SHOEBOX!

Ch 7: 5, 6, 11

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7.4 λ at which emergency calls occur

a) suppose that 2,050 emergency calls are received in a one-year period

911 service
171 calls/month
rate of house fire emergencies was estimated at 171 per month
next month → 153 calls received

Variables λ = Rate of house fire reports (per month)
X_n = time b/w $(n-1)$st and nth fire (months)
Assumptions: House fires occur at random with rate λ; i.e., $X_1, X_2, \ldots$ are independent, and each X_n has an exponential distribution with rate parameter λ

$\frac{2050}{12} \approx 170.833 = \lambda$

b) true value of λ is 171 calls per month
Range of normal variation: $\bar{x} \pm Z_{\alpha/2} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$

$171 * 12 = 2052$

$171 - 170.833 = 0.167$

$0.0009766082 * 2052 = 2.004$

$2052 - 2.004 = 2049.996$

$2052 + 2.004 = 2054.004$

$(2049.996, 2054.004)$

c) $\frac{2049.996}{12} = 170.833$

$(170.833, 171.0003)$

$\frac{2054.004}{12} = 171.0003$

The estimate of the true rate of house fires occuring (λ) is very accurate b/c 171 is within the interval.

d) Less than a year, because 170.833 is already within ± 0.5 from 171.

6. Poisson Distribution

$$Pr(N_t = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

for all $n = 0, 1, 2, \ldots$

a) Show that

$$EN_t = \lambda t$$

and

$$VN_t = \lambda t$$

Mean of distribution $= \lambda t$

$$E(x) = \Sigma\, x \cdot p(x)$$

$$= \Sigma\, n \cdot \frac{e^{-\lambda t}\boxed{\lambda t}^n}{n!} \quad \text{for all } n = 0, 1, 2$$

$$EN_t = \lambda t$$

11.

AD Type	P_{detect}	$P_{acquire}$	P_{hit}
Low	0.90	0.80	0.05
High	0.75	0.95	0.70

The guns can fire 20 shells per minute
Missile installation = 3 per minute

a) Go low, because you have a lower probability of getting hit than if you go high.
$0.498 \times 15 = 7.48 > 1 \leftarrow$ High
$0.036 * 20 = 0.72 \leftarrow$ low

b) $16 - 0.72 = 15.28$
$15.28 * .7 = 10.696$ — 1069% chance of destroying target

c) $\# - 0.72 = \_\_\_$
$\_\_\_ * .7 = .95$
$\_\_\_ = .95/.7$
$\# - 0.72 = .95/.7$
$\# = .95/.7 + 0.72$
$\# = 2.077$

At least 3 bombers to guarantee a 95% chance of mission success.

d)
$.95/.7 + 0.72 = 2.077$
$.95/.63 + 0.72 = 2.2279$
$.95/.77 + 0.72 = 1.96376$

e) P_{detect} is a cubed root of the overall probability, so if this probability goes down, the overall probability of hitting the target only goes down by $\sqrt[3]{\#}$.
For p, probability that a bomber can destroy a target,

it goes down the whole probability.

Documentation of Collaboration
William Kvasnak; he helped me with problems 11 & 6. We worked on them together and found the answers to the problems.