Project 1_ Kaiyue Wu

1. Use the Random Number generators discussed in the class to do the following:

   (a) Using the LGM method generate 10,000 Uniformly distributed random numbers on [0,1] and compute the empirical mean and the standard deviation of the sequence.

   In Q1(a), X_0 is set to 6

   mean : 0.5011609332500776

   standard deviation : 0.29044406111428883

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LGM mean is 0.5011609332500776, LGM standard deviation is 0.29044406111428883
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   (b) Use built-in functions of the software you are using to do the same thing as in (a).

   For the built-in function, I empoly **np.random.uniform** and the result is

   mean is 0.49942371265383917

   deviation is 0.2891098321026936

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Built-in function mean is 0.49942371265383917, Built-in function deviation is
0.2891098321026936
```

   (c) Compare your findings in (a) and (b) and comment (be short, but precise).

   Both mean and standard deviation from the two methods are very close to theorical value that means are around 0.5 and standard deviations are around 0.289.

2. Use the numbers of part (a) of question 1 to do the following:

   (a) Generate 10,000 random numbers with the following distribution:

   $-1 \ with \ probability \ 0.30 \ 0 \ with \ probability \ 0.35 \ 1 \ with \ probability \ 0.20 \ 2 \ with \ probability \ 0.15$

   (b) Draw the histogram and compute the empirical mean and standard deviation of the sequence of 10,000 numbers generated in part (a).

   mean : 0.2038

   standard deviation : 1.0303715640486202

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Empirical mean is 0.2038, Empirical standard deviation is 1.0303715640486202
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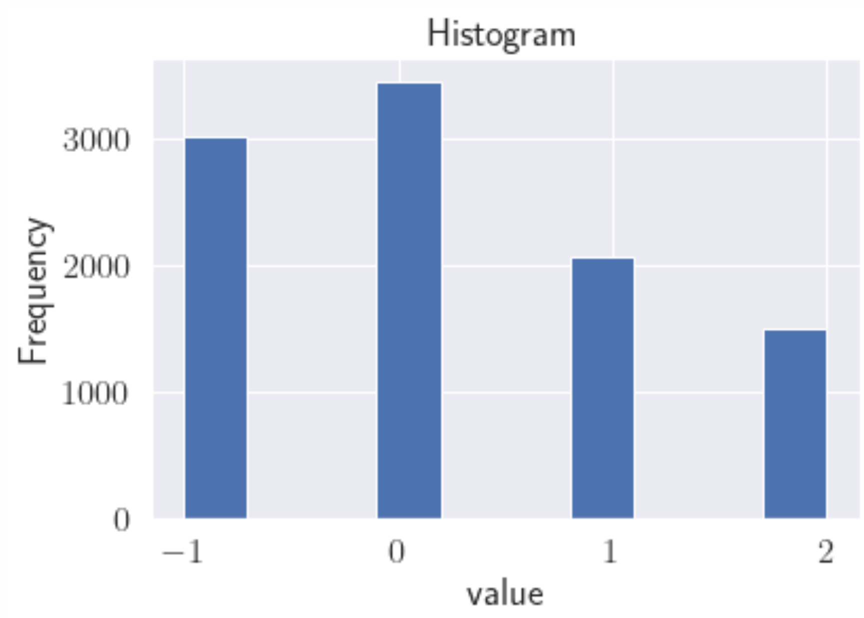
   The plot is

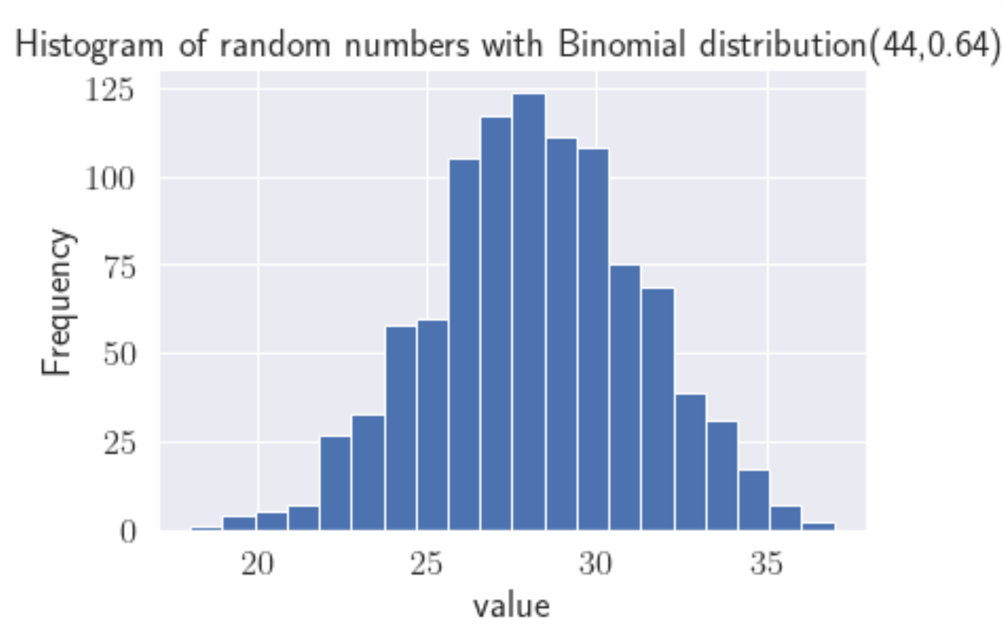


3. Using the LGM method generate Uniformly distributed random numbers on [0,1] to do the following:

   (a) Generate 1,000 random numbers with Binomial distribution with $n = 44$ and $p = 0.64$. (*Hint*: A random variable with Binomial distribution (n, p) is a sum of n Bernoulli (p) distributed random variables, so you will need to generate 44,000 Uniformly distributed random numbers, to start with).

   (b) Draw the histogram. Compute the probability that the random variable X that has Binomial (44, 0.64) distribution, is at least 40: $P(X \geq 40)$. Use any statistics textbook or online resources for the exact number for the above probability and compare it with your finding and comment.

   The plot is



The probability that random variable X has Binomial (44,0.64) distribution P(x>=40) is : 0.0, however, the probability for the exact number should be 0.000048241

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The probability of x >= 40 is 0.0
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P(x>=40) = P(x=40) + P(x=41) + P(x=42) + P(x=43) + P(x=44)

= 0.00004029 + 0.000006987 + 0.0000008873 + 0.00000007337 + 0.000000002964

= 0.000048241

The possible explaination might be we only have 1000 sample, which is relatively samll so that we can's observe x>=40. We cna improve the sample size to test if the possibility wiuld change.

4. Using the LGM method generate Uniformly distributed random numbers on [0,1] to:
   (a) Generate 10,000 Exponentially distributed random numbers with parameter $\lambda = 1.5$.

   For problem 4, I set x_0 in LGM to be 6.

   (b) Compute $P(X \geq 1)$ and $P(X \geq 4)$.

   $P(X \geq 1)$: 0.2256

   $P(X \geq 4)$: 0.0032

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The probability of x >= 1 is 0.2256, The probability of x >= 4 is 0.0032
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   (c) Compute the empirical mean and the standard deviation of the sequence of 10,000 numbers generated in part (a). Draw the histogram by using the 10,000 numbers of part (a).

   mean : 0.6673470598512684

   standard deviation : 0.6751481072709264

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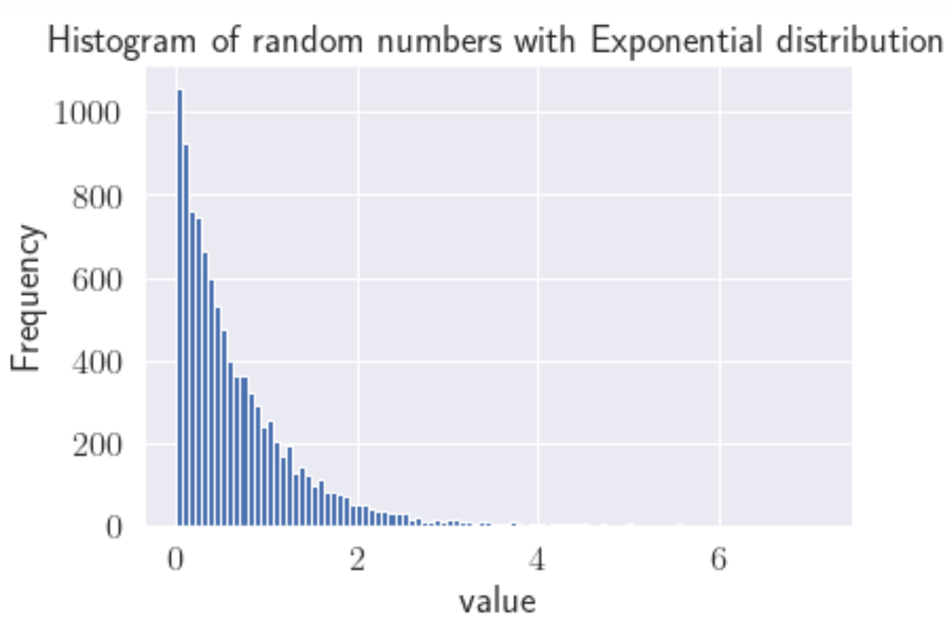
Empirical mean is 0.6673470598512684, Empirical standard deviation is
0.6751481072709264
```

   The plot is



5. Using the LGM method generate Uniformly distributed random numbers on [0,1] to:

(a) Generate 5,000 Normally distributed random numbers with mean 0 and variance 1, by **Box- Muller** Method.

(b) Now use the **Polar-Marsaglia** method to Generate 5,000 Normally distributed random numbers with mean 0 and variance 1.

(c) Now compare the efficiencies of the two above-algorithms, by comparing the execution **times** to generate 5,000 normally distributed random numbers by the two methods. Which one is more efficient? If you do not see a clear difference, you need to increase the number of generated realizations of random variables to 10,000, 20,000, etc.

BM mean: -0.020994735561742948

BM standard deviation: 1.0104714556847032

BM execution time: 0.03935813903808594

PM mean: -0.03441971190072053

PM standard deviation: 0.9988391532719084

PM execution time: 0.030804872512817383

As we can see, the Polar-Marsaglia method has less execution time than Box- Muller Method, indicating it is more efficient becuase not every pair is used in the algorithm.

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The mean for Box- Muller Method normal distributed r.v is -0.020994735561742948
The mean for Polar-MarsagliaMethod normal distributed r.v is -0.03441971190072053

The standard deviation for Box- Muller Method normal distributed r.v is
1.0104714556847032
The standard deviation for Polar-Marsaglia Method normal distributed r.v is
0.9988391532719084

The execution time for Box- Muller Method is 0.03935813903808594
The execution time for Polar-Marsaglia method is 0.030804872512817383
```