Project 7

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code clarity and accuracy will determine the grades.

Submit your codes and a PDF file of your answers to questions (including graphs, histograms, but no codes, in this PDF file) by 3PM PDT on Next Wednesday

1. Consider the following information on the stock of company XYZ: The volatility of the stock price is $\sigma = 20\%$ per annum. Assume the prevailing risk-free rate is $r = 4\%$ per annum. Use the $X = ln(S)$ transformation of the Black-Scholes PDE, and $\Delta t = 0.002,$ with $\Delta X = \sigma\sqrt{\Delta t};\ then\ with\ \Delta X = \sigma\sqrt{3\Delta t};\ then\ with\ \Delta X = \sigma\sqrt{4\Delta t}$, and a *uniform* grid (on X) to price a European Put option with strike price of $K = \$10$, expiration of 0.5-years, and current stock prices ranging from \$4 to \$16; using the specified methods below:

(a) Explicit Finite-Difference method,
(b) Implicit Finite-Difference method,
(c) Crank-Nicolson Finite-Difference method.

Inputs: $K, \sigma, T, \Delta t$
Outputs:

i. Values: Pa, Pb and Pc for the European Put option using each of the methods (a), (b) and (c).
ii. Writeup: compare the three methods from (a), (b) and (c) and comment. To compare, calculate the relative error with respect to the prices derived from Black-Scholes-Merton formula. Do this for current stock prices of \$4 to \$16 in \$1 increments and put them in a table. Put the table and your comments in a .pdf file.

2. Consider the following information on the stock of company XYZ: The volatility of the stock price is $\sigma = 20\%$ per annum. Assume the prevailing risk-free rate is $r = 4\%$ per annum. Use the Black-Scholes PDE (for S) to price American Call and American Put options with strike prices of $K = \$10$, expiration of 0.5-years, and current stock prices for a range from \$4 to \$16; using the specified methods below:

(a) Explicit Finite-Difference method,
(b) Implicit Finite-Difference method,
(c) *Crank-Nicolson Finite-Difference method.*

Choose $\Delta t = 0.002$, with $\Delta S = 0.5$, or with $\Delta S = 1$.

Inputs: $K, \sigma, T, \Delta t$
Outputs:

i. Values: Ca, Cb, Cc, Pa, Pb and Pc and for the American call and put options using each of the methods (a), (b) and (c).
ii. Graphs: Plot the American Call option price as a function of the current stock price from \$4 to \$16 in \$1 increments for methods (a), (b) and (c) on the same graph. Use a color legend or linestyles to differentiate the plots. Do the same for the American Put option in another graph. Place the two graphs in a .pdf file.

3. ***[Optional-NOT for grading]*** Consider the following situation on the stock of company XYZ: The current stock price is \$10, and the volatility of the stock price is $\sigma = 20\%$ per annum. Assume the prevailing risk-free rate is $r = 4\%$ per annum. Assume we are in the Black-Scholes framework. Consider the following options:
American Put options with strike price of $K = \$10$, maturity of 0.5-years, and current stock prices for a range from \$6 to \$14, in increments of \$0.5.

   (a) *Use binomial/trinomial* grid to price above-mentioned options. Use $\Delta t = 0.002, u = e^{\sigma\sqrt{0.25\Delta t}}, or\ u = e^{\sigma\sqrt{\Delta t}},\ or\ u = e^{\sigma\sqrt{4\Delta t}}$.
   (b) Use the *Explicit Finite-difference method*, the *Implicit Finite-Difference Method*, and *Crank-Nicolson Finite-Difference Method* to price to price above-mentioned options.
   (c) Use the *LSMC method* to price the above-mentioned options.

4. ***[Optional-NOT for grading]*** Consider the following situation on the stock of company XYZ: The current stock price is \$10, and the volatility of the stock price is $\sigma = 20\%$ per annum. Assume the prevailing risk-free rate is $r = 4\%$ per annum. Use the transformation of the Black-Scholes PDE to the **Heat Equation**, and $\Delta t = 0.002$, $\Delta X = \sqrt{4\Delta t}$, or $\Delta X = \sqrt{2\Delta t}$, or $\Delta X = \sigma\sqrt{1.9\Delta t}$, and a *uniform* grid to price the following options using the specified methods below:

   Use the *Explicit Finite-difference method*, the *Implicit Finite-Difference Method*, and *Crank-Nicolson Finite-Difference Method* to price an American Call and Put option with strike price of $K = \$10$, maturity of 0.5-years, and current stock prices for a range from \$6 to \$14, in increments of about \$0.5.