

Supervision questions: set 4.

Graph algorithms

There is a method of calculating the diameter of an acyclic graph with undirected, unweighted edges which only requires two breadth-first searches. What is it? Give an example of a graph with cycles where this approach doesn't work.

Betweenness centrality and Newman-Girvan method examples

The following 5-node graphs are described by their edge lists.

- 1. (1 2) (2 3) (3 4) (4 5) (1 5)
- 2. (1 2) (1 3) (1 4) (1 5)
- 3. (1 2) (1 3) (1 4) (1 5) (2 3)
- 4. (1 2) (2 3) (2 4) (3 4) (4 5)
- 5. (1 2) (2 3) (3 4) (4 5) (3 5)
- 6. (1 2) (2 3) (3 4) (2 4) (2 5)
- 7. (1 2) (2 3) (2 4) (3 4) (1 4) (1 5)

For each graph:

What is the betweenness centrality of each node?

Graph \ Node	1	2	3	4	5
1	1	1	1	1	1
2	6	0	0	0	0
3	5	0	0	0	0
4	0	3	0	3	0
5	0	3	4	0	0
6	0	5	0	0	0
7	3	1	0	1	0

Which edge or edges have the highest betweenness centrality?

Graph	Edge(s)
1	All equal
2	All equal
3	(1,5), (1,4)
4	(1,2), (4,5)
5	(2, 3)
6	(1,2), (2,5)
7	(1, 5)

What graphs do you get if you remove the highest betweenness centrality edges (removing multiple edges in the case of ties)?

Graph	Edges
1	No edges
2	No edges
3	(1,2), (1,3), (2,3)
4	(2,3), (2,4), (3,4)
5	(1,2), (3,4), (3,5), (4,5)
6	(2,3), (2,4), (3,4)
7	(1,2), (1,4), (2,3), (2,4), (3,4)

Try working the examples out manually (but check your results using your code).

Random graphs and metrics

In the course, we have discussed a number of attributes of graphs including:

1. (distribution of) degrees of nodes
2. number of connected components
3. (distribution of) shortest paths
4. extent of clustering. We have not formalized this concept (unless you did the starred tick and looked at modularity), but an informal notion is all that is needed here.

The additional notes in [random-graphs.html](#) supplement the brief discussion of random networks in Session 12.

Consider the following types of graph:

1. Erdős–Rényi

2. **Watts-Strogatz**

3. **A collaboration network (e.g., the AMS-derived network discussed in Session 12)**

How do you expect they will vary with respect to the four attributes listed above? (Proofs are not expected!)

Erdős-Rényi:

- Small variance in degrees of nodes
- Several connected components
- Shortest paths will tend to be quite long
- Not much clustering.

Watts-Strogatz:

- Nodes all with similar degree
- Few connected components
- Shortest paths will be quite short
- Not much clustering

Collaboration network:

- Large variance in degree
- Many connected components
- Shortest paths may be quite long
- High clustering