

## Project 4

MGMTMFE 405

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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code clarity and accuracy will determine the grades.

**Submit your codes and a PDF file of your answers to questions (including graphs, histograms, but no codes, in this PDF file) by 3PM PDT on Next Wednesday**

1. Compare the convergence rates of the four methods below by doing the following:

Use the Binomial Method to price a 6-month European Call option with the following information: the risk-free interest rate is 5% per annum; the volatility is 24% per annum; the current stock price is \$32; and the strike price is \$30. Divide the time interval into  $n$  parts to estimate the price of this option. Use  $n = 10, 20, 40, 80, 100, 200$ , and 500 to estimate the price and draw all resulting prices in one graph, where the horizontal axis measures  $n$ , and the vertical one the price of the option.

- (a) Use the binomial method in which

$$u = \frac{1}{d}, \quad d = c - \sqrt{c^2 - 1}, \quad c = \frac{1}{2}(e^{-r\Delta} + e^{(r+\sigma^2)\Delta}), \quad p = \frac{e^{r\Delta} - d}{u - d}$$

- (b) Use the binomial method in which

$$u = e^{r\Delta} \left(1 + \sqrt{e^{\sigma^2\Delta} - 1}\right), \quad d = e^{r\Delta} \left(1 - \sqrt{e^{\sigma^2\Delta} - 1}\right), \quad p = 1/2$$

- (c) Use the binomial method in which

$$u = e^{\left(r - \frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}}, \quad d = e^{\left(r - \frac{\sigma^2}{2}\right)\Delta - \sigma\sqrt{\Delta}}, \quad p = 1/2$$

- (d) Use the binomial method in which

$$u = e^{\sigma\sqrt{\Delta}}, \quad d = e^{-\sigma\sqrt{\Delta}}, \quad p = \frac{1}{2} + \frac{1}{2} \left( \frac{\left(r - \frac{\sigma^2}{2}\right)\sqrt{\Delta}}{\sigma} \right)$$

**Outputs:** Graphs: 4 plots in one graph.

2. Take the current price of AMZN. Use risk-free rate of 1% per annum, and a strike price that is the closest integer to 110% of the current price (divisible by 50). Estimate the price of the American Call option that expires in January of next year, using the Binomial Method. AMZN does not pay dividends. To estimate the historical volatility, use 60 months of historical stock price data on the company. You may use *Bloomberg* or *finance.yahoo.com* to obtain historical prices and the current stock price of AMZN.

- (a) Compare your estimated option price with the one you can get from *Bloomberg* or *finance.yahoo.com* and comment.
- (b) If the two prices in part (a) are different, then find the volatility that would make your estimated price equal to the market price. Comment on your finding (market's expectation) by comparing the historical volatility with the implied volatility.

**Outputs:** Writeup: comments for parts (a) and (b).

3. Consider the following information on the stock of a company and options on it:

$$S_0 = \$49, X = \$50, r = 0.03, \sigma = 0.2, T = 0.3846 \text{ (20 weeks)}, \mu = 0.14.$$

Using the Binomial Method (any one of the parameter choices) estimate the following and draw the graphs:

- (i) Delta of the call option as a function of  $S_0$ , for  $S_0$  ranging from \$20 to \$80, in increments of \$2.
- (ii) Delta of the call option, as a function of  $T$  (time to expiration), from 0 to 0.3846 in increments of 0.01.
- (iii) Theta of the call option, as a function of  $S_0$ , for  $S_0$  ranging from \$20 to \$80 in increments of \$2.
- (iv) Gamma of the call option, as a function of  $S_0$ , for  $S_0$  ranging from \$20 to \$80 in increments of \$2.
- (v) Vega of the call option, as a function of  $S_0$ , for  $S_0$  ranging from \$20 to \$80 in increments of \$2.
- (vi) Rho of the call option, as a function of  $S_0$ , for  $S_0$  ranging from \$20 to \$80 in increments of \$2.

**Outputs:** Graphs: 6 plots all in one graph.

4. Consider 12-month put options on a stock of company XYZ. Assume the risk-free rate is 5%/annum, the volatility of the stock price is 30 % /annum, and the strike price of the option is \$100. Use a Binomial Method to estimate the prices of European and American Put options with current stock prices varying from \$80 to \$120 in increments of \$4. Draw them all in one graph, compare them and comment on your findings.

**Outputs:** i. Graphs: plot in a graph, and ii. writeup: comments.

5. Compare the convergence rates of the two methods, (a) and (b) described below, by doing the following: Use the Trinomial Method to price a 6-month European Call option with the following information: the risk-free interest rate is 5% per annum, the volatility is 24%/annum, the current stock price is \$32, and the strike price is \$30.

Divide the time interval into  $n$  equal parts to estimate the price of this option. Use  $n = 10, 15, 20, 40, 70, 80, 100, 200$  and 500 to estimate the price and draw them all in one graph, where the horizontal axis measures  $n$ , and the vertical one measures the price of the option.

The two methods are in (a) and (b) below:

(a) Use the trinomial method applied to the stock price-process ( $S_t$ ) in which

$$u = \frac{1}{d}, \quad d = e^{-\sigma\sqrt{3\Delta}},$$

$$p_d = \frac{r\Delta(1-u)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(1-d)}, \quad p_u = \frac{r\Delta(1-d)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(u-1)}, \quad p_m = 1 - p_u - p_d$$

(b) Use the trinomial method applied to the Log-stock price-process ( $X_t$ ) in which

$$\Delta X_u = \sigma\sqrt{3\Delta}, \quad \Delta X_d = -\sigma\sqrt{3\Delta}$$

$$p_d = \frac{1}{2} \left( \frac{\sigma^2 \Delta + \left(r - \frac{\sigma^2}{2}\right)^2 \Delta^2}{\Delta X_u^2} - \frac{\left(r - \frac{\sigma^2}{2}\right) \Delta}{\Delta X_u} \right), \quad p_u = \frac{1}{2} \left( \frac{\sigma^2 \Delta + \left(r - \frac{\sigma^2}{2}\right)^2 \Delta^2}{\Delta X_u^2} + \frac{\left(r - \frac{\sigma^2}{2}\right) \Delta}{\Delta X_u} \right), \quad p_m = 1 - p_u - p_d$$

**Outputs:** Graphs: plot in a graph.

6. Use Halton's Low-Discrepancy Sequences to price European Call options. The code should be generic: it will prompt the user to input  $S_0, X, T, r, \sigma, N$  (number of points),  $b_1$  (base 1) and  $b_2$  (base 2). Use the Box-Muller method to generate Standard Normal variates as follows:

$$\begin{cases} Z_1 = \sqrt{-2\ln(H_1)} \cos(2\pi H_2) \\ Z_2 = \sqrt{-2\ln(H_1)} \sin(2\pi H_2) \end{cases}$$

where  $H_1$  and  $H_2$  will be the Halton's numbers with base  $b_1$  and base  $b_2$  respectively.

For the call option price you may use the following formula:

$$C = Ef(W_T) = e^{-(rT)} E \left( \max \left( 0, S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma W_T} - X \right) \right)$$

**Inputs:**  $S_0, X, T, r, \sigma, N, b_1, b_2$

**Outputs:** Values:  $C$

7. *[Optional –NOT for grading]* How much are you willing to pay to play this game: You toss a fair coin. If it is a Tail then you get \$7 in 18 months. If it is a Head then you lose \$2 immediately. The one and two-year zero-coupon rates are 4% and 6% respectively. Would the amount you are willing to pay to play this game increase or decrease if the payoff (in case of Tails) happens in 36-months?
8. *[Optional –NOT for grading]* Value an American Put option that has no maturity (perpetual option). What's the delta of the option if it is at-the-money?