end{bmatrix}^T \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 7 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 16 \\ 10 & 26 \\ 7 & -14 \end{bmatrix}$

3 x **2** **2** x 2 3 x 2