PS4

April 28, 2022

1 PS4

[2]:

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
import yfinance as yf
from datetime import datetime
import yahoo_fin.options as options
from scipy.stats import norm

new_line = '\n'
pd.options.mode.chained_assignment = None
import warnings
warnings.filterwarnings('ignore')

sns.set_style('darkgrid')
sns.set(font_scale=1.5, rc={'text.usetex' : True,})
```

[3]:

```
# def binoEuro_slow(s0,u,d,N,k,r,T,option_type):
#     """
#     s0: stock price
#     u: probability of going up
#     d: probability of going down
#     N: number of periods
#     """
#     dt = T/N
#     p = (np.exp(r*dt)-d)/(u-d)  # risk-neutral probability
#     disc = np.exp(-r*dt)        # discount back

#     S = np.zeros(N+1)           # N -period, every period has N+1 nodes
#     S[N] = s0*d**N              # last botton node

#     for j in range(N-1,-1,-1):  #find the stock prices in last period
#         S[j] = S[j+1]*u/d

#     C = np.zeros(N+1)           #find the call payoff of each price node
```

```
#     for j in range(N+1):
#         if option_type.lower() =='c':
#             C[j] = max(S[j]-k,0)
#         else:
#             C[j] = max(k-S[j],0)

#     for i in range(N,0,-1):     # step backwards, find payoff of previous␣
↪period
#         for j in range(i):
#             C[j] = disc * (p *(C[j]) + (1-p)*C[j+1])

#     return C[0]

# binoEuro_slow(s0=34,u=1.25,d=0.7,N=2,r=0.03,k=30,T=2,option_type='C')
```

[4]:
```
# def binoEuro_fast(s0,u,d,N,k,r,T,option_type):
#     """
#     s0: stock price
#     u: probability of going up
#     d: probability of going down
#     N: number of periods
#     """
#     dt = T/N
#     p = (np.exp(r*dt)-d)/(u-d)  # risk-neutral probability
#     disc = np.exp(-r*dt)        # discount back

#     S =s0* u**np.arange(0,N+1,1)*d**np.arange(N,-1,-1)         # stock prices␣
↪at last step
#     S = S[::-1]

#     if option_type.lower() =='c':
#         S = np.maximum(S-k, np.zeros(N+1))
#     else:
#         S = np.maximum(k-S, np.zeros(N+1))

#     for i in range(N,0,-1):     # step backwards, find payoff of previous␣
↪period
#         S = disc*(p*S[0:i] +(1-p)*S[1:i+1])

#     return S[0]
# binoEuro_fast(s0=34,u=1.25,d=0.7,N=2,r=0.03,k=30,T=2,option_type='C')
```

[5]:
```
# def binoAmer_slow(s0,u,d,N,k,r,T,option_type):
#     dt = T/N
#     p = (np.exp(r*dt)-d)/(u-d)
#     disc = np.exp(-r*dt)
```

```
#     S = np.zeros(N+1)        # stock prices at last step
#     for j in range(0,N+1):
#         S[j] = s0*u**j*d**(N-j)

#     C = np.zeros(N+1)
#     for j in range(0,N+1):
#         if option_type.lower()=='c':
#             C[j] = max(S[j]-k,0)
#         else:
#             C[j] = max(k-S[j],0)

#     for i in np.arange(N-1,-1,-1):
#         for j in range(0,i+1):
#             S = s0 *u**j*d**(i-j)
#             C[j] = disc * (p * C[j+1] + (1-p)* C[j])
#             if option_type.lower()=='c':
#                 C[j] = max(S-k,C[j])
#             else:
#                 C[j] = max(k-S,C[j])

#     return C[0]
```

[6]:

```
# s0=100
# u=1.1
# d=1/u
# N=3
# k=100
# r=0.06
# T=1
# dt = T/N
# p = (np.exp(r*dt)-d)/(u-d)
# disc = np.exp(-r*dt)

# S = np.zeros(shape=(N+1,N+1))
# CV = np.zeros(shape=(N+1,N+1))
# EV = np.zeros(shape=(N+1,N+1))

# S[0][0] = s0

# for i in range(N):
#     for j in range(i,N):
#         S[i][j+1] = S[i][j]*u
#         S[i+1][j+1] = S[i][j]*d
#         EV[i][j+1] = max(k-S[i][j+1],0)
#         EV[i+1][j+1] = max(k-S[i+1][j+1],0)

# for i in range(N,0,-1):
```

```
#     for j in range(i,N+1):
#         CV[i-1][j-1] = disc * (p*EV[i-1][j] + (1-p)* EV[i][j])
```

[7]:
```
# def binoAmer_fast(s0,u,d,N,k,r,T,option_type):
#     dt = T/N
#     p = (np.exp(r*dt)-d)/(u-d)
#     disc = np.exp(-r*dt)

#     S = s0* u**np.arange(0,N+1,1)*d**np.arange(N,-1,-1)       # stock prices␣
↪at last step

#     if option_type.lower()=='c':
#         C = np.maximum(0,S - k)
#     else:
#         C = np.maximum(0,k - S)

#     for i in np.arange(N-1,-1,-1):
#         S = s0* u**np.arange(0,i+1,1)*d**np.arange(i,-1,-1)
#         print(S)
#         C[:i+1] = disc * (p * C[1:i+2] + (1-p)* C[0:i+1])
#         print(C)
#         C = C[:-1]

#         if option_type.lower()=='c':
#             C = np.maximum(C,S - k)
#         else:
#             C = np.maximum(C,k - S)

#     return C[0]
```

[8]:
```
# def AmericanPut_Matrix(s0,u,d,N,k,r,T,option_type):
#     dt = T/N
#     p = (np.exp(r*dt)-d)/(u-d)

#     disc = np.exp(-r*dt)

#     S = np.zeros(shape=(N+1,N+1))
#     CV = np.zeros(shape=(N+1,N+1))
#     EV = np.zeros(shape=(N+1,N+1))

#     S[0][0] = s0

#     for i in range(N):
#         for j in range(i,N):
#             S[i][j+1] = S[i][j]*u
#             S[i+1][j+1] = S[i][j]*d
```

```
#     for i in range(N,0,-1):
#         for j in range(i,N+1):
#             EV[i][j] = max(k-S[i][j],0)
#             CV[i-1][j-1] = disc * (p*EV[i-1][j] + (1-p)* EV[i][j])
#             CV[i][j] = max(CV[i][j],EV[i][j])
#             CV[i-1][j-1] = disc * (p*CV[i-1][j] + (1-p)* CV[i][j])

#     CV[0][0] = disc * (p*CV[0][1] + (1-p)* CV[1][1])

#     return CV[0][0],CV

# AmericanPut_Matrix(s0=100,u=1.1,d=1/1.1,N=3,k=100,r=0.06,T=1,option_type='p')
```

[9]:
```
def parameters(r,sigma,N,T,formula):
    dt = T/N
    if formula == 'a':
        c = 0.5 * (np.exp(-r*dt) + np.exp( (r+sigma**2)*dt) )
        d = c - np.sqrt(c**2 -1)
        u = 1/d
        p = (np.exp(r*dt) -d) / (u-d)
    elif formula == 'b':
        u = np.exp(r*dt)*(1+np.sqrt(np.exp(sigma**2*dt)-1))
        d = np.exp(r*dt)*(1-np.sqrt(np.exp(sigma**2*dt)-1))
        p = 0.5
    elif formula =='c':
        u = np.exp((r-sigma**2/2)*dt + sigma*np.sqrt(dt))
        d = np.exp((r-sigma**2/2)*dt - sigma*np.sqrt(dt))
        p=0.5
    elif formula =='d':
        u = np.exp(sigma*np.sqrt(dt))
        d = np.exp(-sigma*np.sqrt(dt))
        p = 1/2 + 1/2*((r-sigma**2/2)*np.sqrt(dt))/sigma

    return u, d, p, N
```

[10]:
```
def binoEuro_slow(s0,N,k,r,T,sigma,option_type,formula):
    """
    s0: stock price
    u: probability of going up
    d: probability of going down
    N: number of periods
    """
    dt = T/N
    params = parameters(r,sigma,N,T,formula)
    u = params[0]
    d = params[1]
```

```
    p = params[2]

    disc = np.exp(-r*dt)         # discount back

    S = np.zeros(N+1)            # N -period, every period has N+1 nodes
    S[N] = s0*d**N               # last botton node

    for j in range(N-1,-1,-1):  #find the stock prices in last period
        S[j] = S[j+1]*u/d

    C = np.zeros(N+1)            #find the call payoff of each price node
    for j in range(N+1):
        if option_type.lower() =='c':
            C[j] = max(S[j]-k,0)
        else:
            C[j] = max(k-S[j],0)

    for i in range(N,0,-1):      # step backwards, find payoff of previous period
        for j in range(i):
            C[j] = disc * (p *(C[j]) + (1-p)*C[j+1])

    return C[0]
```

[11]:
```
def binoEuro_fast(s0,N,k,r,T,sigma,option_type,formula):
    """
    s0: stock price
    u: probability of going up
    d: probability of going down
    N: number of periods
    """
    dt = T/N
    params = parameters(r,sigma,N,T,formula)
    u = params[0]
    d = params[1]
    p = params[2]

    disc = np.exp(-r*dt)         # discount back

    S =s0* u**np.arange(0,N+1,1)*d**np.arange(N,-1,-1)          # stock prices␣
 ↪at last step
    S = S[::-1]

    if option_type.lower() =='c':
        S = np.maximum(S-k, np.zeros(N+1))
    else:
        S = np.maximum(k-S, np.zeros(N+1))
```

```
    for i in range(N,0,-1):     # step backwards, find payoff of previous period
        S = disc*(p*S[0:i] +(1-p)*S[1:i+1])

    return S[0]
```

[12]:
```
# def AmericanPut_Matrix(s0,N,k,r,T,sigma,option_type,formula):
#     dt = T/N
#     params = parameters(r,sigma,N,T,formula)
#     u = params[0]
#     d = params[1]
#     p = params[2]
#     disc = np.exp(-r*dt)

#     S = np.zeros(shape=(N+1,N+1))
#     CV = np.zeros(shape=(N+1,N+1))
#     EV = np.zeros(shape=(N+1,N+1))

#     S[0][0] = s0

#     for i in range(N):
#         for j in range(i,N):
#             S[i][j+1] = S[i][j]*u
#             S[i+1][j+1] = S[i][j]*d

#     for i in range(N,0,-1):
#         for j in range(i,N+1):
#             EV[i][j] = max(k-S[i][j],0)
#             CV[i-1][j-1] = disc * (p*EV[i-1][j] + (1-p)* EV[i][j])
#             CV[i][j] = max(CV[i][j],EV[i][j])
#             CV[i-1][j-1] = disc * (p*CV[i-1][j] + (1-p)* CV[i][j])

#     CV[0][0] = disc * (p*CV[0][1] + (1-p)* CV[1][1])

#     return CV[0][0]
```

[13]:
```
def binoAmer_fast(s0,N,k,r,T,sigma,option_type,formula):
    dt = T/N
    params = parameters(r,sigma,N,T,formula)
    u = params[0]
    d = params[1]
    p = params[2]
    disc = np.exp(-r*dt)

    S = s0* u**np.arange(0,N+1,1)*d**np.arange(N,-1,-1)       # stock prices at␣
 ↪last step

    if option_type.lower()=='c':
```

```python
        C = np.maximum(0,S - k)
    else:
        C = np.maximum(0,k - S)

    for i in np.arange(N-1,-1,-1):
        S = s0* u**np.arange(0,i+1,1)*d**np.arange(i,-1,-1)

        C[:i+1] = disc * (p * C[1:i+2] + (1-p)* C[0:i+1])

        C = C[:-1]

        if option_type.lower()=='c':
            C = np.maximum(C,S - k)
        else:
            C = np.maximum(C,k - S)

    return C[0]
```

1.1 1. Compare the convergence rates of the four methods below by doing the following:

Use the Binomial Method to price a 6-month European Call option with the following information: the risk-free interest rate is 5% per annum; the volatility is 24% per annum; the current stock price is 32; and the strike price is $30. Divide the time interval into parts to estimate the price of this option. Use = 10,20,40,80,100,200, and 500 to estimate the price and draw all resulting prices in one graph, where the horizontal axis measures , and the vertical one the price of the option.

(a)

$\mu = \frac{1}{d}, \qquad d = c - \sqrt{c^2 - 1} \qquad c = \frac{1}{2}(e^{-r\Delta} + e^{(r+\sigma^2)\Delta}), \qquad p = \frac{e^{r\Delta} - d}{u - d}$

(b)

$\mu = e^{r\Delta}(1 + \sqrt{e^{\sigma^2\Delta} - 1}), \qquad d = e^{r\Delta}(1 - \sqrt{e^{\sigma^2\Delta} - 1}) \qquad p = \frac{1}{2}$

(c)

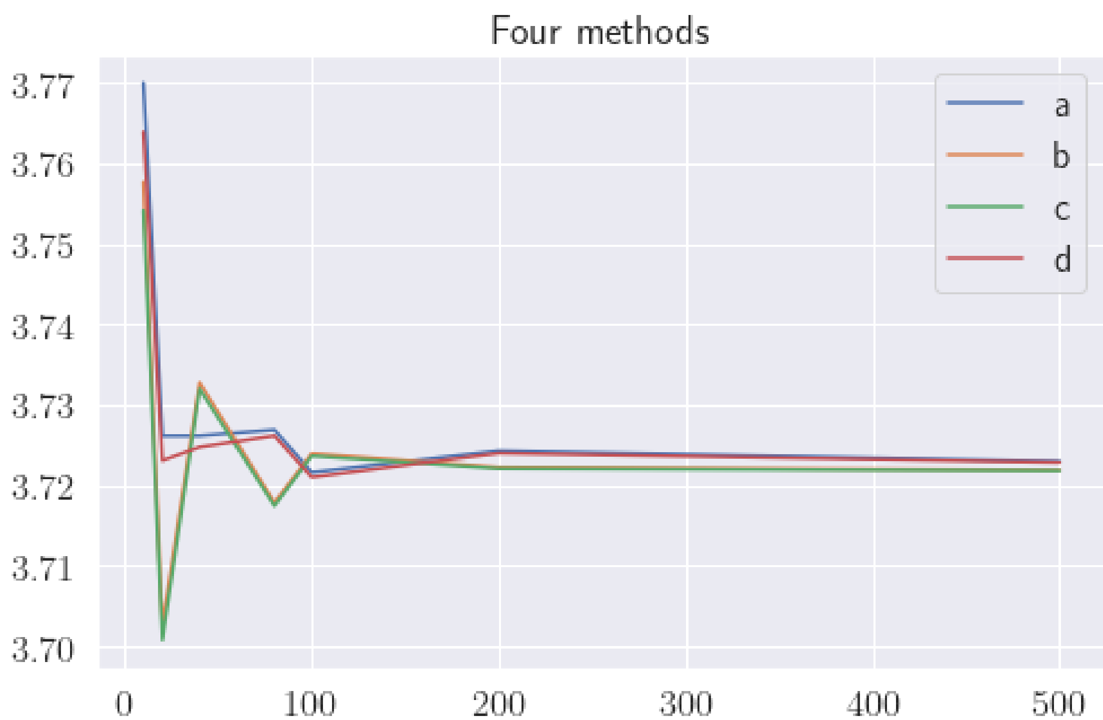
$\mu = e^{(r - \frac{\sigma^2}{2})\Delta + \sigma\Delta} \qquad d = e^{(r - \frac{\sigma^2}{2})\Delta - \sigma\Delta} \qquad p = \frac{1}{2}$

(d)

$\mu = e^{\sigma\sqrt{\Delta}} \qquad d = e^{-\sigma\sqrt{\Delta}} \qquad p = \frac{1}{2} + \frac{1}{2}(\frac{(r - \frac{\sigma^2}{2})\sqrt{\Delta}}{\sigma})$

[14]:
```python
T = 0.5
rf = 0.05
sigma = 0.24
s0 = 32
N = [10,20,40,80,100,200,500]
k=30
```

```
a_call =
 ↪[binoEuro_fast(s0=s0,k=k,T=T,sigma=sigma,r=rf,N=i,option_type='c',formula='a')
 ↪for i in N]
b_call =
 ↪[binoEuro_fast(s0=s0,k=k,T=T,sigma=sigma,r=rf,N=i,option_type='c',formula='b')
 ↪for i in N]
c_call =
 ↪[binoEuro_fast(s0=s0,k=k,T=T,sigma=sigma,r=rf,N=i,option_type='c',formula='c')
 ↪for i in N]
d_call =
 ↪[binoEuro_fast(s0=s0,k=k,T=T,sigma=sigma,r=rf,N=i,option_type='c',formula='d')
 ↪for i in N]

question_1 = pd.DataFrame(list(zip(N,a_call,b_call,c_call,d_call)))
question_1.columns=['Steps','a','b','c','d']
question_1 = question_1.set_index('Steps')

plt.figure(figsize=(8,5))
plt.plot(question_1.iloc[:,:])
plt.legend(['a','b','c','d'])
plt.title('Four methods')

plt.show()
```



[15]:
```
plt.figure()

fig, axs = plt.subplots(2, 2,sharex=True)
fig.set_figheight(5)
fig.set_figwidth(8)

axs[0, 0].plot(a_call)
axs[0, 0].set_title('Method a')

axs[0, 1].plot(b_call)
axs[0, 1].set_title('Method b')

axs[1, 0].plot(c_call)
axs[1, 0].set_title('Method c')

axs[1, 1].plot(d_call)
axs[1, 1].set_title('Method d')


plt.show()
```

```
<Figure size 432x288 with 0 Axes>
```

1.2 2.

Take the current price of AMZN. Use risk-free rate of 1% per annum, and a strike price that is

[4]:
```
# last 60 months data
amzn = yf.download("AMZN", start="2017-04-27", end="2022-04-27")
```

```
[*********************100%***********************]  1 of 1 completed
```

[5]:
```
amzn['return'] = amzn['Adj Close'].pct_change(1)
amzn_std_daily = np.std(amzn['return'])
amzn_std =amzn_std_daily*np.sqrt(252)
```

[18]:
```
rf = 0.01
current_price = amzn.tail(1)['Adj Close'][0]
strike_price = round(current_price*1.1) //10 * 10 -10

# AMZN Jan 2023 1360.000 call expire on 2023-01-20
expire_date = datetime(2023,1,20)
curr_date = amzn.tail(1).index[0]
date_diff = int(str(expire_date - curr_date).split()[0])
N = date_diff
T = N/365

print("AMZN Jan 2023 call expire on 2023-01-20")
print(f"The current price as of April 27 is {current_price}")
print(f"The current volatility as of April 27 is {amzn_std*100}%")
print(f"The strike price we choose is {strike_price}")
print(f"Days until expire is {date_diff}")
```

```
AMZN Jan 2023 call expire on 2023-01-20
The current price as of April 27 is 2787.820068359375
The current volatility as of April 27 is 31.47985505575901%
The strike price we choose is 3050
Days until expire is 269
```

[14]:
```
amzn_estimated =␣
 ↪binoAmer_fast(s0=current_price,k=strike_price,r=rf,N=N,T=T,sigma=amzn_std,option_type='c',f

print(f'The etimated Amazon call price is {amzn_estimated}')
```

```
The etimated Amazon call price is 207.12028952948447
```

I choose the market option price is 302.75 with strike price of 3050, in which the strike price is closest to my strike price 3050.

[15]:
```
def d(s0,k,t,sigma,r):
    d1 = 1/(sigma*np.sqrt(t)) * (np.log(s0/k)+(r+1/2*sigma**2)*t)
    d2 = d1 - sigma*np.sqrt(t)
    return d1, d2
```

```
def BS_call(s0,k,t,sigma,r):
    d1,d2 = d(s0,k,t,sigma,r)
    call = norm.cdf(d1) * s0 - norm.cdf(d2)*k*np.exp(-r*t)

    return call

BS_call(s0=current_price,k=strike_price,t=T,sigma=amzn_std,r=rf)
```

[15]: 207.0038759638444

[19]:
```
target  = 302.75

eps= 0.001
count = 0
max_iter = 1500
vol = amzn_std
i =0
while abs(amzn_estimated-target)>0.01:
    if (amzn_estimated>target):
        vol -= eps
    else:
        vol += eps

    amzn_estimated =␣
 ↪binoAmer_fast(s0=current_price,k=strike_price,r=rf,N=N,T=T,sigma=vol,option_type='c',formul
    i+=1
    if i==max_iter:
        break
```

[20]:
```
new_amzn_std = vol
print(f'The volatility should be about {round(new_amzn_std*100,2)}% to make the␣
 ↪estimated price equal to the market price.')
```

The volatility should be about 41.48% to make the estimated price equal to the market price.

1.3 3. Consider the following information on the stock of a company and options on it:

S0 = 49, x = 50, r = 0.03, = 0.2, T= 0.3846 (20 weeks), = 0.14.
Using the Binomial Method (any one of the parameter choices) estimate the following and draw t

[17]:
```
s0 = 49
k = 50
rf = 0.03
sigma = 0.2
T = 0.3846
```

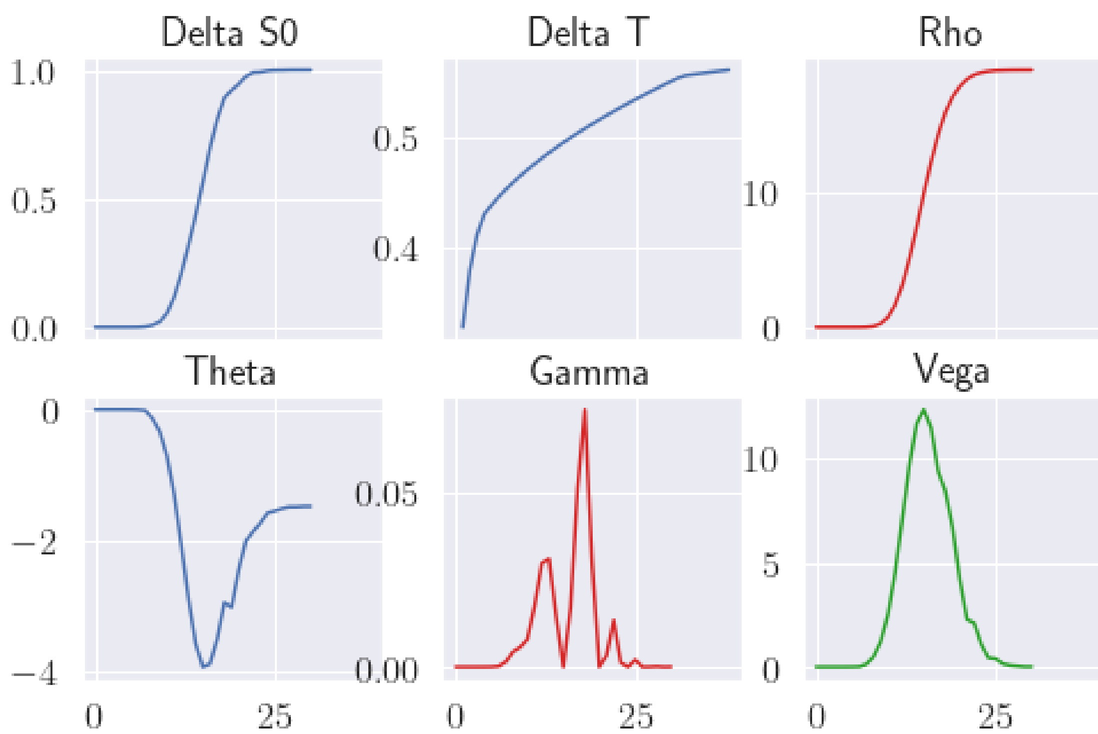
```
mu = 0.14

s0_range= np.arange(20,82,2)
T_range = np.arange(0,0.3846,0.01)
N = len(s0_range)
```

[18]:
```
delta_s0 = np.zeros(N)
delta_t = np.zeros(len(T_range))
gamma = np.zeros(N)
vega = np.zeros(N)
theta = np.zeros(N)
rho = np.zeros(N)

price= np.zeros(N)
for i in range(len(s0_range)):
    price[i] = ␣
 ↪binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a')
    delta_s0[i] =␣
 ↪(binoEuro_fast(s0=s0_range[i]+1,k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a')␣
 ↪- \
                ␣
 ↪binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a'))/
 ↪1
    gamma[i] =␣
 ↪(binoEuro_fast(s0=s0_range[i]+1,k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a')␣
 ↪+\
                 ␣
 ↪binoEuro_fast(s0=s0_range[i]-1,k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a')␣
 ↪- \
           ␣
 ↪2*binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a'))/
 ↪1
    vega[i] = (binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma+0.
 ↪01,r=rf,N=N,option_type='c',formula='a') - \
       ␣
 ↪binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a'))/
 ↪0.01
    theta[i] = -(binoEuro_fast(s0=s0_range[i],k=k,T=T+0.
 ↪01,sigma=sigma,r=rf,N=N,option_type='c',formula='a')- \
                  ␣
 ↪binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a'))/
 ↪0.01
    rho[i] = (binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf+0.
 ↪01,N=N,option_type='c',formula='a') - \
```

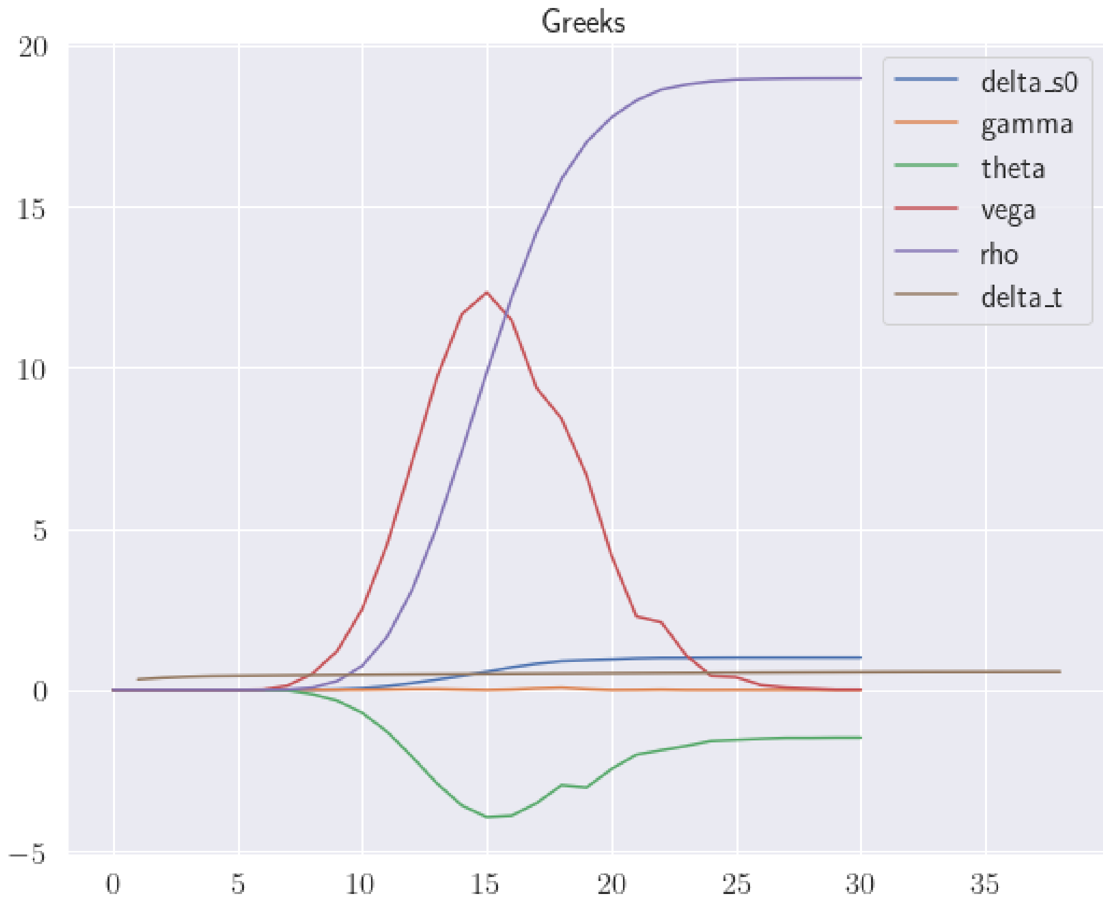
```
                ␣
↪binoEuro_fast(s0=s0_range[i],k=k,T=T,sigma=sigma,r=rf,N=N,option_type='c',formula='a'))/
↪0.01

price= np.zeros(len(T_range))
for i in range(len(T_range)):
    price[i] = ␣
↪binoEuro_fast(s0=s0,k=k,T=T_range[i],sigma=sigma,r=rf,N=N,option_type='c',formula='a')
    delta_t[i] =␣
↪(binoEuro_fast(s0=s0+1,k=k,T=T_range[i],sigma=sigma,r=rf,N=N,option_type='c',formula='a')␣
↪- \
                 price[i])/1
```

[19]:
```
plt.figure()

fig, axs = plt.subplots(2, 3,sharex=True)
fig.set_figheight(5)
fig.set_figwidth(8)

axs[0, 0].plot(delta_s0)
axs[0, 0].set_title('Delta S0')

axs[0, 1].plot(delta_t)
axs[0, 1].set_title('Delta T')

axs[1, 0].plot(theta)
axs[1, 0].set_title('Theta')

axs[1, 1].plot(gamma, 'tab:red')
axs[1, 1].set_title('Gamma')

axs[1, 2].plot(vega, 'tab:green')
axs[1, 2].set_title('Vega')

axs[0, 2].plot(rho, 'tab:red')
axs[0, 2].set_title('Rho')

plt.show()
```

```
<Figure size 432x288 with 0 Axes>
```



[63]:
```
plt.figure(figsize=(10,8))

plt.plot(delta_s0)
plt.plot(gamma)
plt.plot(theta)
plt.plot(vega)
plt.plot(rho)
plt.plot(delta_t)
plt.legend(['delta_s0','gamma','theta','vega','rho','delta_t'])
plt.title('Greeks')

plt.show()
```

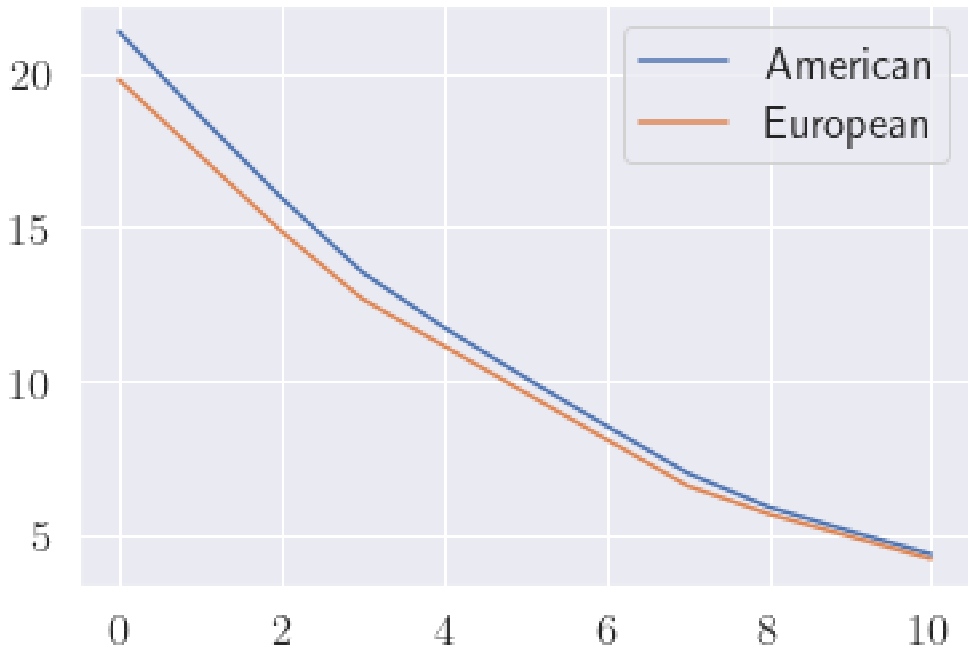


1.4 4. Consider 12-month put options on a stock of company XYZ. Assume the risk-free rate is 5%/annum, the volatility of the stock price is 30 % /annum, and the strike price of the option is 100. Use a Binomial Method to estimate the prices of European and American Put options with current stock prices varying from 80 to 120 in increments of 4. Draw them all in one graph, compare them and comment on your findings.

[64]:
```
rf = 0.05
sigma = 0.3
k = 100
T = 1

s0_range = np.arange(80,124,4)
N = len(s0_range)
```

[65]:
```
am_put =␣
↪[binoAmer_fast(s0=i,N=N,k=k,r=rf,T=T,sigma=sigma,option_type='p',formula='c')␣
↪for i in s0_range]
```

```
eu_put =␣
↪[binoEuro_fast(s0=i,N=N,k=k,r=rf,T=T,sigma=sigma,option_type='p',formula='c')␣
↪for i in s0_range]

plt.figure()

plt.plot(am_put)
plt.plot(eu_put)

plt.legend(['American','European'])
plt.show()
```



1.5 5. Compare the convergence rates of the two methods,

(a) and (b) described below, by doing the following: Use the Trinomial Method to price a 6-mont
the risk-free interest rate is 5% per annum, the volatility is 24%/annum, the current stock pri
Divide the time interval into equal parts to estimate the price of this option. Use n = 10, 1
The two methods are in (a) and (b) below:

[90]:
```
T=0.5
rf = 0.05
sigma = 0.24
s0 = 32
k =30
```

```
N= [10,15,20,40,70,80,100,200,5000]
```

(a) Use the trinomial method applied to the stock price-process ( ) in which

$u = \frac{1}{d}$, $d = e^{-\sigma\sqrt{3\Delta}}$, $p_d = \frac{r\Delta(1-u)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(1-d)}$, $p_u = \frac{r\Delta(1-d)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(u-1)}$, $p_m = 1 - p_u - p_d$

(b) Use the trinomial method applied to the Log-stock price-process (X_t) in which

$\Delta X_u = \sigma\sqrt{3\Delta}$, $\Delta X_d = -\sigma\sqrt{3\Delta}$, $p_u = \frac{1}{2}\left(\frac{\sigma^2\Delta+\left(r-\frac{\sigma^2}{2}\right)^2\Delta^2}{\Delta X_u^2} + \frac{(r-\frac{\sigma^2}{2})\Delta}{\Delta X_u}\right)$, $p_d = \frac{1}{2}\left(\frac{\sigma^2\Delta+\left(r-\frac{\sigma^2}{2}\right)^2\Delta^2}{\Delta X_u^2} - \frac{(r-\frac{\sigma^2}{2})\Delta}{\Delta X_u}\right)$, $P_m = 1 - P_m - P_d$

[91]:
```python
# parameters
def trino_params(sigma,N,T,r):
    dt = T/N

    d = np.exp(-sigma*np.sqrt(3*dt))
    u = 1/d
    pd = (r*dt*(1-u) + (r*dt)**2 + sigma**2*dt)/((u-d)*(1-d))
    pu = (r*dt*(1-d) + (r*dt)**2 + sigma**2*dt)/((u-d)*(u-1))
    pm = 1-pu-pd

    return pu,pd,pm,u,d

def trinoLog_params(sigma,T,N,r):
    dt = T/N
    dxu = sigma*np.sqrt(3*dt)
    dxd = -sigma*np.sqrt(3*dt)
    pu = 1/2 * ((sigma**2*dt + (r-sigma**2/2)**2*dt**2)/dxu**2 + (r-sigma**2/
↪2)*dt/dxu )
    pd = 1/2 * ((sigma**2*dt + (r-sigma**2/2)**2*dt**2)/dxu**2 - (r-sigma**2/
↪2)*dt/dxu )
    pm = 1 - pu - pd

    return pu,pd,pm,dxu,dxd
```

[92]:
```python
#Trinomial slow approach
def trinoEuro_slow(s0,N,k,r,T,sigma,option_type):
    """
    s0: stock price
    u: probability of going up
    d: probability of going down
    N: number of periods
    """

    dt = T/N
    params = trino_params(sigma,N,T,r)
    pu = params[0]
```

```
    pd = params[1]
    pm = params[2]
    u = params[3]
    d = params[4]

    disc = np.exp(-r*dt)        # discount back

    S = np.zeros(2*N+1)
    S[2*N] = s0*(d**N)
    for i in range(2*N-1,-1,-1):
        S[i] = S[i+1] * u
    C = np.zeros(2*N+1)
    for i in range(2*N-1,-1,-1):
        C[i] = max(0,S[i]-k)

    for i in range(N-1,-1,-1):
        for j in range(2*N-1):
            C[j] = (pu*C[j]+pm*C[j+1]+pd*C[j+2])*disc

    return C[0]
#trinoEuro_slow(s0=32,N=10,k=30,r=0.05,T=0.5,sigma=0.24,option_type='c')
```

[93]:

```
# Trinomial fast approach
def trinoEuro_fast(s0,N,k,r,T,sigma,option_type):
    """
    s0: stock price
    u: probability of going up
    d: probability of going down
    N: number of periods
    """

    dt = T/N
    params = trino_params(sigma,N,T,r)
    pu = params[0]
    pd = params[1]
    pm = params[2]
    u = params[3]
    d = params[4]

    disc = np.exp(-r*dt)        # discount back

    S = np.zeros(2*N+1)
    S[2*N] = s0*(d**N)
    for i in range(2*N-1,-1,-1):
        S[i] = S[i+1] * u

    C = np.maximum(S-k,np.zeros(2*N+1))
```

```python
    for i in range(0,N,1):      # step backwards, find payoff of previous period
        C = disc*(pu*C[:-2] +pm*C[1:-1] + pd*C[2:])

    return C[0]

# trinoEuro_fast(s0=32,N=10,k=30,r=0.05,T=0.5,sigma=0.24,option_type='c')
```

[94]:
```python
# Trinomial Log approach
def trinoEuroLog(s0,N,k,r,T,sigma,option_type):
    """
    s0: stock price
    u: probability of going up
    d: probability of going down
    N: number of periods
    """

    dt = T/N
    params = trinoLog_params(sigma,T,N,r)
    pu = params[0]
    pd = params[1]
    pm = params[2]
    dxu = params[3]
    dxd = params[4]

    disc = np.exp(-r*dt)         # discount back

    S = np.zeros(2*N+1)
    S[2*N] = np.exp(np.log(s0)+dxd*N)

    for i in range(2*N-1,-1,-1):
        S[i] = np.exp(np.log(S[i+1])+dxu)

    C = np.maximum(S-k,np.zeros(2*N+1))

    for i in range(0,N,1):      # step backwards, find payoff of previous period
        C = disc*(pu*C[:-2] +pm*C[1:-1] + pd*C[2:])

    return C[0]

# trinoEuroLog(s0=32,N=10,k=30,r=0.05,T=0.5,sigma=0.24,option_type='c')
```

[95]:
```python
a_trino = [trinoEuro_fast(s0=s0,N=i,k=k,r=rf,T=T,sigma=sigma,option_type='c')␣
↪for i in N]
b_trino = [trinoEuroLog(s0=s0,N=i,k=k,r=rf,T=T,sigma=sigma,option_type='c') for␣
↪i in N]
```

[96]:
```
plt.figure(figsize=(8,5))

plt.plot(a_trino)
plt.plot(b_trino)
plt.legend(['Trinomial','Log-Trinomial'])
plt.show()
```

1.6 6. Use Halton's Low-Discrepancy Sequences to price European Call options.

[97]:
```
def halton_oneD(n,base):
    """
    One-dimensional Halton sequence
    (the code is adopted from Brandimarte's "Numerical methods in finance and␣
↪economics:
    a MATLAB-based introduction")

    """
    sequence = np.zeros(n)
    numbits = int(1 + np.ceil(np.log(n)/np.log(base)))
    VetBase = [1/base**(i+1) for i in range(numbits)]
    WorkVet = np.zeros(numbits)
    for i in range(n):
        j=0
        ok = 0
```

```
        while ok==0:
            WorkVet[j] = WorkVet[j]+1
            if WorkVet[j] < base:
                ok = 1
            else:
                WorkVet[j] = 0
                j = j+1
        sequence[i] = np.dot(WorkVet,VetBase)

    return sequence
```

[98]:
```
def Halton_Box_Muller(N,base1,base2):

    H1 = halton_oneD(N//2,base1)
    H2 = halton_oneD(N//2,base2)

    norm_variable = []

    i = 0
    j=N//2

    while i<j:
        u1 = H1[i]
        u2 = H2[i]

        z1 = np.sqrt(-2*np.log(u1))*np.cos(2*np.pi*u2)
        z2 = np.sqrt(-2*np.log(u1))*np.sin(2*np.pi*u2)

        norm_variable.append(z1)
        norm_variable.append(z2)

        i+=1
    return norm_variable
```

[99]:
```
def EuroCall_Halton(s0,k,r,T,N,sigma,base1,base2):
    rv = Halton_Box_Muller(N,base1,base2)
    rv = np.array(rv)
    St = s0*np.exp((r-sigma**2/2)*T + sigma*np.sqrt(T)*rv)

    payoff = np.maximum(St-k, np.zeros(N))
    call = np.exp(-r*T) * np.mean(payoff)

    return call
```

[100]:
```
call_price = EuroCall_Halton(s0=32,k=30,r=0.05,T=0.5,sigma=0.
 ↪24,N=1000,base1=2,base2=5)
print(f"The estimated call price is: {call_price}")
```

```
The estimated call price is: 3.724505550012962
```

[ ]:

```

```