

## Chapter 9. Pricing Asset Backed Securities (ABS) (v.A.2)

In this chapter, we will provide some techniques for pricing Asset Backed Securities (ABS), particularly Mortgage Backed Securities (MBS). Consider a pool of mortgages. The constituents of this pool may have different characteristics, but for tractability, we will assume there is a single lump-sum loan in the pool that has the weighted-average characteristics of the pool of mortgages.

Assume the value of the mortgage at time 0 is  $PV_0$ . Let  $PMT$  be the scheduled (monthly) mortgage payment;  $R$  be the APR (Annual Percentage Rate) of the mortgage; and  $n$  be the number of periods ( $n = 360$  in the case of the constant payment mortgage). Then, the mortgage rate per period (which is a month in this case) is  $r = \frac{R}{12}$ .

In the case of the fixed-rate mortgage, the payments are constant and they form an Ordinary Annuity. The mortgage payments, the original mortgage balance, the monthly mortgage rate, and the number of periods are related by this simple relationship:

$$PV_0 = \frac{PMT}{1+r} + \dots + \frac{PMT}{(1+r)^n} = \frac{PMT}{r} \cdot \left[ 1 - \frac{1}{(1+r)^n} \right]$$

Notice that, this is the formula for finding the present value of an ordinary annuity.

If we assume that the payments grow at the constant rate of  $g$  per period, then, the present value formula would be given as follows:

$$PV_0(g) = \frac{PMT(1+g)}{1+r} + \dots + \frac{PMT(1+g)^n}{(1+r)^n} = \frac{PMT(1+g)}{r-g} \cdot \left[ 1 - \frac{(1+g)^n}{(1+r)^n} \right]$$

Thus, in the case of a constant payment mortgage,  $g = 0$ , and we have

$$PMT = \frac{PV_o \cdot r}{\left[1 - \frac{1}{(1+r)^n}\right]}$$

Generalizing this formula, one can derive a formula for the payment for any period  $t$ , which depends on the outstanding mortgage balance at time  $(t - 1)$ , which is the time when the payment for period  $t$  will be made:

$$PMT_t = \frac{PV_{t-1} \cdot r}{\left[1 - \frac{1}{(1+r)^{n-(t-1)}}\right]}$$

for  $t = 1, 2, \dots, 360$ .

The latter formula can be used to compute not only the payments for the fixed-rate mortgages (the classic mortgage) but also payments for not-traditional mortgages, such as adjustable-rate mortgages (ARMs).

For example, to calculate the monthly payments for a 5/1 ARM, one calculates the monthly payments using the fixed rate for the initial 5 years, assuming that the same payment will be made for the entire 30 year period. After the initial 5 years, or after 60 payments, the mortgage rate will reset to a new one (adjustable, and normally benchmarked to the LIBOR or the 3-m US TSY Bill rates). After the rate reset, one will compute the monthly payments using the new rate for the remaining 300 months. That payment, however, will be made for only 1 year (12 payments). After that one year, the rate will reset again and the calculations will be redone for the next – 7<sup>th</sup> – year. The process will continue until the end of the 30 years.

In general, denote by  $PMT_t$  the monthly payment for the month  $t$ , by  $Interest_t$  the interest portion, and by  $mortization_t$  the principal amortization portion of  $PMT_t$ .

Then,

$$Interest_t = PV_{t-1} \cdot r, \text{ and } Amortization_t = PMT_t - Interest_t$$

### **Mortgage – Imbedded Options**

Majority of mortgages in the US have no pre-payment penalties and carry some kind of default insurance. The borrower, in these mortgages, has the right to prepay (in full or partially) the mortgage's balance prior to its maturity. This right (the prepayment option) is frequently exercised by the borrowers for several reasons, such as the borrower mobility, or lower mortgage rates (refinancing), etc. One of the most important causes of prepayment is the mortgage refinancing due to lower mortgage rates. When mortgage rates are lower, the borrower has financial incentives to pay off the existing mortgage and refinance it with another lender at a lower rate. This type of option (the prepayment option) exercise (in which you pay upon the exercise of the option) is typical to “call-type” options. However, call-type options are exercised when the value of the underlying increases. Thus, the prepayment option is a call-type option, but the underlying of that option is not the mortgage rate, but the value of the mortgage. The mortgage value increases as interest rates (mortgage rates) decrease. The borrower (who has sold the mortgage to the lender) has the right to buy the mortgage back (this right is the prepayment option), which is exercised if the value of the mortgage increases.

Thus, the underlying security (for this option) is the value of the mortgage and not the interest or mortgage rate.

Most mortgages in the US are non-recourse ones, which are of “limited- liability” type. That is, the borrower has the right to give up the property and not to pay back the remaining portion of

the mortgage. The lender can seize the property in case of a default. This right of the borrower can also be viewed as an option. In this case, this “option” is exercised (that is, the borrower defaults on the mortgage and stops paying) if the value of the property declines so much that it becomes much lower than the outstanding loan balance on the property. Upon exercise of this option, the borrower essentially “sells” the property for the amount (outstanding mortgage balance) owed on it. That is, the borrower “gives up” the property (think of it as “selling” it) and in return, the outstanding mortgage balance is nullified (think of it as “getting paid” the remaining mortgage balance). That is, the borrower was supposed to pay the remaining mortgage balance off, but now that mortgage-liability is released in return for forfeiting the property. Thus, the property was “sold” for exactly what the outstanding mortgage balance was.

Thus, this right looks like a put option as the borrower “sells” something and “gets paid”. The underlying security for this option is the value of property.

Thus, there are two imbedded options in most mortgages in the US:

- The prepayment option (“call-type” option where the underlying is the mortgage value);
- The default option (“put-type” option where the underlying is the property value.)

Pricing a Constant Payment Mortgage (the classic mortgage in the US), without the two above-mentioned imbedded options, is like pricing an ordinary annuity, the valuation and analysis of which is a simple task. However, in reality, it is not as easy to value mortgages or pools of mortgages, when the prepayment and default possibilities are taken into account. In what follows, we will price mortgage pools in certain special cases.

As is known, the price of a security is the present value of its expected cash flows (under the risk-neutral measure):  $P_o = E^*(\sum_{t=1}^N PV(CF_t)) = E^*(\sum_{t=1}^N d_t \cdot c_t)$ .

In order to find this price we need to be able to have expressions/formulas for:

(1) The discount factor:  $d_t = \exp\left(-\int_0^t r_u du\right)$  where  $r_u$  is the short-term rate at time  $u$ .

(2) The cash flows:  $c_t$  = cash flow at time  $t$ .

Below we make few notations and review a few properties of ordinary annuities.

Define:

$SP_t$ : Scheduled Principal payment for month  $t$ .

$PP_t$ : Prepayment for month  $t$ .

$TPP_t$ : All Principal Payment, which is Scheduled Principal+ Prepayment, for month  $t$ .

$MP_t$ : Scheduled Mortgage Payment for month  $t$ .

$IP_t$ : Interest Payment for month  $t$ .

$c_t$ : Total Payment, which is Total Principal Payment ( $TPP_t$ ) + Interest Payment ( $IP_t$ ); or

Scheduled Principal ( $SP_t$ ) + Prepayment ( $PP_t$ ) + Interest Payment ( $IP_t$ ).

Thus,  $c_t = SP_t + PP_t + IP_t = TPP_t + IP_t = MP_t + PP_t$ .

Let  $PV_t$  be the principal balance of the mortgage at time  $t$ . ( $PV_0$  is the balance of the mortgage at origination).

The following calculations follow from simple Ordinary Annuity valuation formulas:

- $IP_t = PV_{t-1} \cdot r$  ( $r$  is the rate per month which for a pool of mortgages is calculated as:

$$r = \frac{\text{Weighted Average Coupon}}{12} = \frac{WAC}{12}$$

- $MP_t = \frac{PV_{t-1} \cdot r}{\left[1 - \frac{1}{(1+r)^{N-t+1}}\right]}$

**Note:** Weighted Average Maturity (WAM) is assumed to be  $N$  at origination.

- $PV_t = PV_{t-1} - TPP_t$

Now define pool prepayments:

**Definition 1:** Single Monthly Mortality (SMM) is defined to be the fraction of the beginning-of-month balance that is prepaid during the month  $t$ :

$$SMM_t = \frac{PP_t}{PV_{t-1} - SP_t}$$

Now, define the annualized version of the SMM, which is called the **Conditional Prepayment Rate (CPR)**, as follows:

**Definition 2:** Conditional Prepayment Rate (CPR) is the annualized version of the SMM and is defined as follows:

$$CPR_t = 1 - (1 - SMM_t)^{12}$$

Thus,  $SMM_t = 1 - (1 - CPR_t)^{\frac{1}{12}}$ .

We have

$$PP_t = (PV_{t-1} - SP_t)SMM_t = (PV_{t-1} - SP_t) \left(1 - (1 - CPR_t)^{\frac{1}{12}}\right)$$

Assume that we have an explicit formula for the  $CPR_t$  (to be specified later). Then,

$$c_t = SP_t + PP_t + IP_t = MP_t + PP_t = \frac{PV_{t-1} \cdot r}{\left[1 - \frac{1}{(1+r)^{N-(t-1)}}\right]} + (PV_{t-1} - SP_t) \left(1 - (1 - CPR_t)^{\frac{1}{12}}\right)$$

Since

$$SP_t = MP_t - IP_t = \frac{PV_{t-1} \cdot r}{\left[1 - \frac{1}{(1+r)^{N-(t-1)}}\right]} - PV_{t-1} \cdot r = PV_{t-1} \cdot r \left[ \frac{1}{1 - \frac{1}{(1+r)^{N-(t-1)}}} - 1 \right]$$

Then,

$$c_t = \frac{PV_{t-1} \cdot r}{\left[1 - \frac{1}{(1+r)^{N-(t-1)}}\right]} + \left( PV_{t-1} - PV_{t-1} \cdot r \left[ \frac{1}{1 - \frac{1}{(1+r)^{N-(t-1)}}} - 1 \right] \right) \left(1 - (1 - CPR_t)^{\frac{1}{12}}\right)$$

Assuming that an explicit formula for the  $CPR_t$  is provided, we have an explicit formula for  $c_t$ .

#### Notes:

1. The Interest portion,  $IP_t$ , of  $c_t$  is simply computed as:  $IP_t = PV_{t-1} \cdot r$ .
2. The principal amortization portion,  $TPP_t$ , of  $c_t$  is given by:

$$TPP_t = SP_t + PP_t =$$

$$PV_{t-1} \cdot r \left[ \frac{1}{1 - \frac{1}{(1+r)^{N-(t-1)}}} - 1 \right] + \left( PV_{t-1} - PV_{t-1} \cdot r \left[ \frac{1}{1 - \frac{1}{(1+r)^{N-(t-1)}}} - 1 \right] \right) \left(1 - (1 - CPR_t)^{\frac{1}{12}}\right)$$

Notice that, the total principal amortization portion  $T\mathbf{PP}_t$  has two components: the Scheduled Principal Amortization( $\mathbf{SP}_t$ ), and the Prepayment ( $\mathbf{PP}_t$ ).

The big question now is: *what is an appropriate model for CPR?*

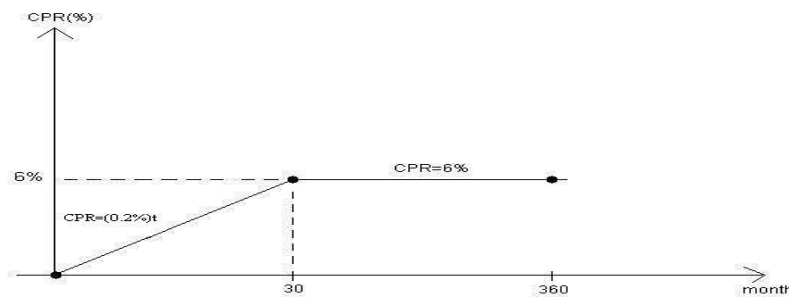
Below we consider two (relatively) simple prepayment models, or models of CPR.

### Models of CPR

We will consider two models for CPR – the Public Securities Association’s Model, and a model developed by Numerix.com.

#### Public Securities Association’s Model (PSA)

Define  $CPR_0 = 0$  and 
$$CPR_t = \begin{cases} CPR_{t-1} + 0.2\% & \text{for } t \leq 30 \text{ months} \\ CPR_{t-1} & \text{for } t > 30 \text{ months} \end{cases}$$



**Figure 1:** Conditional Prepayment Rate of the Public Securities Association’s Model



Most quotes for CPR (e.g. Bloomberg quotes or other market quotes) are given as percentages of PSA.

For example, The CPR for the MBS XYZ for the month of June is 170% PSA. That means whatever PSA prepayment rate would be for an MBS in its  $k - th$  month, the projected prepayment rate for XYZ is 170% of that. In other words, assume the MBS XYZ is currently in its 14<sup>th</sup> month of origination and June is its 15<sup>th</sup> month. Then, the CPR for June, the 15<sup>th</sup> month, is projected to be 170% of  $(0.2\% \times 15) = 170\% \text{ of } 3\% = 5.1\%$

### **Numerix-Prepayment Model** ([www.numerix.com](http://www.numerix.com))

We use a 4- factor model of CPR where the factors are:

- Interest Rate. Lower rates give more financial incentives to borrowers to refinance their mortgages. Therefore lower interest rates imply higher prepayment speeds.
- Burnout. The CPR depends on the path taken in the pool of mortgages. When faced with refinancing opportunities, most aware/savvy borrowers react promptly and prepay/payoff their mortgaged and exit the pool of mortgages. After a while, only those borrowers are left in the pool that will not refinance even if they are faced with larger financial incentives.
- Seasonality. Prepayments are faster during Spring and Summer months than during Fall and Winter. This is due to the mobility of homeowners.
- Seasoning. This factor is also referred to as pool aging. Prepayment rates are lower right after issuance of mortgages, but the prepayment speeds increase over time. This is

because family situations are unlikely to change shortly after the purchase of a house, but they do change after a relatively longer periods of time.

Thus, we model CPR using the 4 factors listed above as follows:

$$CPR_t = (Refi\ Incentive_t)(Burnout_t)(Seasoning_t)(Seasonality_t)$$

or

$$CPR_t = (RI_t)(BU_t)(SG_t)(SY_t)$$

Using historical data and calibrating the above 4-factor model, Numerix offers the following parameters for the 4-factor model of CPR:

- $RI_t = 0.28 + 0.14 \cdot Arctan(-8.57 + 430(R - r_{t-1}(10)))$

where  $R$  is the Mortgage Rate ( $R = 12r$ );

$r_{t-1}(10)$  is the 10-year US TSY Note Yield, observed of the end of  $t - 1$ .

This rate  $- r_{t-1}(10)$  —is a proxy for 30-year fixed mortgage rate.<sup>1</sup>

- $BU_t = 0.3 + 0.7 \frac{PV_{t-1}}{PV_0}$
- $SG_t = \min(1, \frac{t}{30})$
- $SY_t = \{0.94, 0.76, 0.74, 0.95, 0.98, 0.92, 0.98, 1.10, 1.18, 1.22, 1.23, 0.98\}$   
for {Jan, Feb, ... , Dec}, respectively.

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<sup>1</sup> See the historical comparisons of the two rates.

**Comment:** The 10-year risk-free rate,  $r_{t-1}(10)$ , observed at the end of period  $(t - 1)$  can be estimated as follows:

Along each path  $i$ , at time  $(t - 1)$ , we have that the spot rate is given by this formula:

$$r_{t-1}(T)^i = -\frac{1}{T} \ln(P_{t-1}(T)^i) = \frac{1}{T} \int_{t-1}^{t-1+T} r_u^i du$$

Here, one can either use the explicit formula for the price of the bond to estimate the spot rate (if explicit formula exists for the model of rates under consideration), or estimate the spot rate by simulations of the interest rate path for  $T=10$  years out from  $(t - 1)$ .

### **Tranching – CDOs, CMOs, etc.**

In this section, we will consider tranches of securities backed by debt (CDO) or mortgages (CMO).

A CMO structure is a mechanism for reallocating cash flow from a pool of mortgages into multiple classes (tranches) with different priority claims. There are two major types of CMOs:

- 1) Only principal payments are passed through various tranches (examples are: Sequential Pay, PAC – companion, and TAC – companion).
- 2) All payments are passed through various tranches.

The three structures in the first type of tranching, in which only the principal payments are passed through, are described below.

- **Sequential Pay** – this is the most basic structure: A, B,..., Z tranches are created with priority of getting principal payments and prepayments. All tranches receive interest payments (except for the tranche Z), and all the prepayments and principal payments flow to the tranche with the highest priority of cash flows.

Thus, in this case, tranche A will have lower duration than tranche B, tranche C, etc.

- **PAC (Planned Amortization Class)** –in this structure, fixed principal payments are scheduled to be received. Here are a few features of this structure.
  - Buyers of the PAC reduce the Prepayment Risk.
  - Fixed Principal payments are scheduled to be received if the prepayment rate is within a range, [L, U].
  - If the prepayment rates are higher, ( $>U$ ), then PAC's payments will be more.
  - If prepayment rates are lower ( $<L$ ), then PAC's payments are delayed.
  - **TAC (Targeted Amortization Class)** –in this structure, the prepayment risk is reduced as well. Here are a few features of this structure.
    - These, like PACs, offer some protection against the prepayment risk.
    - Here, a single prepayment rate is targeted.
    - The payment schedule is met if the actual prepayment speed is greater than the targeted prepayment rate.

- The excess CF (in case of a higher than targeted speeds) are directed to companion classes.
- If the actual prepayment rate is less than the targeted prepayment rate, then the WAL will be extended.

The two most popular structures of the second type of tranching, are the Principal-Only or Interest-Only tranches, or the **IOs** and **POs**.

- **IOs and POs**

In this case, the behavior of the IO or PO tranches as fixed income securities are substantially different from each other: The prices of PO tranches are inversely related to mortgage rates, like vast majority of fixed income securities, but the prices of IO tranches are directly related with the mortgage rates – unlike most fixed income securities. This property makes the IOs ideal hedge-instruments in fixed income.

Below we will discuss a few techniques that are used in MBS valuation.

In the **Static Valuation method**:

A constant rate is used for discounting across all maturities:  $r_t = r_o = \text{const}$  for  $\forall t$ . This is also referred to as the **0-volatility Option- Adjusted Spread (OAS)** method.

Below we define and explain the Option-Adjusted Spread (OAS).

The value of a fixed income security (including mortgage-backed securities) is usually thought of as the sum of its discounted payments. The payments scheduled to be paid at different times are discounted with different interest rates, the so-called the zero-coupon interest rates. The

Treasury and LIBOR curves are the two most popular discount curves used to get the zero-coupon rates and value fixed income securities. We will first define the Z-Spread for Treasury securities, then for Corporate Securities, and finally apply the concept to define the OAS.

### **The Z-spread**

If we value a Treasury (or Corporate) security by discounting its coupons and its Par Value with the zero-coupon Treasury rates, we normally get a value that is “close” to the observed market price of the security, but not exactly equal to that price.

However, we would like the estimated value to match the market price. Therefore, we ask ourselves the following question:

*What magnitude of parallel shift in the curve of zero-coupon interest rates will result in an estimated value of a security that equals its market price?*

The answer to this question is the **Z-spread** of the security under consideration.

When estimating the Z-Spread for a Corporate Bond, one assumes that there is no default risk. That is, the calculations of a Z-Spread for Treasury Securities is the same as the one for Corporate Bonds.

However, because of the non-trivial credit risk of corporate bonds, the Z-Spread for corporate bonds will be higher than the one of the Treasuries. The higher the credit risk the higher the Z-spread of Corporates to Treasuries.

For Callable Bonds (and for the MBS, which have similar embedded callability feature) the use of a Z-Spread may not be appropriate because of the pricing methods. To value a callable bond

properly we need to use a stochastic model of interest rates (which takes into account the volatility of interest rates) so that the existence of the embedded call option is taken into account.

Assume that the short-term interest rate follows a stochastic model (Vasicek, CIR, Black-Karasinski, etc.).

**Definition.** The OAS (or option adjusted spread) is the constant parallel shift to the zero-coupon interest rates that is needed to ensure that the model-implied estimated value of the bond equals its market price.

**Comment:** For bonds without embedded options, the OAS is the same as the Z-spread (when adjusting for day-count and compounding conventions, etc.).

To better understand callable bond pricing and its OAS, consider separating a callable bond into the bond stripped of the option and a short position in a call option on the stripped bond. We will compare a callable bond to a portfolio with positions in two hypothetical securities:

- A bond stripped of its embedded call option.
- A call option on the stripped bond with the same call schedule as the callable bond.

We will compare a long position in the callable bond with a long position in the stripped bond and a short position in the call option.

Consider the following position:

*Long the callable bond,*

*Short the stripped bond,*

*Long the call option.*

This is a hedged portfolio (net cash flow is zero for all periods) for a Long Callable Bond in all states of the world - all possible interest rates scenarios, in case of credit events (default), or in case when the call option (in the bond) is exercised.

This implies that, the callable bond should be as expensive as the (theoretical) value of the stripped bond minus the (theoretical) value of the call option.

That is:

**Market Price of Callable Bond  $\geq$  Value of Stripped Bond – Value of Call Option**

Is it possible to make a similar statement for the Short Callable Bond?

That is, is it possible to find a perfect hedge for the Short Callable Bond?

Consider the following position:

*Short the callable bond,*

*Long the stripped bond,*

*Short the call option.*

This will be a perfectly hedged position as long as the call option (that was shorted) is not exercised.

However, if the call option is exercised by its holder, then, one would need to call the callable bond, but it can be done only by the long position holder. So, the hedge may not work (because the holder of Long Callable Bond may not exercise the call option at the same time).



Issuing a new bond can be costly and the refinancing cost raises the call price for the bond issuer and thus, reduces the value of the embedded call option.

This is why the **callable bond should be worth more than the (theoretical) value of the stripped bond, minus the (theoretical) value of the call option on the stripped bond.**

In our OAS estimation, we assume that the issuer will call the bond exactly when it is optimal for an outside investor to exercise the call option on the stripped bond. With this assumption, the value of the callable bond becomes **equal to** the value of the stripped bond minus the value of the call option.

That is, our assumption implies:

**Market Price of Callable Bond = Value of Stripped Bond – Value of Call Option**

In MBS-valuation, we set the discount factor to be  $d_t = e^{-\int_0^t r_s ds}$ . With this discount factor, the estimated (or model-implied) price is not always equal to the market price of the security.

Define **OAS** =  $x$  to be the spread over the interest rate such that if we add it to all rate paths, (i.e. make a constant upward shift of interest rates of all maturities), the model-implied price of the MBS will equal to its Market Price. That is, we solve the equation for  $x$  so that:

$$\text{Market Price} = E^Q(C_t \cdot e^{-\int_0^t (r_s + x) ds})$$

Where **Market Price** is the observed price of the MBS (a callable bond). The OAS= $x$  is estimated numerically by using the above equation.

Let  $P$  be the price of the MBS. Define Duration and Convexity for the MBS as follows:

$$D = \text{Duration} = -\frac{1}{P} \frac{\partial P}{\partial y}, \quad C = \text{Convexity} = \frac{1}{P} \frac{\partial^2 P}{\partial y^2}$$

We know that, by taking the first two terms of the price expansion we can write

$$\frac{dP}{P} \approx -D \cdot dy + \frac{1}{2} C (dy)^2$$

This approximate formula can be used as a tool to estimate a fixed income portfolio's interest-rate risk exposure, etc.

Computations of  $D$  and  $C$  have significant importance for understanding and hedging risks of MBS securities. In practice, other measures of duration and convexity are used, which are referred to as the OAS-Duration and the OAS-Convexity.

The OAS-Duration and the OAS-Convexity are defined and estimated as follows:

**OAS – Duration and OAS –Convexity:**

- 1) Compute the OAS of the MBS. Call it  $x$ . Let the market price of the MBS be  $P_o$ .
- 2) Shift the OAS=  $x$  by  $\pm y$  basis points and compute the prices of the MBS with  $OAS = x + y$  and  $OAS = x - y$ . Call these prices  $P_+$  and  $P_-$ , respectively.

**Note:** Normally,  $y = 5bps$  is used.

- 3) Define **OAS-Duration** as follows:  $\frac{P_- - P_+}{2y \cdot P_o}$
- 4) Define **OAS-Convexity** as follows:  $\frac{P_+ + P_- - 2P_o}{2P_o y^2}$ .

## Exercises:

Consider a 30-year MBS with a fixed  $WAC = 8\%$  (monthly cash flows, starting in January). The Notional Amount of the Loan is \$100,000. Use the CIR model of short-term interest rates  $dr_t = \kappa(\bar{r} - r_t)dt + \sigma\sqrt{r_t}dW_t$  with  $r_0 = 0.078, k = 0.6, \bar{r} = 0.08, \sigma = 0.12$ .

1. Consider the *Numerix-Prepayment Model*.
  - (a) Compute the price of the MBS using this model for prepayments. The code should be generic: the user is prompted for inputs and the program runs and gives the output.
  - (b) Compute the price of the MBS for the following ranges of the parameters:  $k$  in 0.3 to 0.9 (in increments of 0.1) and draw the graph of the price vs.  $k$ .
  - (c) Compute the price of the MBS for the following ranges of the parameters:  $\bar{r}$  in 0.03 to 0.09 (in increments of 0.01) and draw the graph of the price vs.  $\bar{r}$ .
2. Consider the *PSA Model* of prepayments.
  - (a) Compute the price of the MBS using the PSA model for Prepayments. The code should be generic: the user is prompted for inputs and the program runs and gives the output.
  - (b) Compute the price of the MBS for the following ranges of the parameters:  $k$  in 0.3 to 0.9 (in increments of 0.1) and draw the graph of the price vs.  $k$ .
3. Compute the Option-Adjusted-Spread (*OAS*) for the Numerix-Prepayment model case with the Market Price of MBS being \$110,000.
4. Compute the *OAS-adjusted Duration and Convexity* of the MBS, considered in the previous question.

5. Consider the MBS described above and the IO and PO tranches. Use the *Numerix-Prepayment Model* and price the IO and PO tranches for:  $\bar{r}$  in 0.03 to 0.09 range, in increments of 0.01.
6. Which is more expensive: (1) A payoff of \$1 if XYZ stock price (that trades at \$15/share today) hits \$20 (at any time in the future); or (2) A payoff of \$1 if ARP stock price (that trades at \$24) hits \$32? Assume  $r = 0$ . Justify your answer.