9a)

(i) A path is a sequence of vertices $v_0 \to v_1 \to v_2 \to \ldots \to v_n$ such that there is an edge $v_i \to v_{i+1}$ for all $0 \leq i < n$, said to have length n. A shortest path between vertices a and b is a path p with $v_0 = a$ and $v_n = b$, such that its length is minimal.

(ii) nPaths $= \binom{6}{3} = 20$

b)

(i) The diameter of a graph is the longest of all shortest paths between any two vertices in that graph.

(ii) The graph above has a diameter of 6.

c)

(i) The betweenness centrality $C_B(v)$ of a vertex v is the sum over every pair of vertices $s, t \neq v$ of the number of shortest paths between s and t that pass through v: $\sigma(s, t, \mid v)$, divided by the total number of shortest paths between s as t: $\sigma(s, t)$. It is similar to, but not the same as, the number of shortest paths that pass through the vertex v.

$$C_B(v) = \sum_{s,t \in V} \frac{\sigma(s, t \mid v)}{\sigma(s, t)}$$

(ii) The corners have the lowest betweenness centrality. The exact result is
$1/20 + 1/10 + 1/4 + 1/10 + 1/6 + 1/3 + 1/4 + 1/3 + 1/2 = 25/12$

d)

(i) A bridge is an edge which connects two nodes which would otherwise be unconnected.

(ii) A local bridge is an edge joining two nodes which have no other neighbours in common.

(iii) There are no bridges in the above graph, since the graph will remain connected after the removal of any individual edge.

(iv) Every edge in the above graph is a local bridge, since no pair of nodes has common neighbours (there is no triadic closure).

e) For real-world social networks, the graph above is unrealistic, since usually those feature large amounts of triadic closure or clustering (friends form groups), neither of which is present in the graph above. For road networks, the same is true, although the above graph could represent a section of a road network in a grid-based city.