Project 3 - Kaiyue Wu

1. Evaluate the following expected values and probabilities:

Inputs: X_0, Y_0

Outputs:

1. **P(Y2 > 5) = 0.97800**
2. **e1 = 0.645733**
3. **e2 = 26.30914**
4. **e3 = 3.893022**

	Question	Answer
0	P(y2>5)	0.978000
1	e1	0.645733
2	e2	26.309194
3	e3	3.893022

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2. Estimate the following expected values:

Inputs: X_0

Outputs:

1. **e1 = 0.978**
2. **e2 = 0.645733**

Question	Answer
e1	0.978000
e2	0.645733

3.

a. European Call options via Monte Carlo simulation

With dt = 0.004 and Antithetic Variates and Euler Method

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MC_EuroCall(s0=15,k=20,sigma=0.25,r=0.04,t=0.5)
The Monte Carlo price is: 0.08025144505413202
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b. European Call options by using the Black-Scholes formula

The Black-Scholes price is: **0.08575224964083072**

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BS_EuroCall(s0=15,k=20,sigma=0.25,r=0.04,t=0.5)
The Black-Scholes price is: 0.08575224964083072
```

c. Estimate the European call option's greeks - delta, gamma, theta, and vega

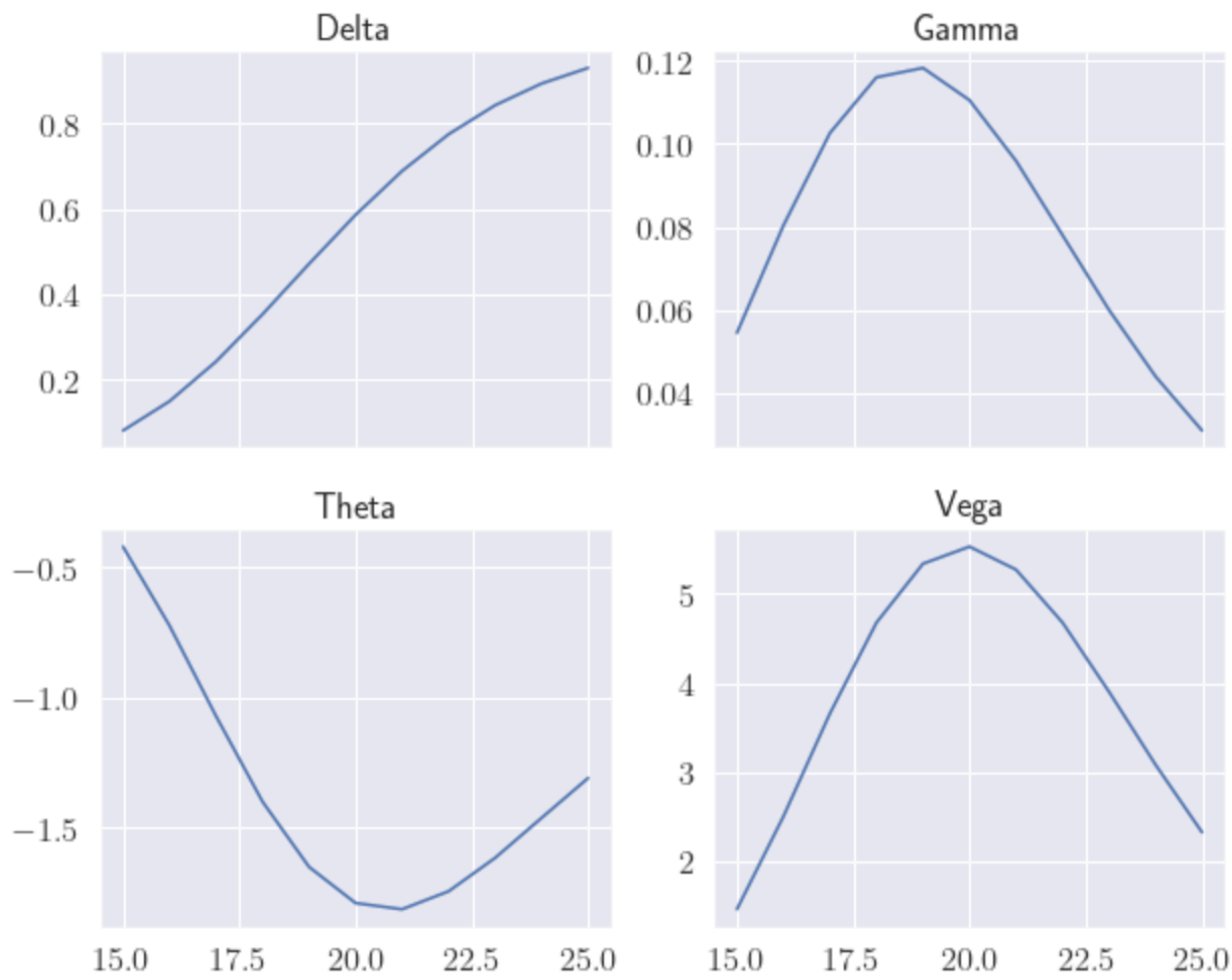
Delta $\Delta = \frac{\partial C}{\partial S_0} = \frac{BS_{call}(S+\Delta S,K,T,r,\sigma)-BS_{call}(S,K,T,r,\sigma)}{\Delta S}$

Gamma $\Gamma = \frac{\partial^2 C}{\partial S_0^2} = \frac{BS_{call}(S+\Delta S,K,T,r,\sigma)-2BS_{call}(S,K,T,r,\sigma)+BS_{call}(S-\Delta S,K,T,r,\sigma)}{(\Delta S)^2}$

Theta $\Theta = \frac{\partial C}{\partial T} = \frac{BS_{call}(S,K,T+\Delta T,r,\sigma)-BS_{call}(S,K,T,r,\sigma)}{\Delta T}$

Vega $\nu = \frac{\partial C}{\partial \sigma} = \frac{BS_{call}(S,K,T,r,\sigma)-BS_{call}(S,K,T,r,\sigma-\Delta\sigma)}{\Delta\sigma}$

Rho $\rho = \frac{\partial C}{\partial r} = \frac{BS_{call}(S,K,T,r+\Delta r,\sigma)-BS_{call}(S,K,T,r,\sigma)}{\Delta r}$



4. Consider the following 2-factor model for stock prices with stochastic volatility:

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Heston Model
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$$\begin{cases} dS_t = rS_t d_t + \sqrt{V_t} S_t dW_t^1 \\ dV_t = \alpha(\beta - V_t)d_t + \sigma\sqrt{V_t} dW_t^2 \end{cases}$$

Three Truncation Methods

$$\hat{S}_{k+1} = \hat{S}_k + r\hat{S}_k\Delta + \sqrt{f_3(\hat{V}_k)}\hat{S}_k\sqrt{\Delta}Z_{k+1}^1$$

$$\hat{V}_{k+1} = f_1(\hat{V}_k) + \alpha(\beta - f_2(\hat{V}_k))\Delta + \sigma\sqrt{f_3(\hat{V}_k)}\sqrt{\Delta}Z_{k+1}^2$$

Scheme	f_1	f_2	f_3
Reflection	$\|x\|$	$\|x\|$	$\|x\|$
Partial Truncation	x	x	x^+
Full Truncation	x	x^+	x^+

Assume K = 20, T = 0.5 for $\rho = -0.6$, $r = 0.03$, $S = \$48$, $V = 0.05$, $\sigma = 0.42$, $\alpha = 5.8$, $\beta = 0.0625$.

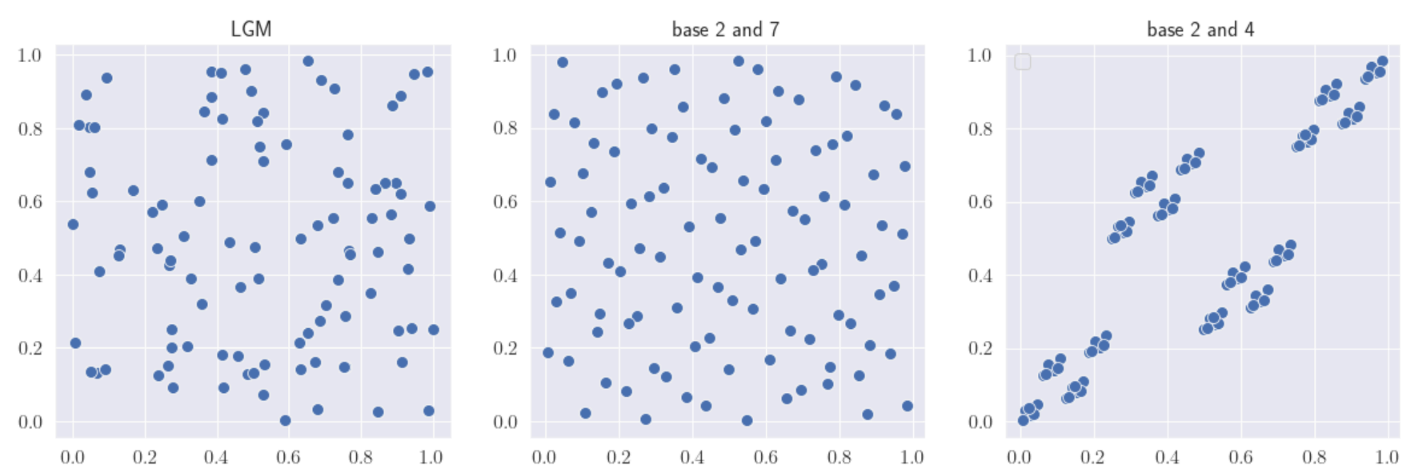
It seems that three methods get similar answer when T is small as 0.5

	T=0.5
Full Truncation	2.558547
Partial Truncation	2.433621
Reflection	2.814669

5. The objective of this exercise is to compare a sample of Pseudo-Random numbers with a sample of Quasi-Monte Carlo numbers of $Uniform[0,1]x[0,1]$:

(a) - (d)

For question (a), I apply LGM method to generate 2-dimensional uniform vectors. From the graph, it seems random variables generated by LGM method is random, but not really evenly distributed. For base 2 and 7, it is very evenly and random distributed. However, the two-dimensional sequences generated with base 2 and 4 is not random and evenly distributed. It has very clear pattern and there are overlapping between numbers. One reason might be because 4 is not a prime number, and numbers with base 2, 4 would have many common factors. .



(e) Use 2-dimensional Halton sequences to compute the following integral:

$I = \int_0^1 \int_0^1 e^{-xy}(\sin 6\pi x + \cos^{\frac{1}{3}} 2\pi y) dxdy$, Use N=10,000 in your simulations. Try different couples for bases: (2,4), (2,7), (5,7).

1. For **base 2,4**, the integral value is **-0.004884**
2. For **base 2,7**, the integral value is **0.026114**
3. For **base 5,7**, the integral value is **0.026164**

Base	Integral Value
2,4	-0.004884
2,7	0.026114
5,7	0.026164