

Project 3

MGMTMFE 405

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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code clarity and accuracy will determine the grades.

Submit your codes and a PDF file of your answers to questions (including graphs, histograms, but no codes, in this PDF file) by 3PM PDT on Next Wednesday.

1. Evaluate the following expected values and probabilities:

$$p1 = P(Y_2 > 5),$$
$$e1 = E\left(X_2^{\frac{1}{3}}\right), \quad e2 = E(Y_3), \quad e3 = E(X_2 Y_2 1(X_2 > 1)),$$

where the Ito's processes X and Y evolve according to the following SDEs:

$$dX_t = \left(\frac{1}{5} - \frac{1}{2}X_t\right)dt + \frac{2}{3}dW_t, \quad X_0 = 1; \quad dY_t = \left(\left(\frac{2}{1+t}\right)Y_t + \frac{1+t^3}{3}\right)dt + \frac{1+t^3}{3}dZ_t, \quad Y_0 = \frac{3}{4},$$

and W, Z are independent Wiener Processes.

Inputs: X_0, Y_0

Outputs: Values: $p1, e1, e2, e3$.

2. Estimate the following expected values:

$$e1 = E(1 + X_3)^{1/3}, \quad e2 = E(X_1 Y_1)$$

where

$$dX_t = \frac{1}{4}X_t dt + \frac{1}{3}X_t dW_t - \frac{3}{4}X_t dZ_t, \quad X_0 = 1; \quad \text{and} \quad Y_t = e^{-0.08t + \frac{1}{3}W_t + \frac{3}{4}Z_t},$$

and W, Z are independent Wiener Processes.

Inputs: X_0

Outputs: Values: $e1, e2$

- 3.
- Write a code to compute prices of European Call options via Monte Carlo simulation. Use variance reduction techniques (e.g. Antithetic Variates) in your estimation. The code should be generic: for any input of the 5 model parameters - S_0, T, X, r, σ - the output is the corresponding price of the European call option.
 - Write code to compute the prices of European Call options by using the Black-Scholes formula. Use the approximation of $N(\cdot)$ described in Chapter 3. The code should be generic: for any input values of the 5 parameters - S_0, T, X, r, σ - the output is the corresponding price of the European call option.
 - Estimate the European call option's greeks - delta, gamma, theta, and vega - and graph them as functions of the initial stock price S_0 . Use $X = 20, \sigma = 0.25, r = 0.04$ and $T = 0.5$ in your estimations. Use the range $[15, 25]$ for S_0 , with a step size of 1. You will have 4 different graphs for each of the 4 greeks.
In all cases, dt (time-step) should be user-defined. Use $dt=0.004$ (a day) as a default value.

Inputs: $dt, S_0, T, X, r, \text{Sigma}$

Outputs: Values: $C1$ for part (a), $C2$ for part (b), D, G, T, V for part (c).

4. Consider the following 2-factor model for stock prices with stochastic volatility:

$$\begin{cases} dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^1 \\ dV_t = \alpha(\beta - V_t)dt + \sigma\sqrt{V_t} dW_t^2 \end{cases}$$

where the Brownian Motion processes above are correlated: $dW_t^1 dW_t^2 = \rho dt$, where the correlation ρ is a constant in $[-1, 1]$.

Estimate the price of a European Call option (via Monte Carlo simulation) that has a strike price of K and matures in T years.

Use the following default parameters of the model: $\rho = -0.6$, $r = 0.03$, $S_0 = \$48$, $V_0 = 0.05$, $\sigma = 0.42$, $\alpha = 5.8$, $\beta = 0.0625$.

Use the Full Truncation, Partial Truncation, and Reflection methods, and provide 3 price estimates by using the tree methods.

Inputs: $\rho, r, S_0, V_0, \sigma, \alpha, \beta$.

Outputs: Values: $C1, C2, C3$.

5. The objective of this exercise is to compare a sample of Pseudo-Random numbers with a sample of Quasi-Monte Carlo numbers of $Uniform[0,1] \times [0,1]$:

- Generate 100 2-dimensional Uniform $[0,1] \times [0,1]$ vectors by using any one of the algorithms for random number generation.
- Generate 100 points of the 2-dimensional Halton sequences, using bases 2 and 7.
- Generate 100 points of the 2-dimensional Halton sequences, using bases 2 and 4. (**Note:** 4 is a non-prime number!).
- Draw all 3 sequences of random numbers on separate graphs. Are there differences in the three sets (visual test only)? Comment on your observations.
- Use 2-dimensional Halton sequences to compute the following integral:

$$I = \int_0^1 \int_0^1 e^{-xy} \left(\sin 6\pi x + \cos^{\frac{1}{3}} 2\pi y \right) dx dy$$

Use $N=10,000$ in your simulations. Try different couples for bases: (2,4), (2,7), (5,7).

Inputs: For e) b1 and b2 (the bases)

Outputs: Values: I for part (e); Graphs: 3 plots for part (d); Writeup: comments for part (d).

6. *OPTIONAL [NOT for grading]*

- You hold two European Call options with similar characteristics, but one (the first) matures in 1 year and the other (the second) matures in 3 months. Which will have higher delta? Higher Gamma? Use simulations to answer.
- Which is more expensive: an ATM European call or an ATM European put on the same stock with the same maturity? Use simulations to answer.
- Which is more expensive: a 5% OTM European call or a 5% OTM European put on the same stock with the same maturity? Use simulations to answer.
- Which is higher: the Gamma of an ATM European call or an ATM European put on the same stock with the same maturity? What if they both are 10% ITM or 10% OTM? Use simulations to answer.