Project 2

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code quality, speed, and accuracy will determine the grades.

Submit your codes and a PDF file of your answers to questions (including graphs, histograms, but no codes, in this PDF file) by 3PM PDT on Next Wednesday.

1. Generate a series $(X_i, Y_i)$for $i = 1, \ldots, n$ of Bivariate-Normally distributed random vectors, with the mean vector of (0,0) and the variance – covariance matrix of $\begin{pmatrix} 3 & a \\ a & 5 \end{pmatrix}$. Compute the following by simulation:

$$\rho(a) = \frac{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \acute{X})(Y_i - \acute{Y})}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \acute{X})^2}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \acute{Y})^2}}$$

where $\acute{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, $\acute{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Take $n = 1000$ and $a = -0.7$.
Inputs: a
Output: ρ.

2. Evaluate the following expected values by using Monte Carlo simulation:

$$E\text{=E}\left[\max(0, (X^3 + \sin(Y) + X^2 Y))\right]$$

where X and Y have $N(0,1)$ distribution and a correlation of $\rho = 0.6$.
Inputs: ρ
Outputs: E

3. (a) Estimate the following expected values by simulation:

$$A(t) = E\left(W_t{}^2 + sin(W_t)\right) \text{ and } B(t) = E\left(e^{\frac{t}{2}}cos(W_t)\right) \text{ for } t = 1,\ 3,\ 5.$$

Here W_t is a Standard Wiener Process.

(b) How are the values of $B(t)$ (for the cases $t = 1,\ 3, 5$) related?

(c) Now use a variance reduction technique (whichever you want) to compute the expected value (t) for the case $t = 5$. Do you see any improvements? Comment on your findings..

Inputs: t
Outputs: 1) $A(t)$ and $B(t)$ for all 3 t's. 2) Writeup: comments for parts (b) and (c).

4. Let S_t be a Geometric Brownian Motion process: $S_t = S_0 e^{\left(\sigma W_t + \left(r - \frac{\sigma^2}{2}\right)t\right)}$, where $r = 0.04, \sigma = 0.2, S_0 = \$88$, W_t is a Standard Brownian Motion process (Standard Wiener process).

(a) Estimate the price **c** of a European Call option on the stock with $T = 5$, $X = \$100$ *by using Monte Carlo simulation*.

(b) Compute the exact value of the option **c** by the Black-Scholes formula.

(c) Now use variance reduction techniques (whichever you want) to estimate the price in part (a) again. Did the accuracy improve? Comment.

Inputs: r , σ, S_0
Outputs: 1) *Ca1 for* parts (a) and (b); 2) Writeup: comments for part (c)

5. (a) For each integer number n from 1 to 10, use 1000 simulations of S_n to estimate ES_n, where S_t is a Geometric Brownian Motion process: $S_t = S_0 e^{\left(\sigma W_t + \left(r - \frac{\sigma^2}{2}\right)t\right)}$, where $r = 0.04, \sigma = 0.18, S_0 =$ \$88. Plot all of the above $E(S_n)$, for n ranging from 1 to 10, in one graph.

(b) Now simulate 6 paths of S_t for $0 \leq t \leq 10$ (defined in part (a)) by dividing up the interval [0, 10] into 1,000 equal parts.

(c) Plot your data from parts (a) and (b) in one graph.

(d) What would happen to the ES_n graph if you increased σ from 18% to 35%? What would happen to the 6 plots of S_t for $0 \leq t \leq 10$, if you increased σ from 18% to 35%?

Inputs: σ
Outputs: 1) Graphs: plots in a .jpg file; 2) writeup: comments in a .pdf file for part (d)

6. Consider the following integral for computing the number π: $4\int_0^1 \sqrt{1-x^2}dx = \pi$.

(a) The integral above can be estimated by a simple numerical integration using, say Euler's discretization (or any other discretization) scheme. Estimate the integral by using the Euler's discretization scheme.
(b) Estimate the integral by Monte Carlo simulation.
(c) Now try the Importance Sampling method to improve the estimate of π in part (b). Comment on errors and improvements.

Inputs: n (number of MC simulations)
Outputs: 1) Values: *Ia* for part (a), *Ib* for part (b); 2) Writeup: comments in a .pdf file