Project_7

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1 Project 7_Kaiyue Wu

[5]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
from scipy.stats import norm
import scipy.stats

new_line = '\n'
pd.options.mode.chained_assignment = None
import warnings
warnings.filterwarnings('ignore')

sns.set(font_scale=1.5, rc={'text.usetex' : True,})
%config InlineBackend.figure_format='retina'
```

1.1 1

[2]:
```
r = 0.04
dt = 0.002
K =10
sigma = 0.2
T = 0.5
S0 = np.arange(4,17,1)
dx1 = sigma*np.sqrt(dt)
dx2 = sigma*np.sqrt(3*dt)
dx3 = sigma*np.sqrt(4*dt)
dX = sigma*np.array([np.sqrt(dt),np.sqrt(3*dt),np.sqrt(4*dt)])
```

[3]:
```
#Black Scholes prices
def BS(S0,K,sigma,r,T,option_type):
    d1 = (np.log(S0/K)+(r+0.5*sigma**2)*T)/(sigma * np.sqrt(T))
    d2 = d1-np.sqrt(T)*sigma

    if option_type.lower() == 'call':
```

```python
        price  = (S0*scipy.stats.norm.cdf(d1,0.0,1.0)-K*np.exp(-r*T)*scipy.
 ↪stats.norm.cdf(d2,0.0,1.0))
    else:
        price  = (K*np.exp(-r*T)*scipy.stats.norm.cdf(-d2, 0.0, 1.0)-S0*scipy.
 ↪stats.norm.cdf(-d1,0.0,1.0))

    return price
#BS(S0=S0[0],K=K,sigma=sigma,r=r,T=T,option_type='put')
```

[4]:
```python
# Pu = dt*(sigma**2/(2*dx1**2)+(r-sigma**2/2)/(2*dx1))
# Pm = 1 - dt*sigma**2/(dx1**2) - r*dt
# Pd = dt*(sigma**2/(2*dx1**2)-(r-sigma**2/2)/(2*dx1))
# steps = int(T/dt)
# gridPrice = np.arange(np.log(16)+dx1,np.log(4)-dx1,-dx1)
# index = np.where(np.exp(gridPrice)<S0[0])[0][0]
# N = len(gridPrice)
# A = np.zeros((N,N))
# pmtx = [Pu,Pm,Pd]
# A[0,[0,1,2]] = pmtx
# A[1,[0,1,2]] = pmtx
# for i in range(2,N-2):
#     A[i,[i-1,i,i+1]] = pmtx
# A[N-2,[N-3,N-2,N-1]] = pmtx
# A[N-1,[N-3,N-2,N-1]] = pmtx
# F = np.maximum(K - np.exp(gridPrice),0)
# B = np.zeros_like(F)
# B[-1] = np.exp(gridPrice[-2])-np.exp(gridPrice[-1])
# for i in range(steps):
#     F = np.dot(A,F)+B
# option = F[index]
# option
```

[5]:
```python
def EFD(K,sigma,T,dt,dX,s0):
    """
    Explicit Finite-Difference method
    """
    steps = int(T/dt)
    Pu = dt*(sigma**2/(2*dX**2)+(r-sigma**2/2)/(2*dX))
    Pm = 1 - dt*sigma**2/(dX**2) - r*dt
    Pd = dt*(sigma**2/(2*dX**2)-(r-sigma**2/2)/(2*dX))
    gridPrice = np.arange(np.log(16)+dX,np.log(4)-dX,-dX)
    index = np.where(np.exp(gridPrice)<s0)[0][0]
    N = len(gridPrice)
    A = np.zeros((N,N))
    pmtx = [Pu,Pm,Pd]
    A[0,[0,1,2]] = pmtx
    A[1,[0,1,2]] = pmtx
```

```python
    for i in range(2,N-2):
        A[i,[i-1,i,i+1]] = pmtx
    A[N-2,[N-3,N-2,N-1]] = pmtx
    A[N-1,[N-3,N-2,N-1]] = pmtx
    F = np.maximum(K - np.exp(gridPrice),0)
    B = np.zeros_like(F)
    B[-1] = np.exp(gridPrice[-2])-np.exp(gridPrice[-1])
    for i in range(steps):
        F = np.dot(A,F)+B
    option = F[index]

    return option

# EFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx1,s0=S0[0])
```

[35]:

```python
def IFD(K,sigma,T,dt,dX,s0):
    """
    Implicit Finite-Difference method
    """
    steps = int(T/dt)
    Pu = -0.5*dt*(sigma**2/(dX**2)+(r-sigma**2/2)/(dX))
    Pm = 1+dt*sigma**2/(dX**2) + r*dt
    Pd = -0.5*dt*(sigma**2/(dX**2)-(r-sigma**2/2)/(dX))
    gridPrice = np.arange(np.log(20)+dX,np.log(1)-dX,-dX)
    index = np.where(np.exp(gridPrice)<s0)[0][0]
    N = len(gridPrice)
    A = np.zeros((N,N))
    pmtx = [Pu,Pm,Pd]
    A[0,[0,1]] = [1,-1]
    for i in range(1,N-1):
        A[i,[i-1,i,i+1]] = pmtx
    A[N-1,[N-2,N-1]] = [1,-1]
    F = np.maximum(K - np.exp(gridPrice),0)
    B = np.zeros_like(F)
    B[1:-1] = F[1:-1]
    B[-1] = np.exp(gridPrice[-2])-np.exp(gridPrice[-1])
#     B[0] = 0
    for i in range(steps):
        F = np.dot(np.linalg.inv(A),B)
        B = np.zeros(N)
        B[1:-1]= F[1:-1]
        B[-1] = np.exp(gridPrice[-2])-np.exp(gridPrice[-1])

    option = F[index]
    return option

# IFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx1,s0=S0[0])
```

[36]:

```python
def CNFD(K,sigma,T,dt,dX,s0):
    """
    Crank-Nicolson Finite-Difference method
    """
    steps = int(T/dt)
    Pu = -1/4*dt*(sigma**2/(dX**2)+(r-sigma**2/2)/(dX))
    Pm = 1+dt*sigma**2/(2*dX**2) + r*dt/2
    Pd = -1/4*dt*(sigma**2/(dX**2)-(r-sigma**2/2)/(dX))

    gridPrice = np.arange(np.log(20)+dX,np.log(1)-dX,-dX)
    index = np.where(np.exp(gridPrice)<s0)[0][0]

    F = np.maximum(K - np.exp(gridPrice),0)

    N = len(gridPrice)
    A = np.zeros((N,N))
    pmtx = [Pu,Pm,Pd]
    A[0,[0,1]] = [1,-1]
    for i in range(1,N-1):
        A[i,[i-1,i,i+1]] = pmtx
    A[N-1,[N-2,N-1]] = [1,-1]

    zpmtx = [-Pu,-(Pm-2),-Pd]
    X = np.zeros((N,N))
    X[0] = 0
    for i in range(1,N-1):
        X[i,[i-1,i,i+1]] = zpmtx

    B = np.dot(np.linalg.inv(A),X)
    B[-1] = np.exp(gridPrice[-2])-np.exp(gridPrice[-1])
    B[0]=0

    for i in range(steps):
        F = np.dot(B,F)

    option = F[index]
    return option

# CNFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx1,s0=S0[0])
```

[37]:

```python
BS_q1 = np.array([BS(S0=i,K=K,sigma=sigma,r=r,T=T,option_type='put') for i in␣
 ↪S0])
```

1.1.1 (a) Explicit Finite-Difference method

[38]:
```
dx1_price_a = np.array([EFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx1,s0=i) for i in S0])
dx2_price_a = np.array([EFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx2,s0=i) for i in S0])
dx3_price_a = np.array([EFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx3,s0=i) for i in S0])

dx1_price_a_error = BS_q1 -dx1_price_a
dx2_price_a_error = BS_q1 -dx2_price_a
dx3_price_a_error = BS_q1 -dx3_price_a

print("Explicit Finite-Difference method")
pd.DataFrame(list(zip(BS_q1,
                      dx1_price_a,dx2_price_a,dx3_price_a,
                      dx1_price_a_error,dx2_price_a_error,dx3_price_a_error)),
            index=S0,columns=['BS price','dx1','dx2','dx3','error dx1','error␣
 ↪dx2','error dx3'])
```

```
Explicit Finite-Difference method
```

[38]:

	BS price	dx1	dx2	dx3	error dx1	error dx2	error dx3
4	5.801987	5.802249	5.833768	5.837862	-0.000262	-0.031781	-0.035875
5	4.801987	4.844540	4.872652	4.888680	-0.042553	-0.070665	-0.086693
6	3.802058	3.820255	3.865616	3.820258	-0.018197	-0.063558	-0.018200
7	2.805357	2.840817	2.873559	2.902368	-0.035460	-0.068201	-0.097010
8	1.844269	1.877162	1.873509	1.877333	-0.032893	-0.029241	-0.033064
9	1.024428	1.062792	1.110251	1.120862	-0.038364	-0.085823	-0.096434
10	0.464695	0.481928	0.508746	0.520756	-0.017234	-0.044051	-0.056061
11	0.171537	0.173751	0.199263	0.173599	-0.002214	-0.027726	-0.002062
12	0.052460	0.058746	0.058100	0.067100	-0.006286	-0.005641	-0.014640
13	0.013651	0.015487	0.016193	0.015572	-0.001836	-0.002541	-0.001920
14	0.003107	0.003119	0.003537	0.003840	-0.000012	-0.000429	-0.000732
15	0.000635	0.000749	0.000880	0.000773	-0.000114	-0.000245	-0.000139
16	0.000119	0.000206	0.000200	0.000199	-0.000087	-0.000082	-0.000081

1.1.2 (b) Implicit Finite-Difference method

[39]:
```
dx1_price_b = np.array([IFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx1,s0=i) for i in S0])
dx2_price_b = np.array([IFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx2,s0=i) for i in S0])
dx3_price_b = np.array([IFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx3,s0=i) for i in S0])

dx1_price_b_error = BS_q1 -dx1_price_b
dx2_price_b_error = BS_q1 -dx2_price_b
dx3_price_b_error = BS_q1 -dx3_price_b

print("Implicit Finite-Difference method")
pd.DataFrame(list(zip(BS_q1,
                      dx1_price_b,dx2_price_b,dx3_price_b,
                      dx1_price_b_error,dx2_price_b_error,dx3_price_b_error)),
```

```
                index=S0,columns=['BS price','dx1','dx2','dx3','error dx1','error␣
 ↪dx2','error dx3'])
```

Implicit Finite-Difference method

[39]:

	BS price	dx1	dx2	dx3	error dx1	error dx2	error dx3
4	5.801987	5.804117	5.808879	5.804115	-0.002131	-0.006892	-0.002128
5	4.801987	4.802333	4.841731	4.846849	-0.000346	-0.039744	-0.044862
6	3.802058	3.823053	3.828397	3.876278	-0.020995	-0.026339	-0.074221
7	2.805357	2.844188	2.830626	2.844260	-0.038831	-0.025269	-0.038903
8	1.844269	1.880884	1.941440	1.946540	-0.036616	-0.097172	-0.102272
9	1.024428	1.065758	1.069562	1.065725	-0.041330	-0.045133	-0.041297
10	0.464695	0.483404	0.481017	0.483142	-0.018709	-0.016322	-0.018448
11	0.171537	0.174368	0.184771	0.193675	-0.002831	-0.013234	-0.022138
12	0.052460	0.059205	0.052892	0.059242	-0.006745	-0.000432	-0.006782
13	0.013651	0.015779	0.014605	0.018595	-0.002127	-0.000954	-0.004944
14	0.003107	0.003260	0.004415	0.003317	-0.000152	-0.001307	-0.000210
15	0.000635	0.000798	0.000800	0.001016	-0.000163	-0.000166	-0.000381
16	0.000119	0.000131	0.000172	0.000176	-0.000013	-0.000053	-0.000058

1.1.3 (c) Crank-Nicolson Finite-Difference method

[11]:
```
dx1_price_c = np.array([CNFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx1,s0=i) for i in␣
 ↪S0])
dx2_price_c = np.array([CNFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx2,s0=i) for i in␣
 ↪S0])
dx3_price_c = np.array([CNFD(K=K,sigma=sigma,T=T,dt=dt,dX=dx3,s0=i) for i in␣
 ↪S0])

dx1_price_c_error = BS_q1 -dx1_price_c
dx2_price_c_error = BS_q1 -dx2_price_c
dx3_price_c_error = BS_q1 -dx3_price_c

print("Crank-Nicolson Finite-Difference method")
pd.DataFrame(list(zip(BS_q1,
                      dx1_price_c,dx2_price_c,dx3_price_c,
                      dx1_price_c_error,dx2_price_c_error,dx3_price_c_error)),
                index=S0,columns=['BS price','dx1','dx2','dx3','error dx1','error␣
 ↪dx2','error dx3'])
```

Crank-Nicolson Finite-Difference method

[11]:

	BS price	dx1	dx2	dx3	error dx1	error dx2	error dx3
4	5.801987	5.804109	5.808871	5.804107	-0.002123	-0.006884	-0.002120
5	4.801987	4.802325	4.841723	4.846841	-0.000338	-0.039736	-0.044854
6	3.802058	3.823039	3.828382	3.876265	-0.020981	-0.026324	-0.074207
7	2.805357	2.844103	2.830539	2.844175	-0.038745	-0.025182	-0.038818
8	1.844269	1.880706	1.941262	1.946362	-0.036437	-0.096993	-0.102093

9	1.024428	1.065789	1.069590	1.065755	-0.041361	-0.045162	-0.041327
10	0.464695	0.483681	0.481294	0.483420	-0.018986	-0.016600	-0.018725
11	0.171537	0.174528	0.184943	0.193856	-0.002992	-0.013406	-0.022319
12	0.052460	0.059186	0.052861	0.059223	-0.006727	-0.000402	-0.006763
13	0.013651	0.015697	0.014523	0.018514	-0.002046	-0.000872	-0.004863
14	0.003107	0.003205	0.004353	0.003263	-0.000097	-0.001246	-0.000155
15	0.000635	0.000772	0.000774	0.000986	-0.000137	-0.000140	-0.000352
16	0.000119	0.000123	0.000163	0.000167	-0.000005	-0.000044	-0.000048

1.2 2

[7]:
```
s0 = np.arange(4, 17, 1)
dt =  0.002
sigma = 0.2
r = 0.04
K = 10
T = 0.5
ds1 = 0.5
ds2 = 1
```

[10]:
```
def BS_PDE(K,sigma,r,T,dt,dS,s0,alpha,option_type):
    """
    alpha = 1: explicit
    alpha != 1: implicit
    alpha = 0: fully implicit
    alpha = 0.5: crank-nicolson
    """

    gridPrice = np.arange(0,20+dS,dS)
    index = np.abs(gridPrice-s0).argmin()
    N = len(gridPrice)
    steps = int(T/dt)
    j = np.arange(1,N-1,1)
    a1 = 0.5*((sigma**2)*(j**2)-r*j)*(1-alpha)
    a2 = -1/dt-((sigma**2)*(j**2)+r)*(1-alpha)
    a3 = 0.5*((sigma**2)*(j**2)+r*j)*(1-alpha)
    b1 = 0.5*((sigma**2)*(j**2)-r*j)*alpha
    b2 = 1/dt-((sigma**2)*(j**2)+r)*alpha
    b3 = 0.5*((sigma**2)*(j**2)+r*j)*alpha

    A = np.zeros((N,N))
    A[0,[0,1]]=[1,-1]
    for i in range(1,N-1):
        A[i,[i-1,i,i+1]] = [a1[i-1],a2[i-1],a3[i-1]]
    A[N-1,[N-2,N-1]]=[1,-1]

    B = np.zeros((N,N))
```

```python
    for i in range(1,N-1):
        B[i,[i-1,i,i+1]] = [-b1[i-1],-b2[i-1],-b3[i-1]]

    eC = np.zeros(N)
    eC[0] = gridPrice[0]-gridPrice[1]
    eP = np.zeros(N)
    eP[0] = gridPrice[-2]-gridPrice[-1]

    if option_type.lower() =='call':
        C = np.maximum(np.round(gridPrice - K,4),0)
        for i in range(steps):
            C = np.dot(np.linalg.inv(A),np.dot(B,C)+eC)
            C = np.maximum(C,np.maximum(np.round(gridPrice - K,4),0))
        return C[index]
    else:
        P = np.maximum(np.round(K - gridPrice,4),0)
        for i in range(steps):
            P = np.dot(np.linalg.inv(A),np.dot(B,P)+eP)
            P = np.maximum(P,np.maximum(np.round(K - gridPrice,4),0))
        return P[index]

#BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=1,s0=s0[0],alpha=1,option_type='call')
```

1.2.1 (a) Explicit Finite-Difference method

[18]:
```python
Ca_ds1 =␣
 ↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds1,s0=i,alpha=1,option_type='call')␣
 ↪for i in S0]
Ca_ds2 =␣
 ↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds2,s0=i,alpha=1,option_type='call')␣
 ↪for i in S0]
Pa_ds1 =␣
 ↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds1,s0=i,alpha=1,option_type='put')␣
 ↪for i in S0]
Pa_ds2 =␣
 ↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds2,s0=i,alpha=1,option_type='put')␣
 ↪for i in S0]
print("Explicit Finite-Difference method")
pd.DataFrame(list(zip(Ca_ds1,Ca_ds2,Pa_ds1,Pa_ds2)),
             index=S0,columns=['Call when ds=0.5','Call when ds=1','Put when␣
 ↪ds=0.5','Put when ds=1'])
```

```
Explicit Finite-Difference method
```

[18]:

	Call when ds=0.5	Call when ds=1	Put when ds=0.5	Put when ds=1
4	1.341755e-08	7.921639e-07	6.000000	6.000000
5	2.626757e-06	2.645102e-05	5.000000	5.000000

6	1.680357e-04	5.069207e-04	4.000000	4.000000
7	4.055082e-03	5.889406e-03	3.000000	3.000000
8	4.206117e-02	4.291649e-02	2.000000	2.000000
9	2.166802e-01	2.010445e-01	1.075747	1.051627
10	6.540399e-01	6.230475e-01	0.473328	0.439979
11	1.364160e+00	1.348123e+00	0.170676	0.154452
12	2.248808e+00	2.244958e+00	0.051867	0.048152
13	3.211448e+00	3.211427e+00	0.013756	0.013876
14	4.200695e+00	4.200989e+00	0.003287	0.003796
15	5.196284e+00	5.196161e+00	0.000726	0.001004
16	6.189832e+00	6.189327e+00	0.000151	0.000260

1.2.2 (b) Implicit Finite-Difference method

[19]:
```
Cb_ds1 =␣
↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds1,s0=i,alpha=0,option_type='call')␣
↪for i in S0]
Cb_ds2 =␣
↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds2,s0=i,alpha=0,option_type='call')␣
↪for i in S0]
Pb_ds1 =␣
↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds1,s0=i,alpha=0,option_type='put')␣
↪for i in S0]
Pb_ds2 =␣
↪[BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds2,s0=i,alpha=0,option_type='put')␣
↪for i in S0]

print("Implicit Finite-Difference method")
pd.DataFrame(list(zip(Cb_ds1,Cb_ds2,Pb_ds1,Pb_ds2)),
             index=S0,columns=['Call when ds=0.5','Call when ds=1','Put when␣
↪ds=0.5','Put when ds=1'])
```

Implicit Finite-Difference method

[19]:

	Call when ds=0.5	Call when ds=1	Put when ds=0.5	Put when ds=1
4	1.901301e-08	8.984022e-07	6.000000	6.000000
5	3.215518e-06	2.871215e-05	5.000000	5.000000
6	1.855986e-04	5.316790e-04	4.000000	4.000000
7	4.209029e-03	6.025739e-03	3.000000	3.000000
8	4.240316e-02	4.323244e-02	2.000000	2.000000
9	2.165805e-01	2.010542e-01	1.075205	1.051252
10	6.534354e-01	6.222878e-01	0.472453	0.439081
11	1.363833e+00	1.347857e+00	0.170268	0.154173
12	2.248829e+00	2.244942e+00	0.051919	0.048255
13	3.211345e+00	3.211035e+00	0.013913	0.014022
14	4.199690e+00	4.199106e+00	0.003390	0.003886
15	5.192673e+00	5.190197e+00	0.000772	0.001046

```
16       6.180231e+00     6.174095e+00          0.000168        0.000277
```

1.2.3 Crank-Nicolson Finite-Difference method

[20]:
```
Cc_ds1 = [BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds1,s0=i,alpha=0.
 ↪5,option_type='call') for i in S0]
Cc_ds2 = [BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds2,s0=i,alpha=0.
 ↪5,option_type='call') for i in S0]
Pc_ds1 = [BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds1,s0=i,alpha=0.
 ↪5,option_type='put') for i in S0]
Pc_ds2 = [BS_PDE(K=K,sigma=sigma,r=r,T=T,dt=dt,dS=ds2,s0=i,alpha=0.
 ↪5,option_type='put') for i in S0]

print("Crank-Nicolson Finite-Difference method")
pd.DataFrame(list(zip(Cc_ds1,Cc_ds2,Pc_ds1,Pc_ds2)),
            index=S0,columns=['Call when ds=0.5','Call when ds=1','Put when␣
 ↪ds=0.5','Put when ds=1'])
```

```
Crank-Nicolson Finite-Difference method
```

[20]:

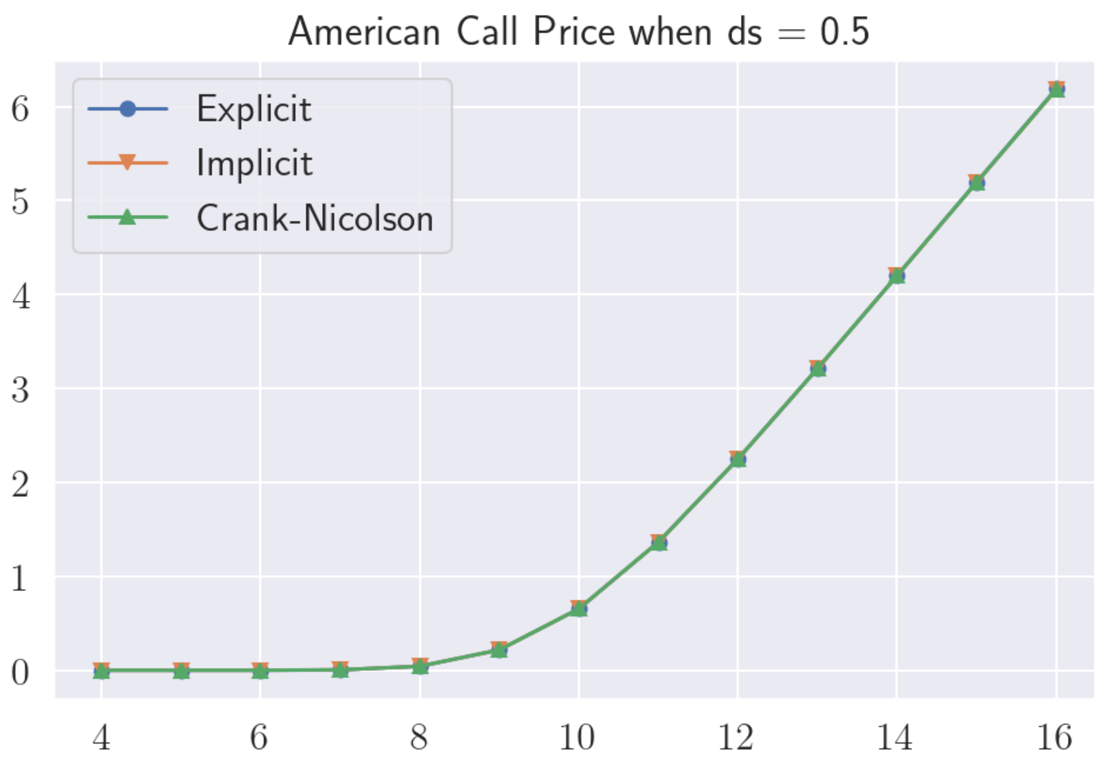
	Call when ds=0.5	Call when ds=1	Put when ds=0.5	Put when ds=1
4	1.604763e-08	8.443094e-07	6.000000	6.000000
5	2.912898e-06	2.757096e-05	5.000000	5.000000
6	1.767506e-04	5.192633e-04	4.000000	4.000000
7	4.132178e-03	5.957630e-03	3.000000	3.000000
8	4.223239e-02	4.307473e-02	2.000000	2.000000
9	2.166300e-01	2.010489e-01	1.075475	1.051439
10	6.537380e-01	6.226676e-01	0.472889	0.439530
11	1.363996e+00	1.347986e+00	0.170470	0.154312
12	2.248818e+00	2.244927e+00	0.051893	0.048203
13	3.211388e+00	3.211110e+00	0.013835	0.013949
14	4.200129e+00	4.199528e+00	0.003339	0.003841
15	5.194172e+00	5.191400e+00	0.000749	0.001025
16	6.183959e+00	6.176737e+00	0.000160	0.000268

[29]:
```
plt.figure(figsize=(8,5))

plt.plot(S0,Ca_ds1,marker='o')
plt.plot(S0,Cb_ds1,marker='v')
plt.plot(S0,Cc_ds1,marker='^')

plt.title('American Call Price when ds = 0.5')
plt.legend(['Explicit','Implicit','Crank-Nicolson'])

plt.show()
```

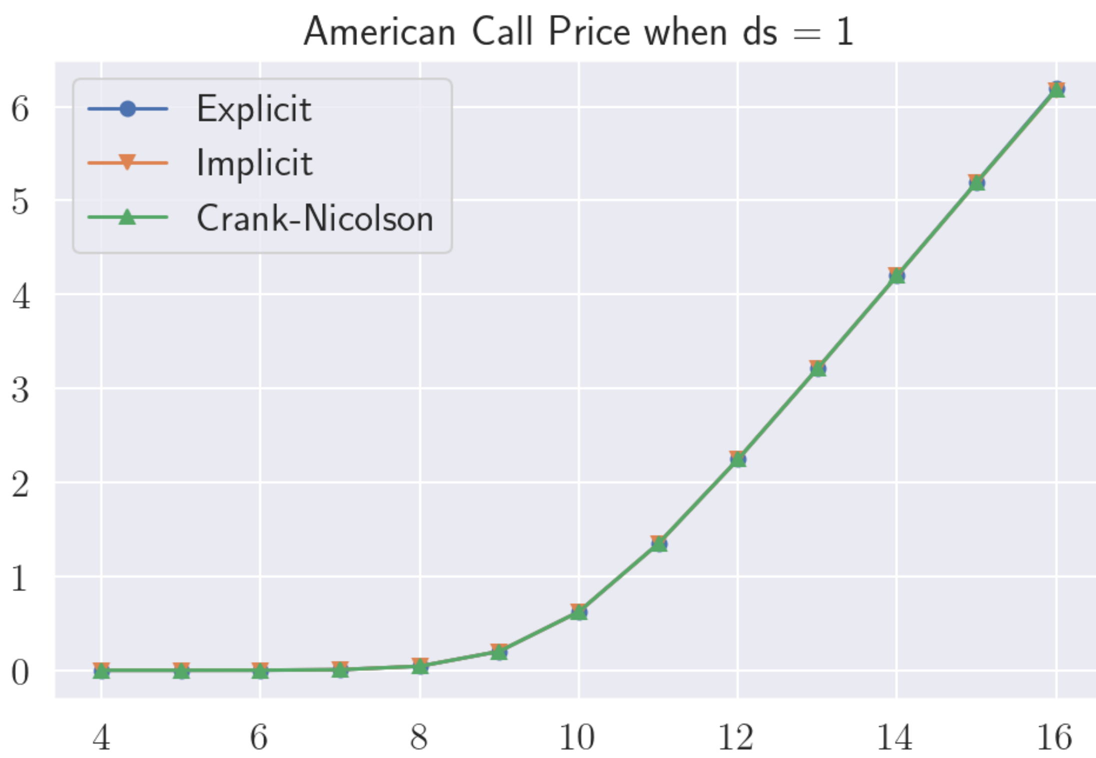


[32]:
```
plt.figure(figsize=(8,5))

plt.plot(S0,Ca_ds2,marker='o')
plt.plot(S0,Cb_ds2,marker='v')
plt.plot(S0,Cc_ds2,marker='^')

plt.title('American Call Price when ds = 1')
plt.legend(['Explicit','Implicit','Crank-Nicolson'])

plt.show()
```

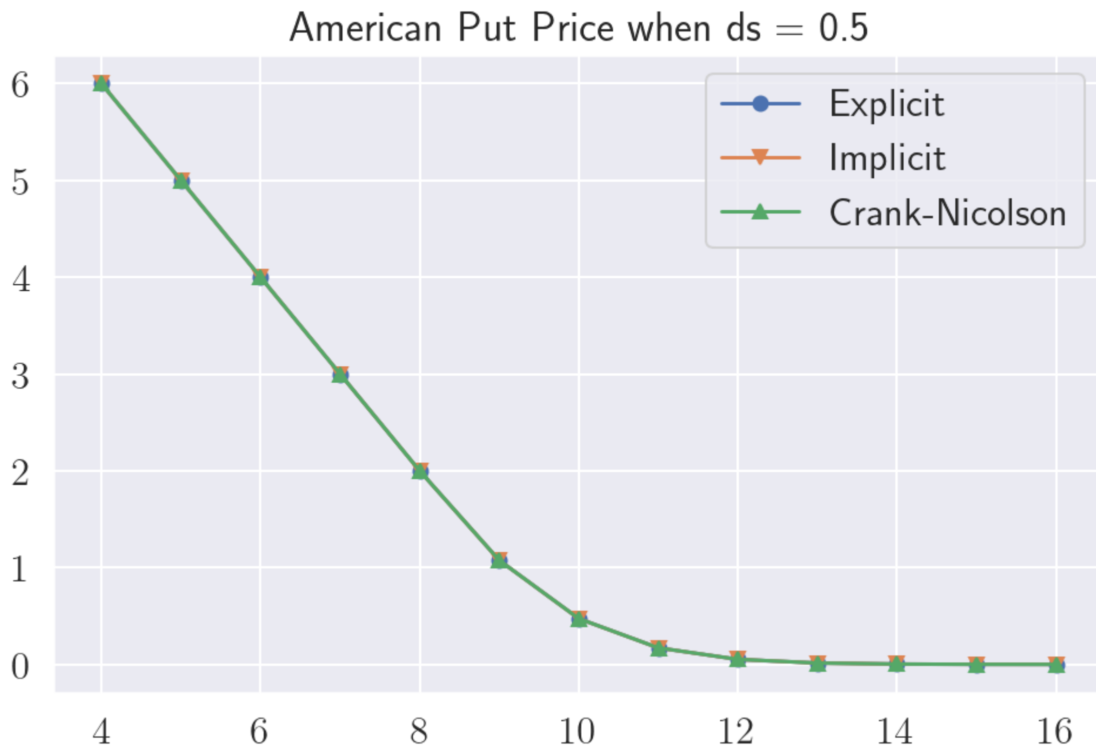


[33]:
```
plt.figure(figsize=(8,5))

plt.plot(S0,Pa_ds1,marker='o')
plt.plot(S0,Pb_ds1,marker='v')
plt.plot(S0,Pc_ds1,marker='^')

plt.title('American Put Price when ds = 0.5')
plt.legend(['Explicit','Implicit','Crank-Nicolson'])

plt.show()
```

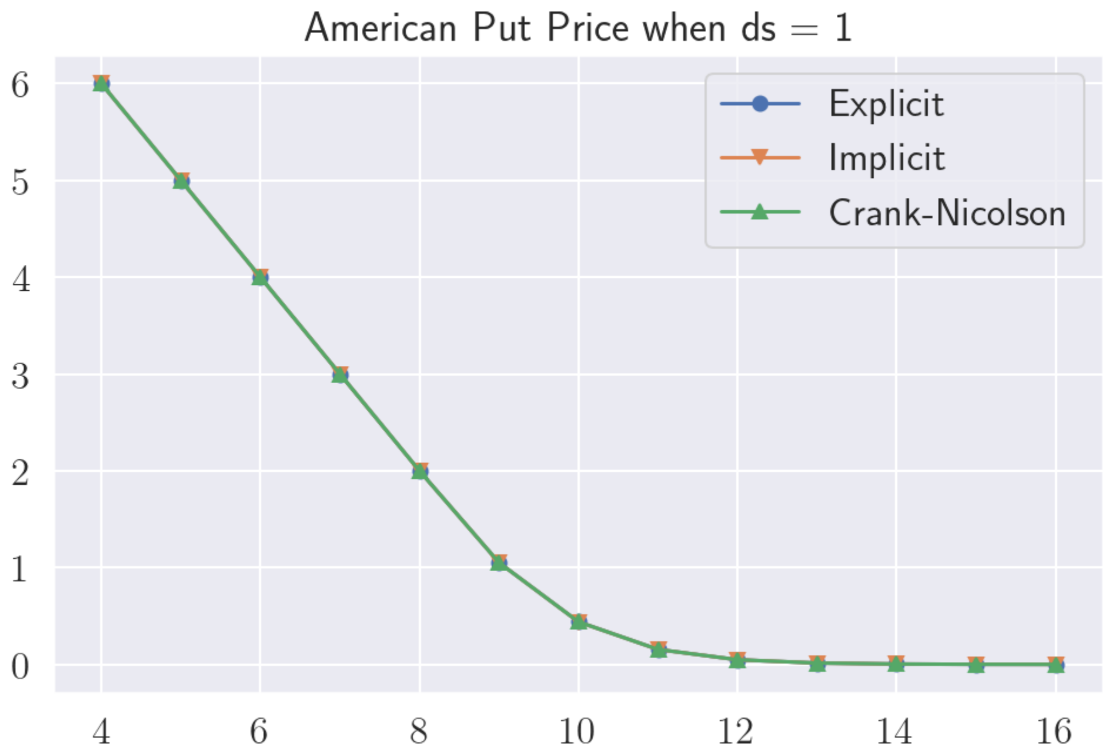


[34]:
```
plt.figure(figsize=(8,5))

plt.plot(S0,Pa_ds2,marker='o')
plt.plot(S0,Pb_ds2,marker='v')
plt.plot(S0,Pc_ds2,marker='^')

plt.title('American Put Price when ds = 1')
plt.legend(['Explicit','Implicit','Crank-Nicolson'])

plt.show()
```


American Put Price when ds = 1
Explicit
Implicit
Crank-Nicolson
6
5
4
3
2
1
0
4
6
8
10
12
14
16

[ ]: