EE523 Assignment 6

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For the Kundur system from the previous homeworks, we want to study the small-signal stability properties. Assume a first order exciter control model with KA=50 and TA = 0.01 sec. Assume Vrmin = -4 and VRmax = +4. Efdmin = 0 and Efdmax = 2.0. For the governor model, assume that Tsg = 100 and Ksg = 1 with Psgmin=0 and Psgmax = 1 pu. R=5%. Assume KD= 2 pu for all three type of models. Then carry out initialization and small-signal analysis for each of Type 1, 2 and 3 models.

1) Starting from the power-flow solution, initialize the steady-state values of all the dynamic variables.
2) Linearize the equations and find the system Jacobian matrix. You can use numerical differencing to compute the Jacobian entries numerically.
3) Find all eigenvalues and eigenvectors.
4) Compute all the participation factors and analyze each mode.
5) Design Power System Stabilizers (PSSs) as needed to render the damping ratios of all modes to be over 5% for each of Type 1 and Type 2 models. Assume KD= 2 pu for Type 1 and Type 2 model.

1.Type 1 model

Type 1 model for Kundur two area system in DAE form can be represented in following form:

$$\dot{x} = f(x,y) = \begin{cases} \dot{\theta_i} = (w_i - 1)w_s \\ \dot{w_i} = \dfrac{1}{2H}(P_{mi} - P_{ei} - K_{Di}(w_i - 1)) \\ \dot{E'_{qi}} = \dfrac{1}{T'_{d0i}}(-E'_{qi} - (X_{di} - X'_{di})I_{di} + E_{fdi}) \\ \dot{E'_{di}} = \dfrac{1}{T'_{q0i}}(-E'_{di} + \left(X_{qi} - X'_{qi}\right)I_{qi}) \\ \dot{V_{Ai}} = \dfrac{1}{T_{Ai}}(-V_{Ai} + K_{Ai}(V_{refi} - V_i)) \\ \dot{P_{sgi}} = \dfrac{1}{T_{sgi}}(-P_{sgi} + K_{sgi}(P_{ci} + \dfrac{1}{R_i}(1 - w_i)) \end{cases}$$

$$0 = g(x,y) = \begin{cases} P_{Gi} - P_{Li} - \sum_{j=1}^{n} V_i V_j Y_{ij} \cos\left(\delta_i - \delta_j - \theta_{ij}\right) = 0 \\ Q_{Gi} - Q_{Li} - \sum_{j=1}^{n} V_i V_j Y_{ij} \sin\left(\delta_i - \delta_j - \theta_{ij}\right) = 0 \end{cases}$$

Here, state variables are

$$x = [\theta_2, \omega_2, E'_{q2}, E'_{d2}, \theta_3, \omega_3, E'_{q3}, E'_{d3}, \theta_4, \omega_4, E'_{q4}, E'_{d4}]^T$$
$$y = [\delta_2, \delta_3 \dots, \delta_{11}, V_2, V_3, \dots, V_{11}]^T$$

1.1 Initialization

The NR power flow solution for Kundur two area system is as shown in table 1.1-1:

Table 1.1-1 NR power flow solution for Kundur system

Bus	*V (p.u.)*	*δ (deg)*	*δ (rad)*	*P_G (pu)*	*Q_G (pu)*
1	1.0300	0	0	6.9125	0.9763
2	1.0100	-9.3423	-0.1631	6.9999	0.2874
3	1.0300	-19.0143	-0.3319	7.1900	1.3188
4	1.0100	-29.0831	-0.5076	6.9999	0.9600
5	1.0203	-6.2953	-0.1099	0.0000	0.0000
6	1.0118	-15.8988	-0.2775	0.0000	0.0000
7	1.0213	-23.6499	-0.4128	0.0000	0.0000

8	1.0095	-31.6624	-0.5526	0.0000	0.0000
9	1.0025	-43.8034	-0.7645	0.0000	0.0000
10	1.0008	-35.7122	-0.6233	0.0000	0.0000
11	1.0153	-25.5954	-0.4467	0.0000	0.0000

Then the command 'fslove' is used to get the initial values for state variables. Following equations are used:

$$\begin{cases} 0 = (w_i - 1)w_s \\ 0 = P_{mi} - P_{ei} - K_{Di}(w_i - 1) \\ 0 = -E'_{qi} - (X_{di} - X'_{di})I_{di} + E_{fdi} \\ 0 = -E'_{di} + \left(X_{qi} - X'_{qi}\right)I_{qi} \\ 0 = (-V_{Ai} + K_{Ai}(V_{refi} - V_i)) \\ 0 = -P_{sgi} + K_{sgi}(P_{ci} + \dfrac{1}{R_i}(1 - w_i) \end{cases}$$

$$\begin{cases} V_{di} = V_i \sin(\theta_i - \delta_i) \\ V_{qi} = V_i \cos(\theta_i - \delta_i) \end{cases}$$

$$\begin{cases} I_{di} = \dfrac{1}{X'_{di}}(E'_{qi} - V_i \cos(\theta_i - \delta_i)) \\ I_{qi} = -\dfrac{1}{X'_{qi}}(E'_{di} - V_i \sin(\theta_i - \delta_i)) \end{cases}$$

For type 1 model

$$P_{ei} = P_{Gi} = V_{di}I_{di} + V_{qi}I_{qi}$$
$$P_{Gi}^{pf} = P_{sgi} = P_{Ci} = P_{mi}$$

The initial values for dynamic variables are as shown in table 1.1-2:

Table 1.1-2 Initial values for dynamic variables of type 1 model

Bus	θ*(p.u.)*	$\omega(p.u.)$	*E'q (p.u.)*	*E'd (p.u.)*	*Efd (p.u.)*	*Eref(p.u.)*	*Psg(p.u.)*
2	0.7254	1	0.8222	0.5302	1.7486	1.0450	6.9999
3	0.4716	1	0.9122	0.5015	1.8977	1.0680	7.1900
4	0.3256	1	0.8715	0.5057	1.8329	1.0467	6.9999

1.2 Linearization

Above equations can be linearized into following form:

$$\begin{cases} \dot{\Delta x} = A\Delta x + B\Delta y \\ 0 = C\Delta x + D\Delta y \end{cases}$$

$$A = \frac{\partial f}{\partial x}|(x_e, y_e)$$

$$B = \frac{\partial f}{\partial y}|(x_e, y_e)$$

$$C = \frac{\partial g}{\partial x}|(x_e, y_e)$$

$$D = \frac{\partial g}{\partial y}|(x_e, y_e)$$

Then, we can get the system Jacobian matrix for small signal analysis:

$$J = A - BD^{-1}C$$

A(18x18):

0.0000	376.9911	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.2167	-0.1538	-0.2010	0.0891	0.0000	0.0085	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.4899	0.0000	-0.7500	0.0000	0.1250	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.3294	0.0000	0.0000	-7.7273	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-100.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-1.8000	0.0000	0.0000	0.0000	-0.0100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	376.9911	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2355	-0.1619	-0.2001	0.1053	0.0000	0.0090	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.4633	0.0000	-0.7500	0.0000	0.1250	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.7379	0.0000	0.0000	-7.7273	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-100.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.8000	0.0000	0.0000	0.0000	-0.0100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	376.9911	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2274	-0.1619	-0.2018	0.1000	0.0000	0.0090
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.4672	0.0000	-0.7500	0.0000	0.1250	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.5505	0.0000	0.0000	-7.7273	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-100.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.8000	0.0000	0.0000	0.0000	-0.0100

B(18x20):

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0016	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2167	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3941	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4899	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0568	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.3294	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-5000.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2355	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.4339	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4633	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	3.7623	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.7379	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-5000.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2274	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.4203	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4672	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	3.8687	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.5505	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-5000.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

C(20x18):

25.3584	0.0000	23.5154	-10.4226	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	26.1731	0.0000	22.2398	-11.7012	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	25.2719	0.0000	22.4250	-11.1145	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.1918	0.0000	19.1080	12.8266	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0387	0.0000	21.4523	12.1308	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0764	0.0000	20.3767	12.2318	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

D(20x20)

-6.7408	0.0000	0.0000	0.0000	-6.9181	0.0000	0.0000	0.0000	0.0000	0.0000	-86.2624	0.0000	0.0000	0.0000	60.9041	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-7.0182	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-7.0814	0.0000	-88.4946	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	62.3215
0.0000	0.0000	-6.8550	0.0000	0.0000	0.0000	0.0000	0.0000	-6.9943	0.0000	0.0000	0.0000	-85.5035	0.0000	0.0000	0.0000	0.0000	0.0000	60.2316	0.0000
0.0000	0.0000	0.0000	-4.0409	-2.7572	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-103.6606	40.9967	0.0000	0.0000	0.0000	0.0000	0.0000
6.9307	0.0000	0.0000	10.6364	-14.0255	-3.5848	0.0000	0.0000	0.0000	0.0000	60.9041	0.0000	0.0000	39.6324	-203.2999	102.7634	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	23.6579	-12.8719	-1.1109	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	100.0035	-127.9587	27.9552	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	6.5007	-4.5453	-2.0455	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	27.1785	-45.3737	18.1953	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	5.5601	-11.7308	23.7982	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	17.4289	-114.3831	96.9542	0.0000
0.0000	0.0000	6.9307	0.0000	0.0000	0.0000	0.0000	-4.1365	-13.8726	10.8643	0.0000	0.0000	60.2316	0.0000	0.0000	0.0000	0.0000	99.7506	-198.8933	38.9111
0.0000	6.9806	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.1047	-4.0211	0.0000	62.3215	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	40.3249	-102.6463
-82.5947	0.0000	0.0000	0.0000	60.1916	0.0000	0.0000	0.0000	0.0000	0.0000	-6.8082	0.0000	0.0000	0.0000	7.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-85.4128	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	61.3805	0.0000	-7.2287	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	7.1900
0.0000	0.0000	-83.3461	0.0000	0.0000	0.0000	0.0000	0.0000	60.1828	0.0000	0.0000	0.0000	-6.9236	0.0000	0.0000	0.0000	0.0000	0.0000	7.0000	0.0000
0.0000	0.0000	0.0000	-101.5947	40.5171	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	4.1231	2.7898	0.0000	0.0000	0.0000	0.0000	0.0000
60.3011	0.0000	0.0000	38.8426	-200.9217	100.6177	0.0000	0.0000	0.0000	0.0000	-7.0000	0.0000	0.0000	-10.8527	14.1915	3.6612	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	98.8336	-124.3078	27.6911	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-23.9379	22.8164	1.1215	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	26.6110	-44.9450	18.1493	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-6.6394	4.5887	2.0507	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	17.2642	-113.0964	96.8757	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-5.6131	29.4306	-23.8175	0.0000
0.0000	0.0000	59.6352	0.0000	0.0000	0.0000	0.0000	99.4984	-198.7322	38.3236	0.0000	0.0000	-7.0000	0.0000	0.0000	0.0000	0.0000	4.1470	13.8839	-11.0308
0.0000	60.5063	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	40.2922	-101.0965	0.0000	-7.1900	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.1072	4.0828

Jacobian for Type 1 model:

0	376.991118	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.0953029	-0.1538462	-0.1093832	0.02528548	0	0.00854701	0.00774128	0	-0.0009767	-0.0077456	0	0	0.00768511	0	-0.0041386	-0.009918	0	0
-0.2256716	0	-0.355265	-0.0096484	0.125	0	0.02726841	0	0.02969839	-0.0084886	0	0	0.04184211	0	0.04088636	-0.01616	0	0
1.36007624	0	0.49499962	-5.3835483	0	0	-0.0198939	0	0.33837701	0.21039422	0	0	0.12996192	0	0.5727729	0.21578138	0	0
128.877452	0	-2358.9078	-1691.1693	-100	0	-122.80178	0	-401.02067	-113.3593	0	0	-307.57076	0	-631.46626	-78.65609	0	0
0	-1.8	0	0	0	-0.01	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	376.991118	0	0	0	0	0	0	0	0	0	0
0.00897084	0	0.00325885	-0.0070317	0	0	-0.070166	-0.1619433	-0.0981201	0.00953441	0	0.00899685	0.04809463	0	0.00959814	-0.0408883	0	0
0.02086291	0	0.03768089	0.00354377	0	0	-0.1707241	0	-0.2908867	-0.0114403	0.125	0	0.12650309	0	0.16312278	-0.025284	0	0
-0.1144721	0	0.22019466	0.26276015	0	0	0.82975062	0	0.62919282	-4.6687672	0	0	-0.4862871	0	1.10198571	1.12882631	0	0
-37.064802	0	-360.87052	-200.57742	0	0	376.962889	0	-2983.0718	-2042.067	-100	0	-367.81537	0	-1664.6357	-636.70555	0	0
0	0	0	0	0	0	0	-1.8	0	0	0	-0.01	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	376.991118	0	0	0	0
0.01314333	0	0.00931393	-0.0073018	0	0	0.05574182	0	0.03186002	-0.0337143	0	0	-0.0870069	-0.1619433	-0.1110671	0.01806195	0	0.00899685
0.02923956	0	0.05975426	0.00955662	0	0	0.09790824	0	0.17492241	0.0082521	0	0	-0.1582444	0	-0.2805396	-0.0193352	0.125	0
-0.1848111	0	0.22614131	0.33871496	0	0	-1.0225667	0	0.5038737	1.23573587	0	0	1.54652723	0	1.17401331	-5.0844966	0	0
-26.47648	0	-481.56688	-291.19797	0	0	197.152604	0	-1297.7817	-919.19993	0	0	-243.46849	0	-3356.7959	-1766.8545	-100	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1.8	0	0	0	-0.01

1.3 Eigenvalues and Eigenvectors

-93.6195 + 0.0000i
-97.1773 + 0.0000i
-97.8485 + 0.0000i
-0.4754 + 6.9219i
-0.4754 - 6.9219i
-0.3480 + 6.0437i
-0.3480 - 6.0437i
-0.2443 + 2.9043i
-0.2443 - 2.9043i
-4.8685 + 2.0241i
-4.8685 - 2.0241i
-6.0667 + 0.0000i
-5.6557 + 0.0000i
-2.4757 + 0.0000i
-1.8254 + 0.0000i
-0.0100 + 0.0000i
-0.0100 + 0.0000i
-0.0100 + 0.0000i

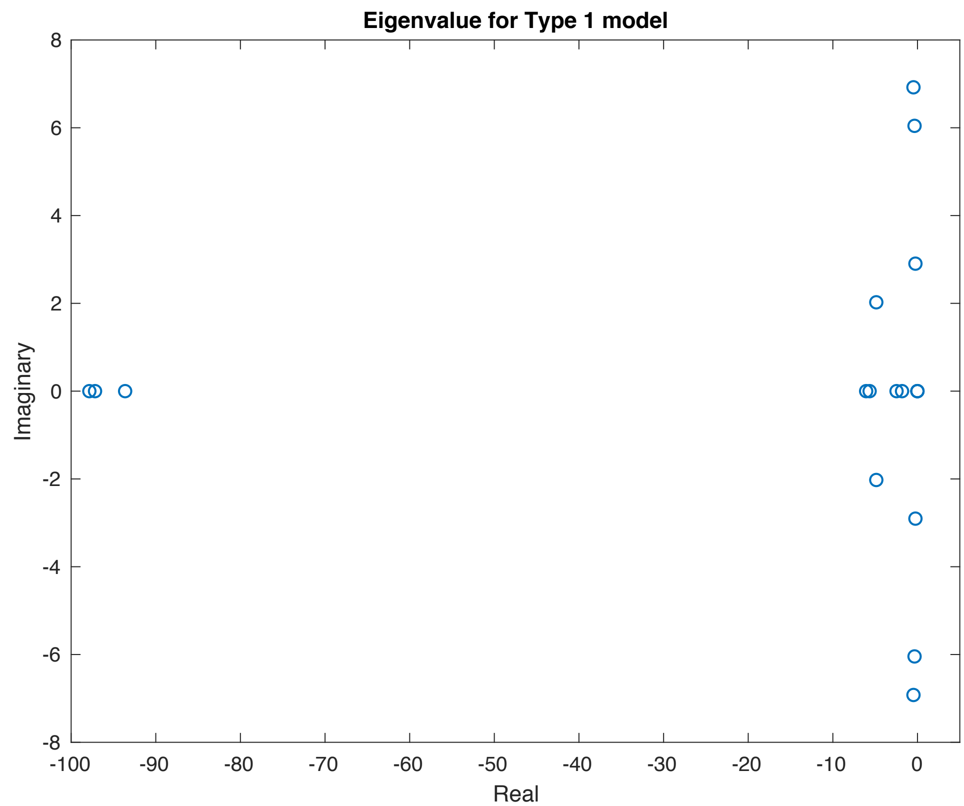


Figure1.3-1 Eigenvalues for Type1 Model

As we can see from the above figure, all eigenvalues of type 1 model locate at the left-half plane, so the system is small signal stable.

1.4 Participation Factors and Modal Analysis

Participation factors for each mode is as shown in figure 1.4-1

0.0000	0.0000	0.0000	0.0183	0.0183	0.9984	0.9984	0.1174	0.1174	0.0151	0.0151	0.0000	0.0478	0.0779	0.0006	0.0003	0.0006	0.0000
0.0000	0.0000	0.0000	0.0183	0.0183	1.0000	1.0000	0.1180	0.1180	0.0155	0.0155	0.0001	0.0491	0.0829	0.0007	0.0000	0.0000	0.0000
0.0087	0.0292	0.0009	0.0065	0.0065	0.0544	0.0544	0.0549	0.0549	0.1298	0.1298	0.0025	0.1675	1.0000	0.0039	0.0000	0.0000	0.0000
0.0002	0.0001	0.0000	0.0009	0.0009	0.0713	0.0713	0.0205	0.0205	0.1547	0.1547	0.0143	1.0000	0.2756	0.0006	0.0000	0.0000	0.0000
0.1275	1.0000	0.0413	0.0005	0.0005	0.0014	0.0014	0.0017	0.0017	0.0071	0.0071	0.0002	0.0096	0.0242	0.0001	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0004	0.0004	0.0002	0.0002	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.1662	1.0000	0.0009
0.0000	0.0000	0.0000	0.7079	0.7079	0.0532	0.0532	0.9950	0.9950	0.0349	0.0349	0.0024	0.0010	0.0167	0.0232	0.0021	0.0000	0.0004
0.0000	0.0000	0.0000	0.7091	0.7091	0.0533	0.0533	1.0000	1.0000	0.0359	0.0359	0.0025	0.0010	0.0179	0.0253	0.0000	0.0000	0.0000
0.0519	0.0026	0.0221	0.0704	0.0704	0.0030	0.0030	0.0470	0.0470	0.9989	0.9989	0.1296	0.0250	0.0142	0.8752	0.0000	0.0000	0.0000
0.0008	0.0000	0.0000	0.0393	0.0393	0.0028	0.0028	0.0418	0.0418	0.9740	0.9740	0.8205	0.0822	0.0125	0.1747	0.0000	0.0000	0.0000
0.7612	0.0884	1.0000	0.0065	0.0065	0.0002	0.0002	0.0011	0.0011	0.0547	0.0547	0.0078	0.0014	0.0003	0.0149	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0002	0.0002	0.0000	0.0000	0.0019	0.0019	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	1.0000	0.0615	0.9137
0.0000	0.0000	0.0000	0.9983	0.9983	0.0029	0.0029	0.8694	0.8694	0.1188	0.1188	0.0074	0.0001	0.0048	0.0333	0.0020	0.0000	0.0005
0.0000	0.0000	0.0000	1.0000	1.0000	0.0029	0.0029	0.8738	0.8738	0.1222	0.1222	0.0076	0.0002	0.0051	0.0363	0.0000	0.0000	0.0000
0.0682	0.0003	0.0198	0.1150	0.1150	0.0008	0.0008	0.0901	0.0901	1.0000	1.0000	0.1037	0.0034	0.0010	1.0000	0.0000	0.0000	0.0000
0.0007	0.0000	0.0001	0.1086	0.1086	0.0000	0.0000	0.0458	0.0458	0.9447	0.9447	1.0000	0.0181	0.0015	0.1939	0.0000	0.0000	0.0000
1.0000	0.0092	0.8946	0.0058	0.0058	0.0001	0.0001	0.0022	0.0022	0.0548	0.0548	0.0061	0.0002	0.0000	0.0170	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0017	0.0017	0.0001	0.0001	0.0000	0.0000	0.0000	0.0002	0.9795	0.0230	1.0000

Figure1.4-1 Participation Factors for Modes (Type1 model)

Table 1.4-1 Mode Analysis for Type 1 Model

#	Mode Type	λ	Frequency (Hz)	Damping ratio	Dominant States

1		-93.6195 + 0.0000i	0	1.0000	$E_{fd2}, E'_{q3}, E_{fd3}, E'_{q4}, E_{fd4}$
2		-97.1773 + 0.0000i	0	1.0000	E'_{q2} , E_{fd2}, E_{fd3}
3		-97.8485 + 0.0000i	0	1.0000	$E_{fd2}, E'_{q3}, E_{fd3}, E'_{q4}, E_{fd4}$
4	Local Area mode	-0.4754 + 6.9219i	1.1017	0.0685	$\theta 2$, w2, $\theta 3$, w3, E'_{q3}, E'_{d3}, $\theta 4$, w4, E'_{q4}, E'_{d4}
5	Local Area mode	-0.4754 - 6.9219i	1.1017	0.0685	$\theta 2$, $\theta 3$, w3, $\theta 4$, w4
6	Intro Area mode	-0.3480 + 6.0437i	0.9619	0.0575	$\theta 2$, w2
7	Intro Area mode	-0.3480 - 6.0437i	0.9619	0.0575	$\theta 2$, w2
8	Inter Area mode	-0.2443 + 2.9043i	0.4622	0.0838	$\theta 3$, w3, $\theta 4$, w4
9	Inter Area mode	-0.2443 - 2.9043i	0.4622	0.0838	$\theta 3$, w3, $\theta 4$, w4
10	Inter Area mode	-4.8685 + 2.0241i	0.3221	0.9234	$E'_{q3}, E'_{d3}, E'_{q4}, E'_{d4}$
11	Inter Area mode	-4.8685 - 2.0241i	0.3221	0.9234	$E'_{q3}, E'_{d3}, E'_{q4}, E'_{d4}$
12		-6.0667 + 0.0000i	0	1.0000	E'_{d3}, E'_{d4}
13		-5.6557 + 0.0000i	0	1.0000	E'_{d2}
14		-2.4757 + 0.0000i	0	1.0000	E'_{q2}
15		-1.8254 + 0.0000i	0	1.0000	E'_{q3}, E'_{q4}
16		-0.0100 + 0.0000i	0	1.0000	P_{m3},P_{m4}
17		-0.0100 + 0.0000i	0	1.0000	P_{m2}
18		-0.0100 + 0.0000i	0	1.0000	P_{m3},P_{m4}

2. Type 2 model

On the basis of Type-1 model, there's two more assumptions for type 2 model:
1) Represent load as constant impedance load model

$$Y_{Li} = \frac{{S_{Li}}^{*}}{{V_i}^{2}}$$

2) No saliency on generators, so for G2, G3 and G4

$$X'_{di} = X'_{qi} = \frac{X'_{di} + X'_{qi}}{2} = X'$$

For type 2 model, we define

$$E'\angle\gamma = \sqrt{E'^2_{di} + E'^2_{qi}}\angle(arctan\left(\frac{E'_{qi}}{E'_{di}}\right) + \theta_i - \frac{\pi}{2})$$

And following equations can be derived:

$$P_{ei} = \sum_{j=1}^{N_{G+1}} y_{Gij}\, E'_i E'_j \cos\,(\gamma_i - \gamma_j - \theta_{Gij})$$

$$I_{di} = \sum_{j=1}^{N_{G+1}} y_{Gij}\, E'_j \sin\,(\theta_i - \gamma_j - \theta_{Gij})$$

$$I_{qi} = \sum_{j=1}^{N_{G+1}} y_{Gij}\, E'_j \cos(\theta_i - \gamma_j - \theta_{Gij})$$

2.1 Initialization

Here equations for initialization is the same as Type1 model, but $X'_d = X'_q = \frac{X'_d + X'_q}{2}$. The initial values for type 2 model are as shown below:

Bus	*θ(p.u.)*	*ω(p.u.)*	*E'q (p.u.)*	*E'd (p.u.)*	*Efd (p.u.)*	*Vref(p.u.)*	*Psg(p.u.)*
2	0.7252	1	0.8996	0.5878	1.7489	1.0450	7.0000
3	0.4715	1	0.9944	0.5560	1.8978	1.0680	7.1900
4	0.3255	1	0.9518	0.5605	1.8332	1.0467	7.0000

2.2 Linearization

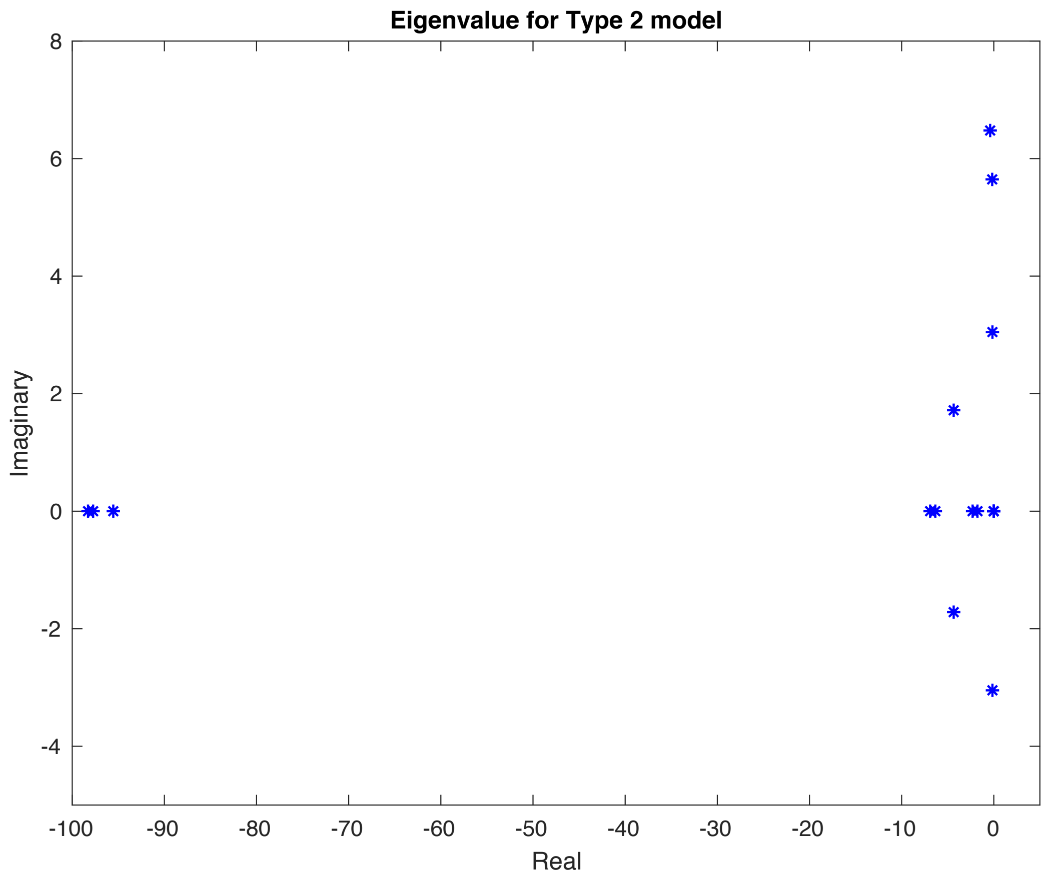
Using ‘jacobian’ command in Matlab, Jacobian matrix(18x18) of type 2 model can be achieved, as shown below:

0.0000	376.9911	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0820	-0.1538	-0.0990	0.0265	0.0000	0.0085	0.0051	0.0000	-0.0039	-0.0073	0.0000	0.0000	0.0046	0.0000	-0.0071	-0.0091	0.0000	0.0000
-0.1736	0.0000	-0.3260	-0.0264	0.1250	0.0000	0.0196	0.0000	0.0083	-0.0151	0.0000	0.0000	0.0259	0.0000	0.0077	-0.0227	0.0000	0.0000
1.6719	0.0000	0.4897	-6.2271	0.0000	0.0000	0.0037	0.0000	0.2805	0.1531	0.0000	0.0000	0.1008	0.0000	0.4212	0.1421	0.0000	0.0000
489.0692	0.0000	-1839.8994	-1745.8668	-100.0000	0.0000	-154.9811	0.0000	-209.4516	38.7525	0.0000	0.0000	-254.2314	0.0000	-277.6603	103.5851	0.0000	0.0000
0.0000	-1.8000	0.0000	0.0000	0.0000	-0.0100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	376.9911	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0080	0.0000	-0.0005	-0.0092	0.0000	0.0000	-0.0622	-0.1619	-0.0909	0.0117	0.0000	0.0090	0.0432	0.0000	-0.0035	-0.0475	0.0000	0.0000
0.0168	0.0000	0.0146	-0.0092	0.0000	0.0000	-0.1305	0.0000	-0.2811	-0.0315	0.1250	0.0000	0.0880	0.0000	0.0740	-0.0489	0.0000	0.0000
-0.1426	0.0000	0.1707	0.2701	0.0000	0.0000	1.1094	0.0000	0.5833	-5.3958	0.0000	0.0000	-0.7980	0.0000	0.9071	1.3727	0.0000	0.0000
-76.1625	0.0000	-206.9234	-50.5413	0.0000	0.0000	587.7720	0.0000	-2410.9695	-1939.0835	-100.0000	0.0000	-372.9047	0.0000	-1070.6074	-238.7251	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.8000	0.0000	0.0000	0.0000	-0.0100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	376.9911	0.0000	0.0000	0.0000	0.0000
0.0118	0.0000	0.0011	-0.0124	0.0000	0.0000	0.0507	0.0000	0.0110	-0.0448	0.0000	0.0000	-0.0790	-0.1619	-0.1103	0.0180	0.0000	0.0090
0.0220	0.0000	0.0216	-0.0103	0.0000	0.0000	0.0726	0.0000	0.0850	-0.0255	0.0000	0.0000	-0.1285	0.0000	-0.3136	-0.0555	0.1250	0.0000
-0.2481	0.0000	0.1917	0.4011	0.0000	0.0000	-1.3037	0.0000	0.4735	1.5758	0.0000	0.0000	1.8627	0.0000	1.0286	-5.9982	0.0000	0.0000
-60.6316	0.0000	-274.4132	-111.9042	0.0000	0.0000	39.1745	0.0000	-940.1655	-565.0476	0.0000	0.0000	149.7286	0.0000	-2301.9030	-1512.9853	-100.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.8000	0.0000	0.0000	0.0000	-0.0100

2.3 Eigenvalues

Eigenvalues for type 2 model is as shown below:

-97.7295 + 0.0000i
-98.2833 + 0.0000i
-95.5443 + 0.0000i
-0.3952 + 6.4780i
-0.3952 - 6.4780i
-0.1730 + 5.6460i
-0.1730 - 5.6460i
-0.1475 + 3.0481i
-0.1475 - 3.0481i
-6.9080 + 0.0000i
-6.3423 + 0.0000i
-4.3540 + 1.7177i
-4.3540 - 1.7177i
-2.3017 + 0.0000i
-1.7709 + 0.0000i
-0.0100 + 0.0000i
-0.0100 + 0.0000i
-0.0100 + 0.0000i



As we can see from the above figure, all eigenvalues of type 2 model also locate at the left-half plane, so the system is small signal stable.

2.4 Participation Factors and Modal Analysis

Participation factors for each mode of type 2 model is as shown in following figure:

0.0000	0.0000	0.0000	0.0091	0.0091	0.9991	0.9991	0.1014	0.1014	0.0002	0.0313	0.0106	0.0106	0.0476	0.0020	0.0003	0.0006	0.0000
0.0000	0.0000	0.0000	0.0091	0.0091	1.0000	1.0000	0.1017	0.1017	0.0002	0.0321	0.0109	0.0109	0.0508	0.0021	0.0000	0.0000	0.0000
0.0233	0.0005	0.0040	0.0035	0.0035	0.0600	0.0600	0.0856	0.0856	0.0010	0.0885	0.0442	0.0442	1.0000	0.0591	0.0000	0.0000	0.0000
0.0001	0.0000	0.0001	0.0009	0.0009	0.0881	0.0881	0.0217	0.0217	0.0032	1.0000	0.0538	0.0538	0.2192	0.0095	0.0000	0.0000	0.0000
1.0000	0.0305	0.0866	0.0002	0.0002	0.0043	0.0043	0.0027	0.0027	0.0001	0.0057	0.0020	0.0020	0.0223	0.0010	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0005	0.0002	0.0002	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.1659	1.0000	0.0008
0.0000	0.0000	0.0000	0.6673	0.6673	0.0400	0.0400	0.9971	0.9971	0.0195	0.0001	0.0919	0.0919	0.0094	0.0090	0.0019	0.0000	0.0004
0.0000	0.0000	0.0000	0.6684	0.6684	0.0401	0.0401	1.0000	1.0000	0.0200	0.0001	0.0949	0.0949	0.0101	0.0098	0.0000	0.0000	0.0000
0.0014	0.0142	0.0467	0.0812	0.0812	0.0043	0.0043	0.0454	0.0454	0.0788	0.0107	0.9877	0.9877	0.0533	0.7696	0.0000	0.0000	0.0000
0.0000	0.0001	0.0006	0.0644	0.0644	0.0027	0.0027	0.0480	0.0480	0.7167	0.0328	1.0000	1.0000	0.0085	0.1326	0.0000	0.0000	0.0000
0.0619	0.8108	1.0000	0.0064	0.0064	0.0003	0.0003	0.0015	0.0015	0.0056	0.0007	0.0467	0.0467	0.0012	0.0127	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0017	0.0017	0.0000	0.0000	0.0001	0.0001	0.0000	0.0001	1.0000	0.0641	0.8747
0.0000	0.0000	0.0000	0.9984	0.9984	0.0041	0.0041	0.7875	0.7875	0.0129	0.0000	0.1791	0.1791	0.0018	0.0179	0.0018	0.0000	0.0005
0.0000	0.0000	0.0000	1.0000	1.0000	0.0041	0.0041	0.7898	0.7898	0.0132	0.0000	0.1850	0.1850	0.0020	0.0196	0.0000	0.0000	0.0000
0.0000	0.0175	0.0424	0.0523	0.0523	0.0010	0.0010	0.0489	0.0489	0.0585	0.0020	0.8203	0.8203	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0001	0.0004	0.1376	0.1376	0.0001	0.0001	0.0517	0.0517	1.0000	0.0035	0.7816	0.7816	0.0003	0.1638	0.0000	0.0000	0.0000
0.0019	1.0000	0.9074	0.0016	0.0016	0.0001	0.0001	0.0007	0.0007	0.0040	0.0001	0.0387	0.0387	0.0000	0.0165	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0004	0.0004	0.0000	0.0000	0.0014	0.0014	0.0000	0.0000	0.0001	0.0001	0.0000	0.0001	0.9411	0.0219	1.0000

Table 2.4-1Mode Analysis for Type 2 Model

#	Mode Type	λ	Frequency (Hz)	Damping ratio	Dominant States
1		-97.7295 + 0.0000i	0	1.0000	E_{fd2}
2		-98.2833 + 0.0000i	0	1.0000	E_{fd3}, E_{fd4}
3		-95.5443 + 0.0000i	0	1.0000	E_{fd3}, E_{fd4}
4	Local mode	-0.3952 + 6.4780i	1.0310	0.0609	$\theta3$, w3, $\theta4$, w4
5	Local mode	-0.3952 - 6.4780i	1.0310	0.0609	$\theta3$, w3, $\theta4$, w4
6	Intra area mode	-0.1730 + 5.6460i	0.8986	0.0306	$\theta2$, w2
7	Intra area mode	-0.1730 - 5.6460i	0.8986	0.0306	$\theta2$, w2
8	Inter area mode	-0.1475 + 3.0481i	0.4851	0.0483	$\theta3$, w3, $\theta4$, w4
9	Inter area mode	-0.1475 - 3.0481i	0.4851	0.0483	$\theta3$, w3, $\theta4$, w4
10		-6.9080 + 0.0000i	0	1.0000	E'_{d3}, E'_{d4}
11		-6.3423 + 0.0000i	0	1.0000	E'_{d2}
12	Inter area mode	-4.3540 + 1.7177i	0.2734	0.9302	E'_{q3}, E'_{d3}, E'_{q4} E'_{d4}
13	Inter area mode	-4.3540 - 1.7177i	0.2734	0.9302	E'_{q3}, E'_{d3}, E'_{q4} E'_{d4}
14		-2.3017 + 0.0000i	0	1.0000	E'_{q2}
15		-1.7709 + 0.0000i	0	1.0000	E'_{q3}, E'_{q4}
16		-0.0100 + 0.0000i	0	1.0000	P_{m3},P_{m4}
17		-0.0100 + 0.0000i	0	1.0000	P_{m2}
18		-0.0100 + 0.0000i	0	1.0000	P_{m3},P_{m4}

Damping ratio for modes 6,7,8,9 are smaller than 5%, through adding PSS to type 2 model, damping ratio can be improved to above 5%. Dynamic equations for PSS model are as follows:

$$\dot{V_W} = \frac{1}{T_W}(-V_W - \dot{w_\iota} T_W K_{stab})$$
$$\dot{V_{PSS}} = \frac{1}{T_2}(-V_{PSS} + V_W + \dot{V_W} T_1)$$
Correspondingly, make change to dynamic equation of exciter:

$$\dot{V_{A\iota}} = \frac{1}{T_{Ai}}(-V_{Ai} + K_{Ai}(V_{refi} - V_i - V_{PSSi}))$$
Here, parameters of PSS are as follows：

$$K_{stab} = 5，T_1 = 0.06，\ T_2 = 0.02，T_W = 10$$
With PSS, all of the damping ratio is improved to above 5%：

1
1
1
1
1
1
0.0707
0.0707
0.049
0.049
1
1
0.1397
0.1397
0.9207
0.9207
1
1
1
1
1
1
1
1

3. Type 3 model (classical model)

The type-3 model (ODE):

On the basis of type 2 model, in type 3 model, we have two more assumptions;
1) E'_{di} and E'_{qi} are constants
2) $\gamma_i = \theta_i$
Then we get the classical model:

$$\dot{x} = f(x)$$

$$\text{x}= [\theta_2, w_2, \theta_3, w_3, \theta_4, w_4]^T$$

$$f(x) = \begin{cases} \dot{\theta_i} = (w_i - 1)w_s \\ 2H\dot{w_i} = P_{mi} - P_{ei} - K_{Di}(w_i - 1) \end{cases}, i = 2{,}3{,}4$$

$$P_{ei} = \sum_{j=1}^{N_{G+1}} y_{Gij}\, E'_i E'_j \cos\left(\gamma_i - \gamma_j - \theta_{Gij}\right)$$

3.1 Initialization
The initial values for type 3 model is as shown below:

	E'	θ
G1	1.0300	0
G2	1.0746	0.1464
G3	1.1392	-0.0383
G4	1.1046	-0.2068

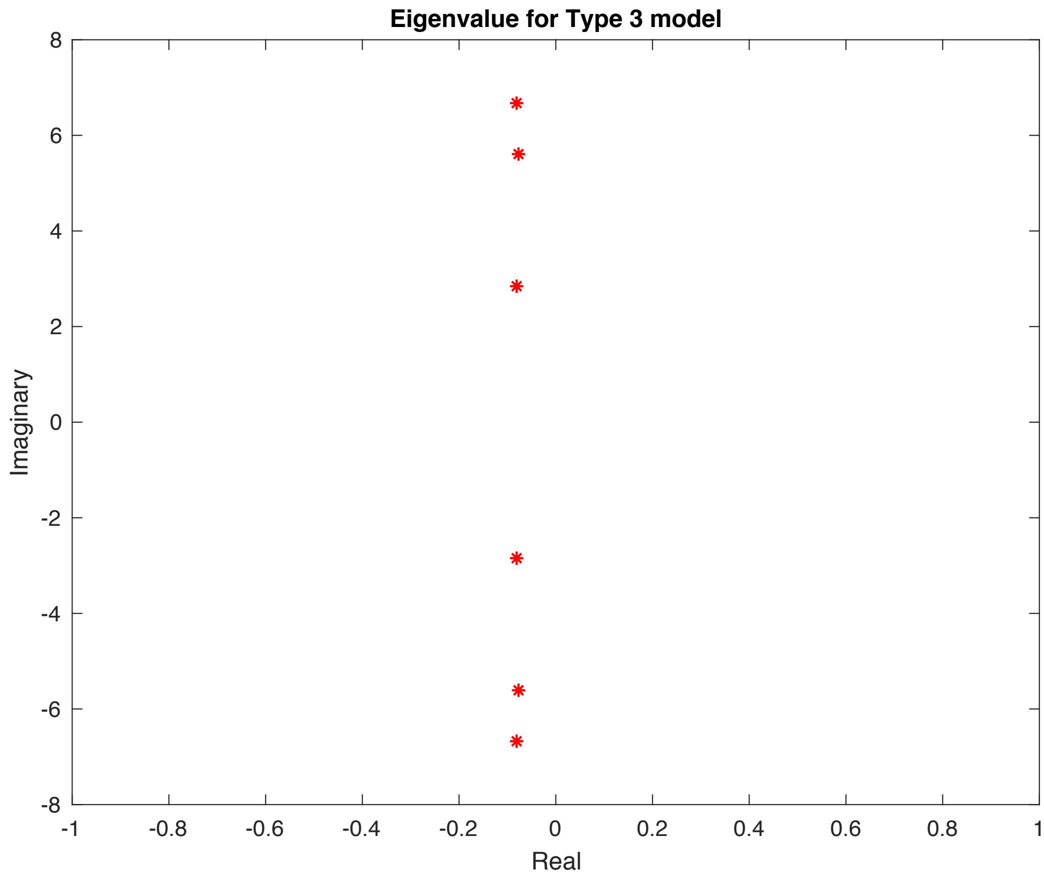
3.2 Linearization
Jacobian matrix of type 3 model is as shown below:

0.0000	376.9911	0.0000	0.0000	0.0000	0.0000
-0.0820	-0.1538	0.0051	0.0000	0.0046	0.0000
0.0000	0.0000	0.0000	376.9911	0.0000	0.0000
0.0080	0.0000	-0.0622	-0.1619	0.0432	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	376.9911
0.0118	0.0000	0.0507	0.0000	-0.0790	-0.1619

3.3 Eigenvalues
Eigenvalues for type 3 model are as follows:

-0.0810 + 6.6747i
-0.0810 - 6.6747i
-0.0770 + 5.6087i
-0.0770 - 5.6087i

-0.0809 + 2.8449i
-0.0809 - 2.8449i



As we can see from the above figure, all eigenvalues of type 3 model also locate at the left-half plane, so the system is small signal stable.

3.4 Participation Factors and Mode Analysis

Participation factors for each mode of type 3 model is as shown in following figure:

0.0026	0.0026	1.0000	1.0000	0.0437	0.0437
0.0026	0.0026	1.0000	1.0000	0.0437	0.0437
0.6873	0.6873	0.0229	0.0229	1.0000	1.0000
0.6873	0.6873	0.0229	0.0229	1.0000	1.0000
1.0000	1.0000	0.0043	0.0043	0.7076	0.7076
1.0000	1.0000	0.0043	0.0043	0.7076	0.7076

#	Mode Type	λ	Frequency (Hz)	Damping ratio	Dominant States
1	Local mode	-0.0810 + 6.6747i	1.0623	1.0623	E_{fd2}
2	Local mode	-0.0810 - 6.6747i	1.0623	1.0623	E_{fd3}, E_{fd4}
3	Intra area mode	-0.0770 + 5.6087i	0.8927	0.8927	E_{fd3}, E_{fd4}
4	Intra area mode	-0.0770 - 5.6087i	0.8927	0.8927	$\theta 3$, w3, $\theta 4$, w4
5	Inter area mode	-0.0809 + 2.8449i	0.4528	0.4528	$\theta 3$, w3, $\theta 4$, w4
6	Inter area mode	-0.0809 - 2.8449i	0.4528	0.4528	$\theta 2$, w2

Appendix

Following the main scripts for this report are appended.

1.main script for type-1

```
%% homework 6
%generator parameters, changing rating from 900MVA to 100MVA
%p---prime; pp---double prime
clc
clear all
 format shortEng
 format compact
X_d(2:4,1)=1.8*100/900;
X_q(2:4,1)=1.7*100/900;
X_l(2:4,1)=0.2*100/900;
X_dp(2:4,1)=0.3*100/900;
X_qp(2:4,1)=0.55*100/900;
X_dpp(2:4,1)=0.25*100/900;
X_qpp(2:4,1)=0.25*100/900;
R_a(2:4,1)=0;
T_d0p(2:4,1)=8.0;
T_q0p(2:4,1)=0.4;
T_d0pp(2:4,1)=0.03;
T_q0pp(2:4,1)=0.05;
H(1)=6.5*900/100;
H(2)=6.5*900/100;
H(3)=6.175*900/100;
H(4)=6.175*900/100;
K_D(2:4,1)=2*900/100;

X=(X_d+X_q)/2;
X_p=(X_dp+X_qp)/2;
ws=2*pi*60;
no_of_states = 6;
%% Exciter parameters
KA=50;
TA=0.01;
VRmin=-4;
VRmax=4;
Efdmin=0;
Efdmax=2.0;

%% Governer parameters
Tsg=100;
Ksg=1;
Psgmin=0;
Psgmax=1;
R=0.05/9;
%% read grid data from file
file_name='b_kundur_system.txt';
[S_Base,No_of_Buses,No_of_Lines,Bus_data,Line_data]=read_data(file_name);
[PQ,nPQ,PV,nPV,Y_ang,Y_mag,V_mag,V_Delta,P_gen_cal,Q_gen_cal, V_result,
P_load, Q_load]=
NR_power_flow_type1(S_Base,No_of_Buses,No_of_Lines,Bus_data,Line_data);

%% solve equilibrium points
V_bus0=V_mag;
Delta0=V_Delta;
```

```
for genbus=1:4 %generator buses, including slack bus
    x0=[0;1;1;0.5;1;1;1.7;7;1;7;7]; %
    options=optimoptions('fsolve','algorithm','levenberg-
marquardt','display','off');
    [solution,~,exitflag]  = fsolve(@(x)
type1_equilibrium_points(x,genbus,ws,K_D,X_d,X_dp,X_q,X_qp,KA,P_gen_cal,
Q_gen_cal, V_bus0,Delta0,R),x0,options);
    theta_0(genbus,1)=solution(1);
    omega_0(genbus,1)=solution(2);
    E_qp0(genbus,1)=solution(3);
    E_dp0(genbus,1)=solution(4);
    E_fd0(genbus,1)=solution(7);
    Pm_0(genbus,1)=solution(8);
    V_ref(genbus,1)=solution(9);
    Pc(genbus,1)=solution(10);
    Pe_0(genbus,1)=solution(11);
end
%% consider load as ZIP model
% initialize ZIP lodad model
P_p=0.5; P_q=0.5; %constant power percent
I_p=0; I_q=0;   %constant current percent
Z_p=0.5; Z_q=0.5; % constant impedance percent

PL0=P_p* P_load;
M0=(I_p* P_load)./V_mag;
G0=(Z_p*P_load)./(V_mag.^2);
QL0=P_q* Q_load;
H0=(I_q* Q_load)./V_mag;
B0=(Z_q*Q_load)./(V_mag.^2);


V_bus=sym('V_bus', [1,11]);
% V_bus_mag= sym('V_bus_mag', [1,11]);
Delta=sym('Delta',[1,11]);
theta=sym('theta',[1,4]); %rotor angle
omega=sym('omega',[1,4]);
Eqp=sym('Eqp',[1,4]);
Edp=sym('Edp',[1,4]);
Efd=sym('Efd',[1,4]);
Psg=sym('Psg',[1,4]);

Pg= zeros(1,11)*omega(1);
Qg= zeros(1,11)*omega(1);

V_bus_mag=sqrt(real(V_bus).^2+imag(V_bus).^2);
for i=1:4
Vd(i)=V_bus_mag(i)*sin(theta(i)-Delta(i));
Vq(i)=V_bus_mag(i)*cos(theta(i)-Delta(i));

Id(i)=(Eqp(i)-V_bus_mag(i)*cos(theta(i)-Delta(i)))/X_dp(i);
Iq(i)=(-1/X_qp(i))*(Edp(i)-V_bus_mag(i)*sin(theta(i)-Delta(i)));
Pg(i)=Vd(i)*Id(i)+Vq(i)*Iq(i);
Qg(i)=Vq(i)*Id(i)-Vd(i)*Iq(i);
end

for i=2:4
```

```
    f(1+6*(i-2))=(omega(i)-1)*ws;
    f(2+6*(i-2))=(Psg(i)-Pg(i)-K_D(i)*(omega(i)-1))/(2*H(i));
    f(3+6*(i-2))=(-Eqp(i)-(X_d(i)-X_dp(i))*Id(i)+Efd(i))/T_d0p(i);
    f(4+6*(i-2))=(-Edp(i)+(X_q(i)-X_qp(i))*Iq(i))/T_q0p(i);
    f(5+6*(i-2))=(-Efd(i)+KA*(V_ref(i)-V_bus_mag(i)))/TA;
    f(6+6*(i-2))=(-Psg(i)+Ksg*(Pc(i)+(1-omega(i))/R))/Tsg;

end


x0_A=[theta_0(2:4)',omega_0(2:4)',E_qp0(2:4)',E_dp0(2:4)',E_fd0(2:4)',Pm_0(2:
4)',Delta0(2:4)',V_bus0(2:4)'];
 A=jacobian(f,[theta(2),omega(2),Eqp(2),Edp(2),Efd(2),Psg(2),...
    theta(3),omega(3),Eqp(3),Edp(3),Efd(3),Psg(3),...
    theta(4),omega(4),Eqp(4),Edp(4),Efd(4),Psg(4)]);
A=double(subs(A,[theta(2:4),omega(2:4),Eqp(2:4),Edp(2:4),Efd(2:4),Psg(2:4),De
lta(2:4),V_bus(2:4)],x0_A));
x0_B=[theta_0(2:4)',omega_0(2:4)',E_qp0(2:4)',E_dp0(2:4)',E_fd0(2:4)',Pm_0(2:
4)',Delta0(2:11)',V_bus0(2:11)'];
B= jacobian(f,[V_bus(2:11),Delta(2:11)]);
B=
double(subs(B,[theta(2:4),omega(2:4),Eqp(2:4),Edp(2:4),Efd(2:4),Psg(2:4),Delt
a(2:11),V_bus(2:11)],x0_B));

P_load=PL0'+(M0').*V_bus_mag + (G0').*(V_bus_mag.^2);
Q_load=QL0'+(H0').*V_bus_mag + (B0').*(V_bus_mag.^2);
P_temp= zeros(1,11)*omega(1);
Q_temp= zeros(1,11)*omega(1);
for i=2:11
    for j=1:11
    P_temp(i)=P_temp(i)+V_bus_mag(i)*V_bus_mag(j)*Y_mag(i,j)*cos(Delta(i)-
Delta(j)-Y_ang(i,j));
    Q_temp(i)=Q_temp(i)+V_bus_mag(i)*V_bus_mag(j)*Y_mag(i,j)*sin(Delta(i)-
Delta(j)-Y_ang(i,j));
    end
    g(i-1)=Pg(i)-P_load(i)-P_temp(i);
    g(i+9)=Qg(i)-Q_load(i)-Q_temp(i);
end
x0_C=[theta_0(2:4)',omega_0(2:4)',E_qp0(2:4)',E_dp0(2:4)',E_fd0(2:4)',Pm_0(2:
4)',Delta0(1:11)',V_bus0(1:11)'];
C= jacobian(g,[theta(2),omega(2),Eqp(2),Edp(2),Efd(2),Psg(2),...
    theta(3),omega(3),Eqp(3),Edp(3),Efd(3),Psg(3),...
    theta(4),omega(4),Eqp(4),Edp(4),Efd(4),Psg(4)]);
C=
double(subs(C,[theta(2:4),omega(2:4),Eqp(2:4),Edp(2:4),Efd(2:4),Psg(2:4),Delt
a(1:11),V_bus(1:11)],x0_C))

x0_D=[theta_0(2:4)',omega_0(2:4)',E_qp0(2:4)',E_dp0(2:4)',E_fd0(2:4)',Pm_0(2:
4)',Delta0(1:11)',V_bus0(1:11)'];
D=jacobian(g,[V_bus(2:11),Delta(2:11)]);
D=
double(subs(D,[theta(2:4),omega(2:4),Eqp(2:4),Edp(2:4),Efd(2:4),Psg(2:4),Delt
a(1:11),V_bus(1:11)],x0_D));

J_type1=A-B*inv(D)*C;
[right_EV,Eigen]=eig(J_type1);      %% Right eigen vector and eigenvalues
left_EV=inv(right_EV);               %% Left eigen vector
```

```
EG=eig(J_type1)
plot(EG,'o')
axis([-100 5 -8 8])
xlabel('Real')
ylabel('Imaginary')
title('Eigenvalue for Type 1 model')

%% participation factor matrix
for i=1:length(J_type1)
    for k=1:length(J_type1)
        Participation_matrix(k,i)=right_EV(k,i)*left_EV(i,k);
    end
end

Max_Participation_matrix=max(abs(Participation_matrix)); % Normalaizing the
Participation matrix by dividing by the maximum valueof each column

for e=1:length(J_type1)

Participation_matrix(:,e)=abs(Participation_matrix(:,e))/Max_Participation_ma
trix(e);
end
%% Calculating the Frequency
frequency = abs(imag(EG))./(2*pi)

%% Calculating the damping ratio
damping_ratio = -real(EG)./abs(EG)
```

2.main script for type-2

```
%% homework 6
%generator parameters, changing rating from 900MVA to 100MVA
%p---prime; pp---double prime
clc
clear all
 format shortEng

%  format compact
X_d(2:4,1)=1.8*100/900;
X_q(2:4,1)=1.7*100/900;
X_l(2:4,1)=0.2*100/900;
X_dp(2:4,1)=0.3*100/900;
X_qp(2:4,1)=0.55*100/900;
X_dpp(2:4,1)=0.25*100/900;
X_qpp(2:4,1)=0.25*100/900;
R_a(2:4,1)=0;
T_d0p(2:4,1)=8.0;
T_q0p(2:4,1)=0.4;
T_d0pp(2:4,1)=0.03;
T_q0pp(2:4,1)=0.05;
psi_t1(2:4,1)=0.9;
H(1)=6.5*900/100;
H(2)=6.5*900/100;
H(3)=6.175*900/100;
H(4)=6.175*900/100;
K_D(2:4,1)=2*900/100;
```

```
ws=2*pi*60;
no_of_states = 6;

% for type2 and type 3
X_p=(X_dp+X_qp)/2;
X_dp=X_p;
X_qp=X_p;

%% Exciter parameters
KA=50;
TA=0.01;
VRmin=-4;
VRmax=4;
Efdmin=0;
Efdmax=2.0;

%% Governer parameters
Tsg=100;
Ksg=1;
Psgmin=0;
Psgmax=1;
R=0.05/9
%% read grid data from file
file_name='b_kundur_system.txt';
[S_Base,No_of_Buses,No_of_Lines,Bus_data,Line_data]=read_data(file_name);
[PQ,nPQ,PV,nPV,Y_mat,V_mag,V_Delta,P_gen_cal,Q_gen_cal, V_result]=
NR_power_flow(S_Base,No_of_Buses,No_of_Lines,Bus_data,Line_data);

%% revise y matrix, and form y_gen
file_name_2='b_kundur_system_extended.txt';
[S_Base_2,No_of_Buses_2,No_of_Lines_2,Bus_data_2,Line_data_2]=read_data(file_
name_2);
[Y_mat_ex,Theta_ex,Y_mag_ex,B_ex,G_ex]=y_bus(Bus_data_2,Line_data_2,No_of_Bus
es_2,No_of_Lines_2);
[Y_gen,Y_gen_mag,Y_gen_angle]=Ygen(Y_mat_ex,4,10);

V_bus0=V_mag;
Delta0=V_Delta;
for genbus=1:4 %generator buses, including slack bus
    x0=[0;1;1;0.5;1;1;1.7;7;1;7;7];  %
    [solution,~,exitflag]  = fsolve(@(x)
type2_equilibrium_points(x,genbus,ws,K_D,X_d,X_dp,X_q,X_qp,KA,P_gen_cal,
Q_gen_cal, V_bus0,Delta0,R) ,x0,optimset('algorithm','levenberg-
marquardt','display','off'));
    theta_0(genbus,1)=solution(1);
    omega_0(genbus,1)=solution(2);
    E_qp0(genbus,1)=solution(3);
    E_dp0(genbus,1)=solution(4);
    E_fd0(genbus,1)=solution(7);
    Pm_0(genbus,1)=solution(8);
    V_ref(genbus,1)=solution(9);
    Pc(genbus,1)=solution(10);
    Pe_0(genbus,1)=solution(11);
end

%% linearization
```

```matlab
V_bus=sym('V_bus', [1,4]);
V_bus_mag= sym('V_bus_mag', [1,4]);
% Delta=sym('Delta',[1,11]);
theta=sym('theta',[1,4]); %rotor angle
omega=sym('omega',[1,4]);
Eqp=sym('Eqp',[1,4]);
Edp=sym('Edp',[1,4]);
Efd=sym('Efd',[1,4]);
Psg=sym('Psg',[1,4]);
Pe=zeros(4,1)*omega(1);
Id=zeros(4,1)*omega(1);
Iq=zeros(4,1)*omega(1);

E_p= sym('E_p',[1,4]);
E_p_mag= sym('E_p_mag',[1,4]);
gama= sym('gama',[1,4]);
for i=1:4
    E_p(i)=sqrt((Edp(i))^2+(Eqp(i))^2)*exp(1j*(atan(Eqp(i)/Edp(i))+theta(i)-
pi/2));
    E_p_mag(i)=sqrt(Edp(i)^2+Eqp(i)^2);
    gama(i)=atan(Eqp(i)/Edp(i))+theta(i)-pi/2;
end

for i=1:4
    for j=1:4
        Pe(i)=Pe(i)+Y_gen_mag(i,j)*E_p_mag(i)*E_p_mag(j)*cos(gama(i)-gama(j)-
Y_gen_angle(i,j));
        Id(i)=Id(i)+Y_gen_mag(i,j)*E_p_mag(j)*sin(theta(i)-gama(j)-
Y_gen_angle(i,j));
        Iq(i)=Iq(i)+Y_gen_mag(i,j)*E_p_mag(j)*cos(theta(i)-gama(j)-
Y_gen_angle(i,j));

    end
end

I_gen=zeros(4,1)*omega(1);
for i=1:4
for m=1:4
    I_gen(i)=I_gen(i)+Y_gen(i,m)*E_p(m);
end
    V_bus(i)=E_p(i)-I_gen(i)*(1j*X_p(i));

    V_bus_mag(i)=sqrt(real(V_bus(i))^2+imag(V_bus(i))^2);
end

for i=2:4

    f(1+6*(i-2))=(omega(i)-1)*ws;
    f(2+6*(i-2))=(Psg(i)-Pe(i)-K_D(i)*(omega(i)-1))/(2*H(i));
    f(3+6*(i-2))=(-Eqp(i)-(X_d(i)-X_dp(i))*Id(i)+Efd(i))/T_d0p(i);
    f(4+6*(i-2))=(-Edp(i)+(X_q(i)-X_qp(i))*Iq(i))/T_q0p(i);
    f(5+6*(i-2))=(-Efd(i)+KA*(V_ref(i)-V_bus_mag(i)))/TA;
    f(6+6*(i-2))=(-Psg(i)+Ksg*(Pc(i)+(1-omega(i))/R))/Tsg;

end
```

```
x0_A=[theta_0(1:4)',omega_0(1:4)',E_qp0(1:4)',E_dp0(1:4)',E_fd0(1:4)',Pm_0(1:
4)'];
 J_type2=jacobian(f,[theta(2),omega(2),Eqp(2),Edp(2),Efd(2),Psg(2),...
    theta(3),omega(3),Eqp(3),Edp(3),Efd(3),Psg(3),...
    theta(4),omega(4),Eqp(4),Edp(4),Efd(4),Psg(4)]);
J_type2=double(subs(J_type2,[theta(1:4),omega(1:4),Eqp(1:4),Edp(1:4),Efd(1:4)
,Psg(1:4)],x0_A));


[right_EV,Eigen]=eig(J_type2);      %% Right eigen vector and eigenvalues
left_EV=inv(right_EV);               %% Left eigen vector

EG=eig(J_type2)
plot(EG,'b*')
axis([-100 5 -5 8])
xlabel('Real')
ylabel('Imaginary')
title('Eigenvalue for Type 2 model')


%% participation factor matrix
for i=1:length(J_type2)
    for k=1:length(J_type2)
        Participation_matrix(k,i)=right_EV(k,i)*left_EV(i,k);
    end
end
% Normalaizing the Participation matrix by dividing by the maximum valueof
each column

Max_Participation_matrix=max(abs(Participation_matrix));

for e=1:length(J_type2)

Participation_matrix(:,e)=abs(Participation_matrix(:,e))/Max_Participation_ma
trix(e)
end
writematrix(Participation_matrix,'Participation2.csv')
%% Calculating the Frequency
frequency = abs(imag(EG))./(2*pi)

%% Calculating the damping ratio
damping_ratio = -real(EG)./abs(EG)
```

3.main script for type-3

```
%% homework 6
%generator parameters, changing rating from 900MVA to 100MVA
%p---prime; pp---double prime
clc
clear all
format short
X_d(2:4,1)=1.8*100/900;
X_q(2:4,1)=1.7*100/900;
X_l(2:4,1)=0.2*100/900;
X_dp(2:4,1)=0.3*100/900;
X_qp(2:4,1)=0.55*100/900;
X_dpp(2:4,1)=0.25*100/900;
X_qpp(2:4,1)=0.25*100/900;
R_a(2:4,1)=0;
T_d0p(2:4,1)=8.0;
T_q0p(2:4,1)=0.4;
T_d0pp(2:4,1)=0.03;
T_q0pp(2:4,1)=0.05;

psi_t1(2:4,1)=0.9;
H(1)=6.5*900/100;
H(2)=6.5*900/100;
H(3)=6.175*900/100;
H(4)=6.175*900/100;
K_D(2:4,1)=2*900/100;

X=(X_d+X_q)/2;
X_p=(X_dp+X_qp)/2;
ws=2*pi*60;
no_of_states = 2;

%% read grid data from file
file_name='b_kundur_system.txt';
[S_Base,No_of_Buses,No_of_Lines,Bus_data,Line_data]=read_data(file_name);
[PQ,nPQ,PV,nPV,Y_mat,V_mag,V_Delta,P_gen_cal,Q_gen_cal, V_result]=
NR_power_flow(S_Base,No_of_Buses,No_of_Lines,Bus_data,Line_data)

%% revise y matrix, and form y_gen
file_name_2='b_kundur_system_extended.txt';
[S_Base_2,No_of_Buses_2,No_of_Lines_2,Bus_data_2,Line_data_2]=read_data(file_
name_2);
[Y_mat_ex,Theta_ex,Y_mag_ex,B_ex,G_ex]=y_bus(Bus_data_2,Line_data_2,No_of_Bus
es_2,No_of_Lines_2);
[Y_gen,Y_gen_mag,Y_gen_angle]=Ygen(Y_mat_ex,4,10);

%% solving for equilibrium point at t=0
S=complex(P_gen_cal, Q_gen_cal);
I=conj(S./(V_mag.*cos(V_Delta)+1i.*V_mag.*sin(V_Delta)));
I_mag=abs(I);
I_angle=angle(I);
E_p=zeros(4,1);
for i=1:4
E_p(i)=complex(V_mag(i)*cos(V_Delta(i)),V_mag(i).*sin(V_Delta(i)))+I(i).*(R_a
(i)+1i.*X_p(i))
end
E_p_mag=abs(E_p);
```

```
E_p_angle=angle(E_p);
w_0(PV)=1; %initialize omega
[Pe_0]=Pe_type3(Y_gen_mag,Y_gen_angle,E_p_mag,E_p_angle,nPV,PV);
Pm_0=Pe_0;

%% calculate Jacobian for model analysis
J_type3=[]
for e=1:nPV
     for k=1:nPV
         i=PV(e);
         j=PV(k);

[A]=Jacobian_type3(i,j,ws,Y_gen_mag,Y_gen_angle,E_p_mag,E_p_angle,H,K_D)
        row = no_of_states*(e-1)+1;
        col = no_of_states*(k-1)+1;
        J_type3((row:(row+no_of_states-1)),(col:(col+no_of_states-1))) =
A;  %% Store in the Jacobian structure
    end
end

[right_EV,Eigen]=eig(J_type3);     %% Right eigen vector and eigenvalues
left_EV=inv(right_EV);              %% Left eigen vector

EG=eig(J_type3)
plot(EG,'r*')
axis([-1 1 -8 8])
xlabel('Real')
ylabel('Imaginary')
title('Eigenvalue for Type 3 model')
%% participation factor matrix
for i=1:length(J_type3)
    for k=1:length(J_type3)
        Participation_matrix(k,i)=right_EV(k,i)*left_EV(i,k);
    end
end
% Normalaizing the Participation matrix by dividing by the maximum valueof
each column

Max_Participation_matrix=max(abs(Participation_matrix));

for e=1:length(J_type3)

Participation_matrix(:,e)=abs(Participation_matrix(:,e))/Max_Participation_ma
trix(e);
end

%% Calculating the Frequency
frequency = abs(imag(EG))./(2*pi)

%% Calculating the damping ratio
damping_ratio = -real(EG)./abs(EG)
```