Computer Science Tripos Part IA and IB

2019-2020
Exam Question
Cover sheet

Student BGN2023A**…..……**

Paper3**……**

Question number9**……**

How did you answer this question?

Timed	☐	Open Book	☑
Untimed	☑	Closed Book	☐

Questions

List all the questions you have answered for this paper here. 1,4,6,8,9

Computer Science Tripos Honour Code

1. We take it as a principle that maintaining the integrity and fairness of examinations should be regarded as a collaboration between students and the Department.
2. The students undertake that they will not help others in examinations and will not receive any help from others (students or non-students).
3. Students will actively contribute to ensuring that all students adhere to the code.
4. Students will keep to the conditions of the assessment and will accurately report those conditions when asked.
5. The Department will not make any attempt at remote invigilation of online examinations.

I **undertake to respect the Computer Science Tripos honour code**

Tick the box to confirm ☑

9a)

(i) A path is a sequence of vertices $v_0 \to v_1 \to v_2 \to \ldots \to v_n$ such that there is an edge $v_i \to v_{i+1}$ for all $0 \leq i < n$, said to have length n. A shortest path between vertices a and b is a path p with $v_0 = a$ and $v_n = b$, such that its length is minimal.

(ii) nPaths $= \binom{6}{3} = 20$

b)

(i) The diameter of a graph is the longest of all shortest paths between any two vertices in that graph.

(ii) The graph above has a diameter of 6.

c)

(i) The betweenness centrality $C_B(v)$ of a vertex v is the sum over every pair of vertices $s, t \neq v$ of the number of shortest paths between s and t that pass through v: $\sigma(s, t, \mid v)$, divided by the total number of shortest paths between s as t: $\sigma(s, t)$. It is similar to, but not the same as, the number of shortest paths that pass through the vertex v.

$$C_B(v) = \sum_{s,t \in V} \frac{\sigma(s, t \mid v)}{\sigma(s, t)}$$

(ii) The corners have the lowest betweenness centrality. The exact result is
$1/20 + 1/10 + 1/4 + 1/10 + 1/6 + 1/3 + 1/4 + 1/3 + 1/2 = 25/12$

d)

(i) A bridge is an edge which connects two nodes which would otherwise be unconnected.

(ii) A local bridge is an edge joining two nodes which have no other neighbours in common.

(iii) There are no bridges in the above graph, since the graph will remain connected after the removal of any individual edge.

(iv) Every edge in the above graph is a local bridge, since no pair of nodes has common neighbours (there is no triadic closure).

e) For real-world social networks, the graph above is unrealistic, since usually those feature large amounts of triadic closure or clustering (friends form groups), neither of which is present in the graph above. For road networks, the same is true, although the above graph could represent a section of a road network in a grid-based city.