Question 2: Enzyme Kinetics

8.1

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \overset{k_3}{\rightarrow} E + P$$

According to the law of mass action:

$$\vartheta_{ES} = k_1[S][E] - k_2[ES] - k_3[ES]$$

$$\vartheta_P = k_3[ES]$$

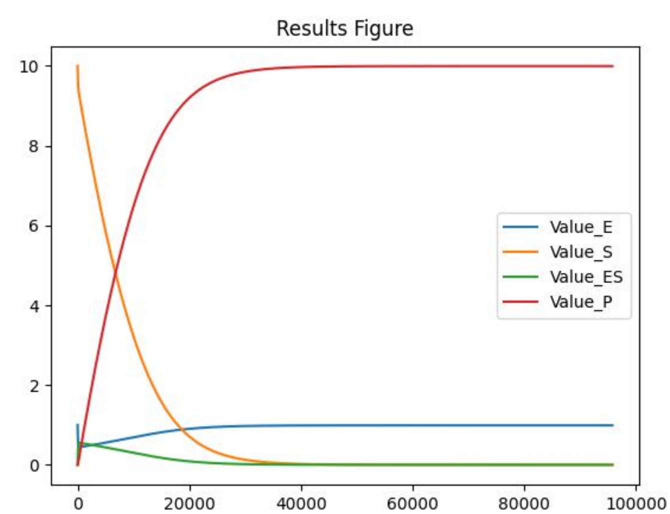
$$\vartheta_E = -k_1[S][E] + k_2[ES] + k_3[ES]$$

$$\vartheta_S = -k_1[S][E] + k_2[ES]$$

8.2

I wrote a Python code to numerically solve the four equations above.

The results are as follows:



8.3

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_3} E + P$$

$$\underset{[Et]-[ES]}{\uparrow} \quad \underset{[S]}{\uparrow} \qquad \underset{[ES]}{\uparrow}$$

let Et represents the total amount of enzymes in the system

the generation rate of ES : $v_1 = k_1[S][E]$

the decomposition rate of ES : $v_2 = k_2[ES] + k_3[ES]$

when the reaction is in a constant state : $v_1 = v_2$

$$k_1[S][E] = k_2[ES] + k_3[ES]$$

$$k_1[S]\left([Et]-[ES]\right) = k_2[ES] + k_3[ES]$$

$$\therefore \frac{[Et][S]-[ES][S]}{[ES]} = \frac{k_2+k_3}{k_1}$$

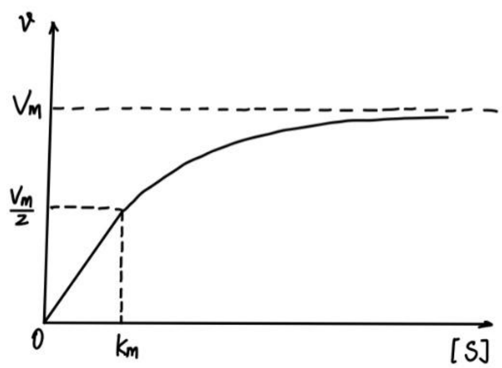
let k_m represent $\frac{k_2+k_3}{k_1}$: $k_m = \frac{k_2+k_3}{k_1}$

$\therefore \ k_m[ES] + [ES][S] = [Et][S]$

$\therefore \ [ES] = \frac{[Et][S]}{k_m+[S]}$

the rate of the enzymatic reaction is determined by $[ES]$: $v = k_3[ES] = \frac{k_3[Et][S]}{k_m+[S]}$

when $[S] \gg [Et]$, $[Et] = [ES]$, v reaches its maximum value V_m



$$V_m = k_3[ES] = k_3[Et]$$

$$v = \frac{V_m[S]}{k_m+[S]}$$

when $v = \frac{V_m}{2}$, $\frac{V_m}{2} = \frac{V_m[S]}{K_m+[S]}$

$\therefore K_m = [S]$