Project 7_Kaiyue Wu

Question 1

a. Values of three different methods:

`Explicit Finite-Difference method`

	BS price	dx1	dx2	dx3	error dx1	error dx2	error dx3
4	5.801987	5.802249	5.833768	5.837862	-0.000262	-0.031781	-0.035875
5	4.801987	4.844540	4.872652	4.888680	-0.042553	-0.070665	-0.086693
6	3.802058	3.820255	3.865616	3.820258	-0.018197	-0.063558	-0.018200
7	2.805357	2.840817	2.873559	2.902368	-0.035460	-0.068201	-0.097010
8	1.844269	1.877162	1.873509	1.877333	-0.032893	-0.029241	-0.033064
9	1.024428	1.062792	1.110251	1.120862	-0.038364	-0.085823	-0.096434
10	0.464695	0.481928	0.508746	0.520756	-0.017234	-0.044051	-0.056061
11	0.171537	0.173751	0.199263	0.173599	-0.002214	-0.027726	-0.002062
12	0.052460	0.058746	0.058100	0.067100	-0.006286	-0.005641	-0.014640
13	0.013651	0.015487	0.016193	0.015572	-0.001836	-0.002541	-0.001920
14	0.003107	0.003119	0.003537	0.003840	-0.000012	-0.000429	-0.000732
15	0.000635	0.000749	0.000880	0.000773	-0.000114	-0.000245	-0.000139
16	0.000119	0.000206	0.000200	0.000199	-0.000087	-0.000082	-0.000081

`Implicit Finite-Difference method`

	BS price	dx1	dx2	dx3	error dx1	error dx2	error dx3
4	5.801987	5.804117	5.808879	5.804115	-0.002131	-0.006892	-0.002128
5	4.801987	4.802333	4.841731	4.846849	-0.000346	-0.039744	-0.044862
6	3.802058	3.823053	3.828397	3.876278	-0.020995	-0.026339	-0.074221
7	2.805357	2.844188	2.830626	2.844260	-0.038831	-0.025269	-0.038903
8	1.844269	1.880884	1.941440	1.946540	-0.036616	-0.097172	-0.102272
9	1.024428	1.065758	1.069562	1.065725	-0.041330	-0.045133	-0.041297
10	0.464695	0.483404	0.481017	0.483142	-0.018709	-0.016322	-0.018448
11	0.171537	0.174368	0.184771	0.193675	-0.002831	-0.013234	-0.022138
12	0.052460	0.059205	0.052892	0.059242	-0.006745	-0.000432	-0.006782
13	0.013651	0.015779	0.014605	0.018595	-0.002127	-0.000954	-0.004944
14	0.003107	0.003260	0.004415	0.003317	-0.000152	-0.001307	-0.000210
15	0.000635	0.000798	0.000800	0.001016	-0.000163	-0.000166	-0.000381
16	0.000119	0.000131	0.000172	0.000176	-0.000013	-0.000053	-0.000058

Crank-Nicolson Finite-Difference method

	BS price	dx1	dx2	dx3	error dx1	error dx2	error dx3
4	5.801987	5.804109	5.808871	5.804107	-0.002123	-0.006884	-0.002120
5	4.801987	4.802325	4.841723	4.846841	-0.000338	-0.039736	-0.044854
6	3.802058	3.823039	3.828382	3.876265	-0.020981	-0.026324	-0.074207
7	2.805357	2.844103	2.830539	2.844175	-0.038745	-0.025182	-0.038818
8	1.844269	1.880706	1.941262	1.946362	-0.036437	-0.096993	-0.102093
9	1.024428	1.065789	1.069590	1.065755	-0.041361	-0.045162	-0.041327
10	0.464695	0.483681	0.481294	0.483420	-0.018986	-0.016600	-0.018725
11	0.171537	0.174528	0.184943	0.193856	-0.002992	-0.013406	-0.022319
12	0.052460	0.059186	0.052861	0.059223	-0.006727	-0.000402	-0.006763
13	0.013651	0.015697	0.014523	0.018514	-0.002046	-0.000872	-0.004863
14	0.003107	0.003205	0.004353	0.003263	-0.000097	-0.001246	-0.000155
15	0.000635	0.000772	0.000774	0.000986	-0.000137	-0.000140	-0.000352
16	0.000119	0.000123	0.000163	0.000167	-0.000005	-0.000044	-0.000048

b. If we compared the put option price calculated by three methods with the value calculated from the Black Scholes formula, we can see from the graph that all the three methods give a good approximation of the Black-Scholes values. The Crank-Nicolson Method seems outperforms the other two within smallest errors, giving a much closer estimation of the option price.

Question 2:

a.

Explicit Finite-Difference method

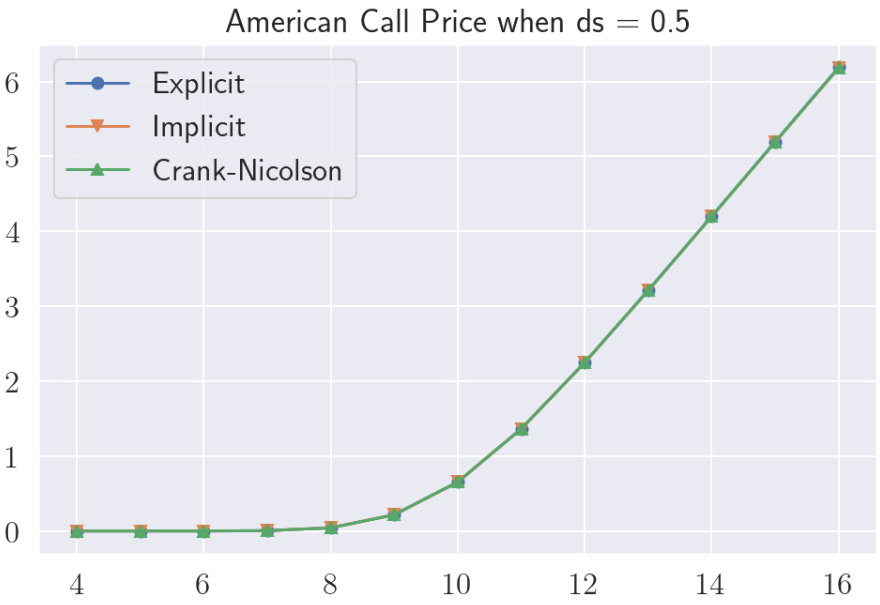
	Call when ds=0.5	Call when ds=1	Put when ds=0.5	Put when ds=1
4	1.341755e-08	7.921639e-07	6.000000	6.000000
5	2.626757e-06	2.645102e-05	5.000000	5.000000
6	1.680357e-04	5.069207e-04	4.000000	4.000000
7	4.055082e-03	5.889406e-03	3.000000	3.000000
8	4.206117e-02	4.291649e-02	2.000000	2.000000
9	2.166802e-01	2.010445e-01	1.075747	1.051627
10	6.540399e-01	6.230475e-01	0.473328	0.439979
11	1.364160e+00	1.348123e+00	0.170676	0.154452
12	2.248808e+00	2.244958e+00	0.051867	0.048152
13	3.211448e+00	3.211427e+00	0.013756	0.013876
14	4.200695e+00	4.200989e+00	0.003287	0.003796
15	5.196284e+00	5.196161e+00	0.000726	0.001004
16	6.189832e+00	6.189327e+00	0.000151	0.000260

Implicit Finite-Difference method

	Call when ds=0.5	Call when ds=1	Put when ds=0.5	Put when ds=1
4	1.901301e-08	8.984022e-07	6.000000	6.000000
5	3.215518e-06	2.871215e-05	5.000000	5.000000
6	1.855986e-04	5.316790e-04	4.000000	4.000000
7	4.209029e-03	6.025739e-03	3.000000	3.000000
8	4.240316e-02	4.323244e-02	2.000000	2.000000
9	2.165805e-01	2.010542e-01	1.075205	1.051252
10	6.534354e-01	6.222878e-01	0.472453	0.439081
11	1.363833e+00	1.347857e+00	0.170268	0.154173
12	2.248829e+00	2.244942e+00	0.051919	0.048255
13	3.211345e+00	3.211035e+00	0.013913	0.014022
14	4.199690e+00	4.199106e+00	0.003390	0.003886
15	5.192673e+00	5.190197e+00	0.000772	0.001046
16	6.180231e+00	6.174095e+00	0.000168	0.000277

Crank-Nicolson Finite-Difference method

	Call when ds=0.5	Call when ds=1	Put when ds=0.5	Put when ds=1
4	1.604763e-08	8.443094e-07	6.000000	6.000000
5	2.912898e-06	2.757096e-05	5.000000	5.000000
6	1.767506e-04	5.192633e-04	4.000000	4.000000
7	4.132178e-03	5.957630e-03	3.000000	3.000000
8	4.223239e-02	4.307473e-02	2.000000	2.000000
9	2.166300e-01	2.010489e-01	1.075475	1.051439
10	6.537380e-01	6.226676e-01	0.472889	0.439530
11	1.363996e+00	1.347986e+00	0.170470	0.154312
12	2.248818e+00	2.244927e+00	0.051893	0.048203
13	3.211388e+00	3.211110e+00	0.013835	0.013949
14	4.200129e+00	4.199528e+00	0.003339	0.003841
15	5.194172e+00	5.191400e+00	0.000749	0.001025
16	6.183959e+00	6.176737e+00	0.000160	0.000268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Explicit
Implicit
Crank-Nicolson
0
1
2
3
4
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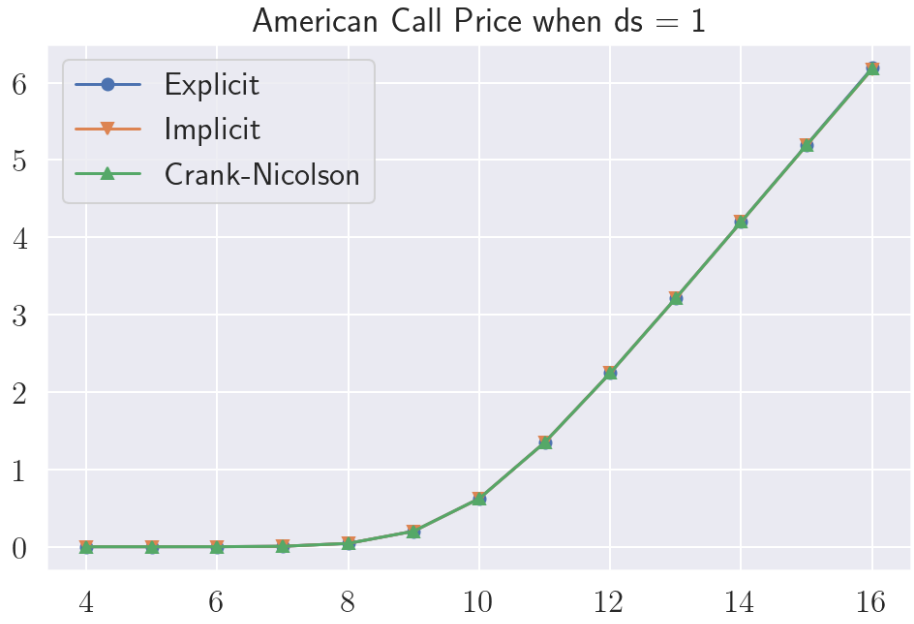
12
14
16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Explicit
Implicit
Crank-Nicolson
0
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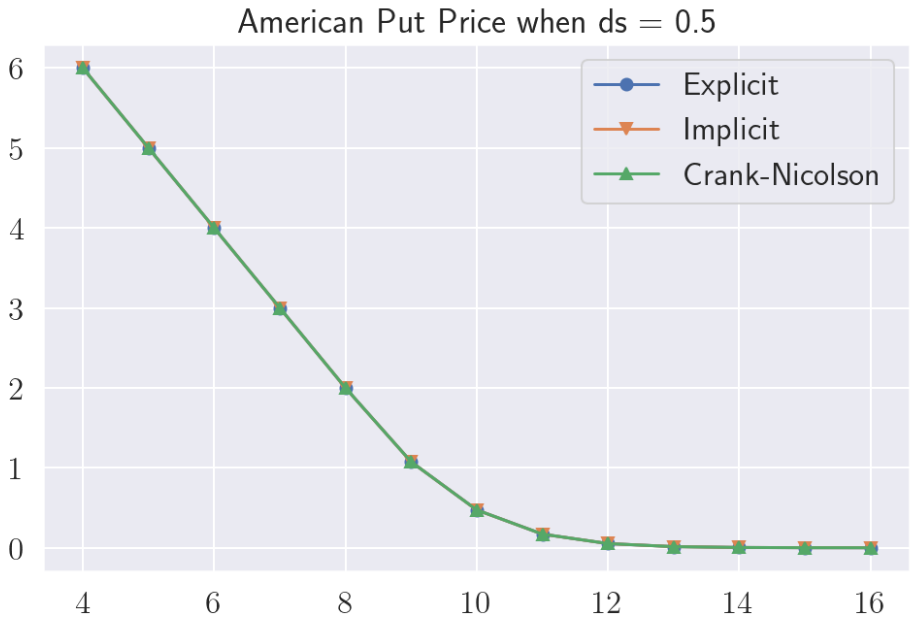
American Put Price when ds = 0.5
Explicit
Implicit
Crank-Nicolson
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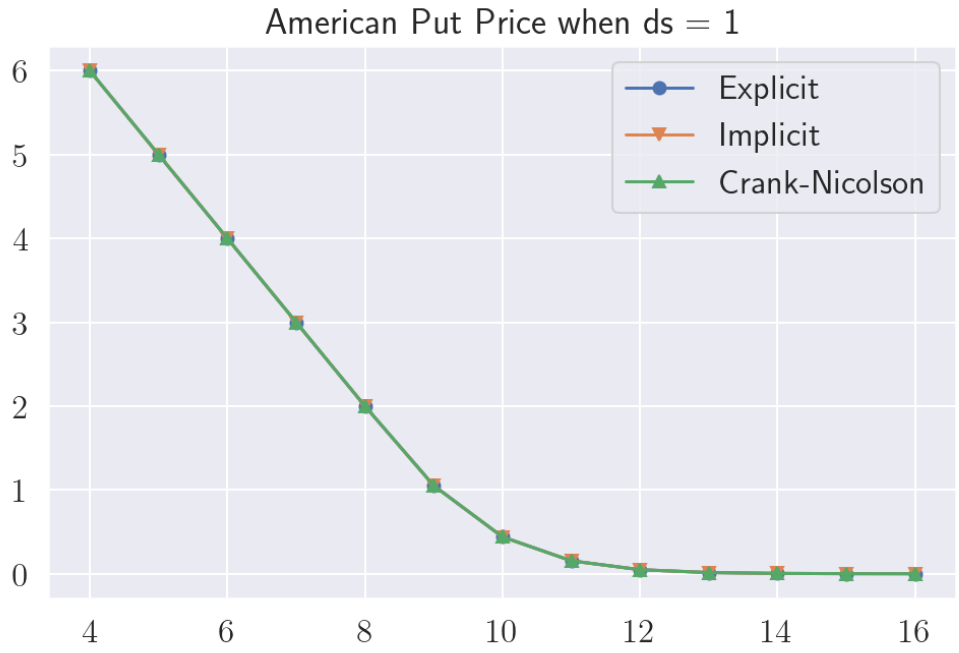
American Put Price when ds = 1
Explicit
Implicit
Crank-Nicolson
0
1
2
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