Solar House and Socks
QEA 3 Project 1
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Group 42
Ellie Ramos & Mateo Macias

Introduction

Heating your home in Massachusetts is vital for surviving the colder seasons. Most homes use gas or electricity to create a heat source, which adds excess heat to our environment. Currently, heating homes makes up about 50% of the energy we consume and 40% of emissions globally (Cole). Passive solar houses, a more sustainable option, only use solar energy to heat the space within the home. Solar energy enters through windows and is absorbed by a heat storage unit inside the house, which then heats up the air over time.

To design an ideal home for Massachusetts, we decided to use the dimensions from the QEA Compendium as a basis. The U.S. Department of Energy defines a "well-designed" solar house as being able to reduce heating and cooling loads using energy efficiency and then meets those loads with solar energy. With these constants, we were able to do parameter sweeps and find the optimal wall and heat storage thicknesses.

Modeling

To simplify our model, we made a few assumptions about how our system operates. For the heat storage unit, we assumed that its heat capacity was greater than the air inside the house and that the entire storage unit is a uniform temperature. We are also modeling our house so that it is only in contact with air, which means that the only heat transfer out of the storage unit is to the air.

We also decided to neglect the minimal air flow in and out of the house. The only radiation taken into account is the solar radiation entering through the window. Additionally, the window does not absorb any energy, so solar radiation is completely absorbed by the heat storage unit. We also assumed there is uniform heat loss from all sides of the house, including the base.

We modeled our house similarly to the heat flow diagram of the tea in the QEA Compendium (Fig. 1a). In our model, we have heat flow through two places: the walls and the window of the house

(Fig. 1b). Due to the pattern of the sun, the best location for the window in a solar house is the south side of the building - so we modeled our house assuming that solar energy is entering the house from the south for optimal exposure.

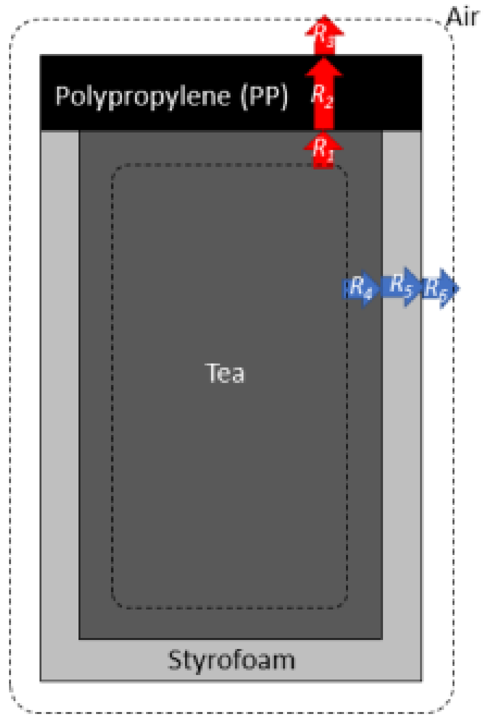

Figure 1a. Heat Flow Diagram from QEA III Book

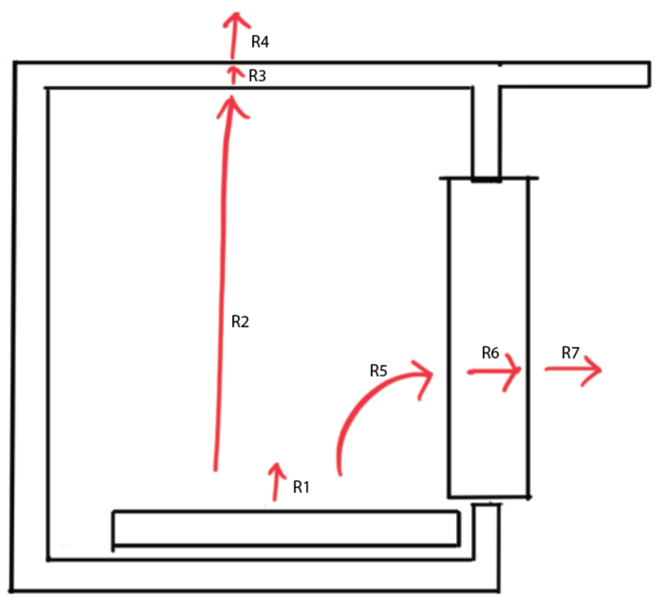

Figure 1b. Heat Flow Diagram of Passive Solar House

Solar energy is entering our system through the window, and we can model that heat transfer from the sun as

$$Q_{in_{sun}} = q_{sun} * A_{win}$$

Where

$$q_{sun} = -361cos(\frac{\pi t}{12*3600}) + 224cos(\frac{\pi t}{12*3600}) + 210$$

and A_{win} is the area of the window. We model it this way because we assume that all radiation passing through the window is absorbed by the heat storage unit.

We can then model the heat transfer to the absorber (which we refer to in our equations as the "floor") as

$$Q_{net_{floor}} = m_{floor} c_{floor} \frac{dT}{dt}$$

Where $Q_{net_{floor}}$ is the energy of the absorber. m_{floor} is the mass of the absorber. c_{floor} is the heat capacity of the absorber. $\frac{dT}{dt}$ is the change in temperature over time.

To model the heat transfer out of our system, we use

$$Q_{out} = \frac{T_{floor} - T_{air\ outdoor}}{TR_{total}}$$

Where T_{floor} is the temperature of the absorber, $T_{air\ outdoor}$ is the constant air temperature of the outside air, and TR_{total} is the total thermal resistance of the system.

We can get the total thermal resistance by combining the thermal resistance of the floor to the air with the sum of the thermal resistances of the house.

$$TR_{total} = TR_{floor\ to\ air} + TR_{house}$$

We are treating the thermal resistance of the house as a system in parallel.

$$TR_{house} = ((R_1 + R_2 + R_3)^{-1} + (R_4 + R_5 + R_6)^{-1})^{-1}$$

$TR_{floor\ to\ air}$ is the initial transfer from the absorber to the air. $TR_{air\ to\ wall\ in}$,TR_{wall}, and $TR_{air\ to\ wall\ out}$ are the parallel transfers of the air to the wall, the transfer within the wall, and the transfer from the wall to the outside air respectively. $TR_{air\ to\ win\ in}$, TR_{win}, and $TR_{air\ to\ win\ out}$ are parallel transfers of the air to the window, transfer within the window, and the window to the outside air respectively.

To find the individual thermal resistance values between a fluid (in this case, air) and solid surface, we find the thermal convection resistance R_{conv} using convection h and the surface area A of the solid.

$$R_{conv} = \frac{1}{hA}$$

To find the individual thermal resistance values of the walls, we find the thermal conduction resistance R_{cond} using thermal conductivity k, the cross-sectional area A, and thickness L of the material.

$$R_{cond} = \frac{L}{kA}$$

Now we can find the total thermal resistance of the house and the floor to the air, where

$$TR_{floor\ to\ air} = \frac{1}{h_{inside} * A_{floor}}$$

$$R_1 = TR_{wall} = \frac{L_{wall}}{k_{wall} * A_{wall\ in}}$$

$$R_2 = TR_{air\ to\ wall\ out} = \frac{1}{h_{outside} * A_{wall\ in}}$$

$$R_3 = TR_{air\ to\ wall\ in} = \frac{1}{h_{inside} * A_{wall\ in}}$$

$$R_4 = TR_{win} = \frac{1}{h_{win} * A_{win}}$$

$$R_5 = TR_{air\ to\ win\ in} = \frac{1}{h_{inside} * A_{win}}$$

$$R_6 = TR_{air\ to\ win\ out} = \frac{1}{h_{outside} * A_{win}}$$

With the information above, we can find the temperature inside the house because we know that the energy flow through our system stays constant. Heat flow from the absorber to the inside air will be the same as heat flow from the absorber to the outside air, which means that:

$$heat\ flow = \frac{T_{floor} - T_{air\ outdoor}}{TR_{total}} = \frac{T_{floor} - T_{air\ indoor}}{TR_{floor\ to\ air}}$$

$$T_{air\ indoor} = T_{floor} - \frac{TR_{floor\ to\ air}(T_{floor} - T_{air\ outdoor})}{TR_{total}}$$

We put together all of the above information, to develop our final ODE:

$$\frac{dT}{dt} = \frac{Q_{in_{sun}}}{m_{floor} * c_{floor}} - \frac{T_{floor} - T_{air\ outdoor}}{TR_{total} * m_{floor} * c_{floor}}$$

Results

We decided to test different thicknesses of the walls and the heat storage unit by doing a parameter sweep from 0.1 meters to 0.9 meters for each material. The thickness of the walls determine the magnitude of heat transfer from our system to the environment. A thinner wall means a lower heat capacity, and so there is a larger heat transfer going from the wall into the environment, and vice versa. For our heat transfer unit, a thicker tile meant a larger heat capacity, and so more energy is required to heat the entire unit before there is a heat transfer out of the tile and into the air.

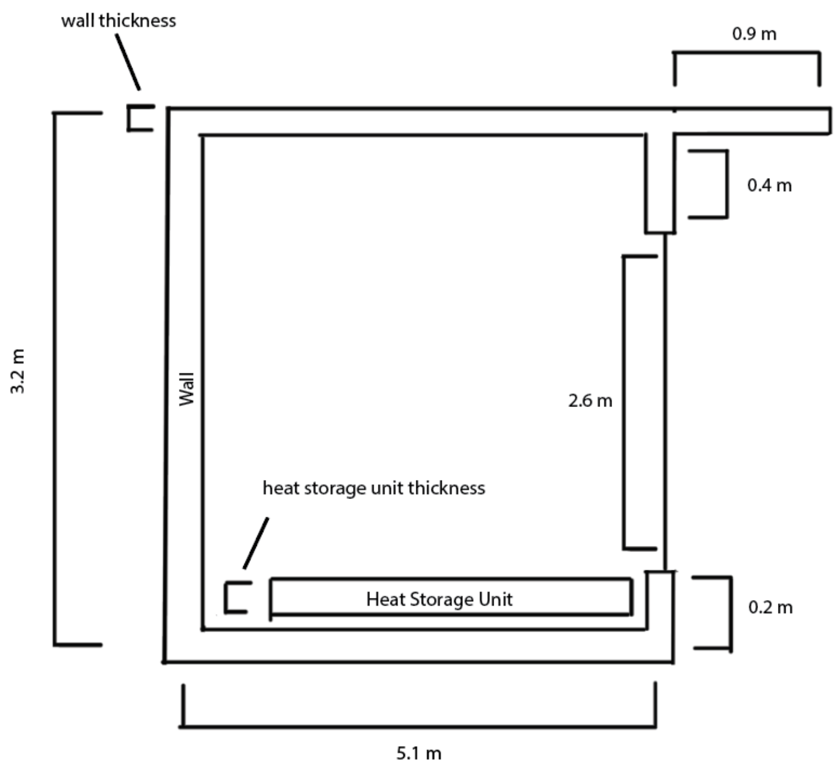

Figure 2. Dimensions of Passive Solar House Design

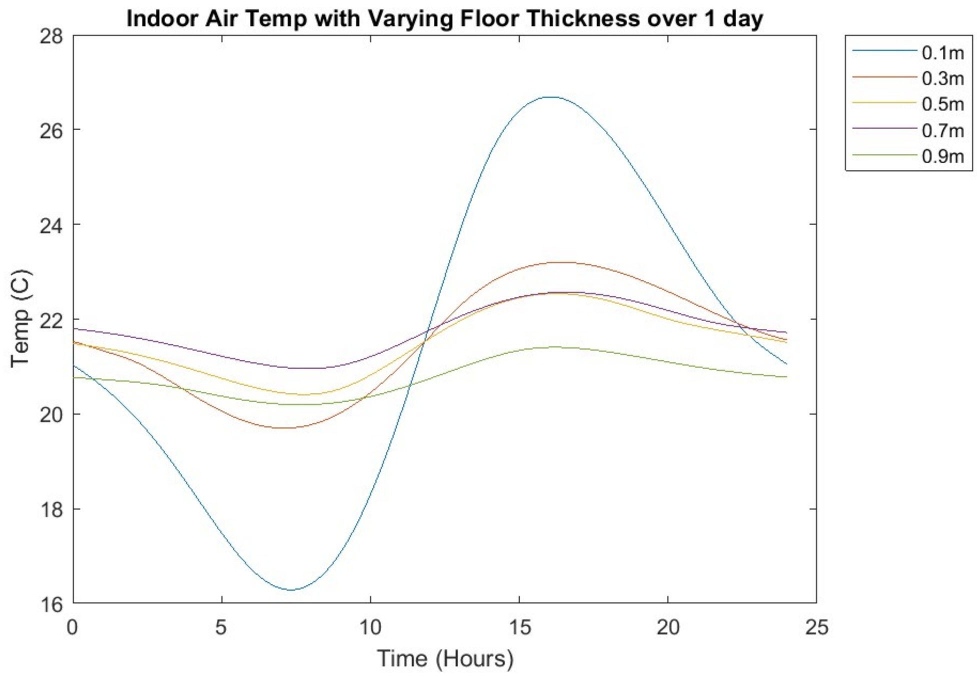

Figure 3b. Plot of the Indoor Air Temperature for Varying Floor (Absorber) Thicknesses Over the Course of 1 Day after Reaching Equilibrium

Changing the thickness of the absorber changes the amplitude of the indoor air curve. The smaller the absorber, the more the temperature varies throughout the day. Changing the thickness does not significantly change the average temperature per day though.

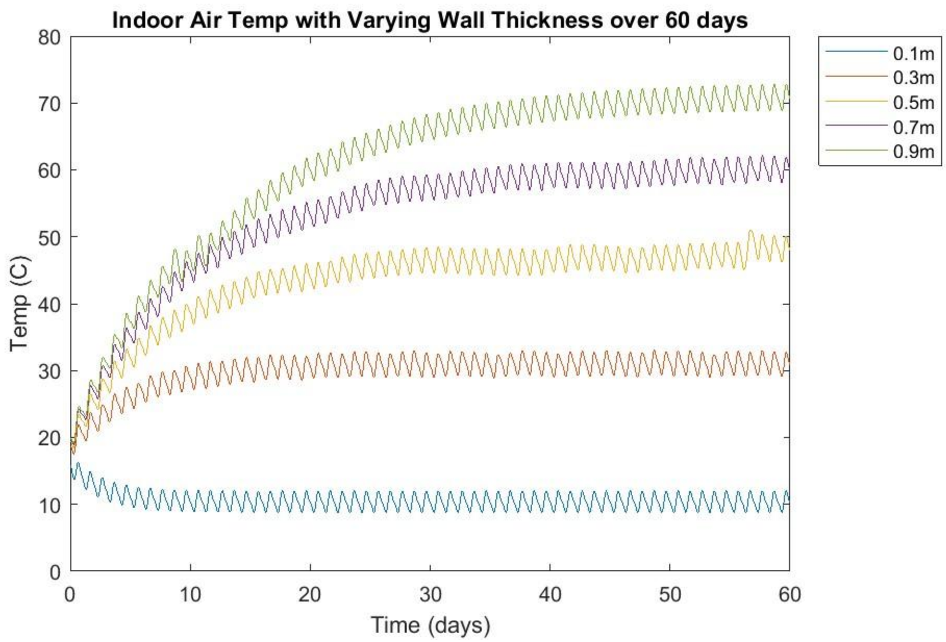

Figure 4a (Left). Plot of the Indoor Air Temperature for Varying Wall Thicknesses Over the Course of 60 Days

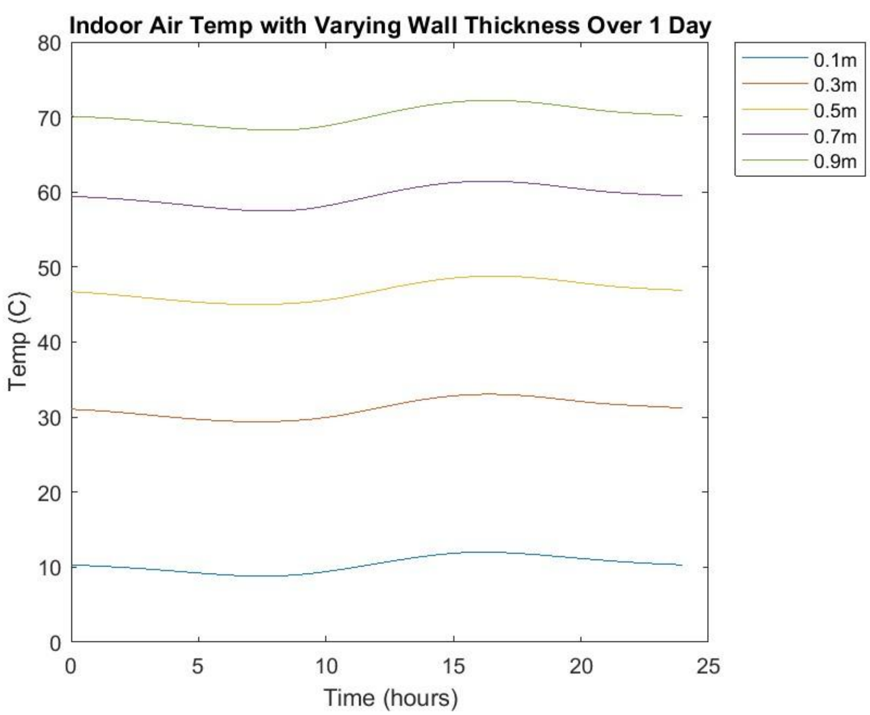

Figure 4b (Left). Plot of the Indoor Air Temperature for Varying Wall Thicknesses Over the Course of 1 Day after Reaching Equilibrium

Changing the wall thickness affects the temperature at which the system reaches equilibrium. This makes sense since the wall thickness affects the thermal resistance of the wall. The thicker the wall, the hotter the air in the house will become, because less heat is transferred to the outside air. The wall thickness determines the average daily temperature of the house.

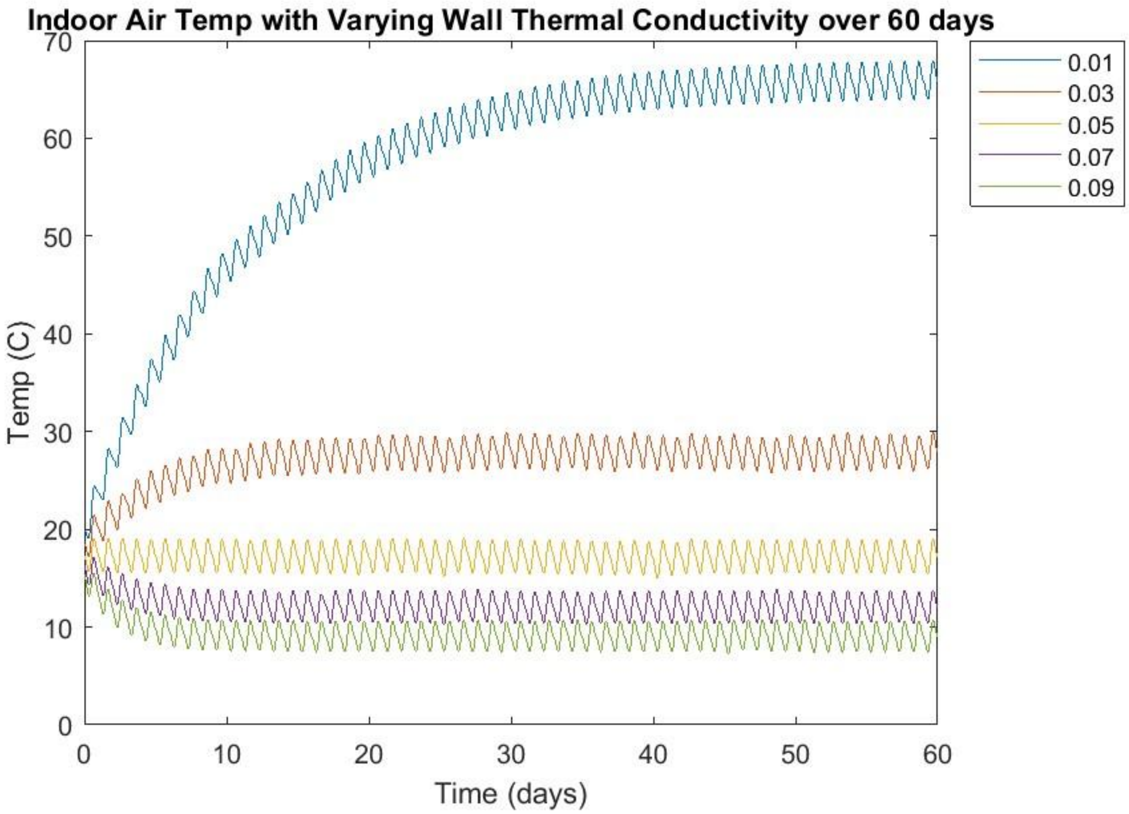

Figure 5a. Plot of the Indoor Air Temperature for Varying Wall Thermal Conductivity Over the Course of 60 Days

We modeled different thermal conductivity of the wall so that we could see how the system might behave with different materials. A wall material with a low thermal conductivity (blue line, figure 5a) is going to cause the indoor air to be hot. A wall material with a higher thermal conductivity (green line, figure 5a) is going to loose heat to the outside air and cause the indoor air to become cold.

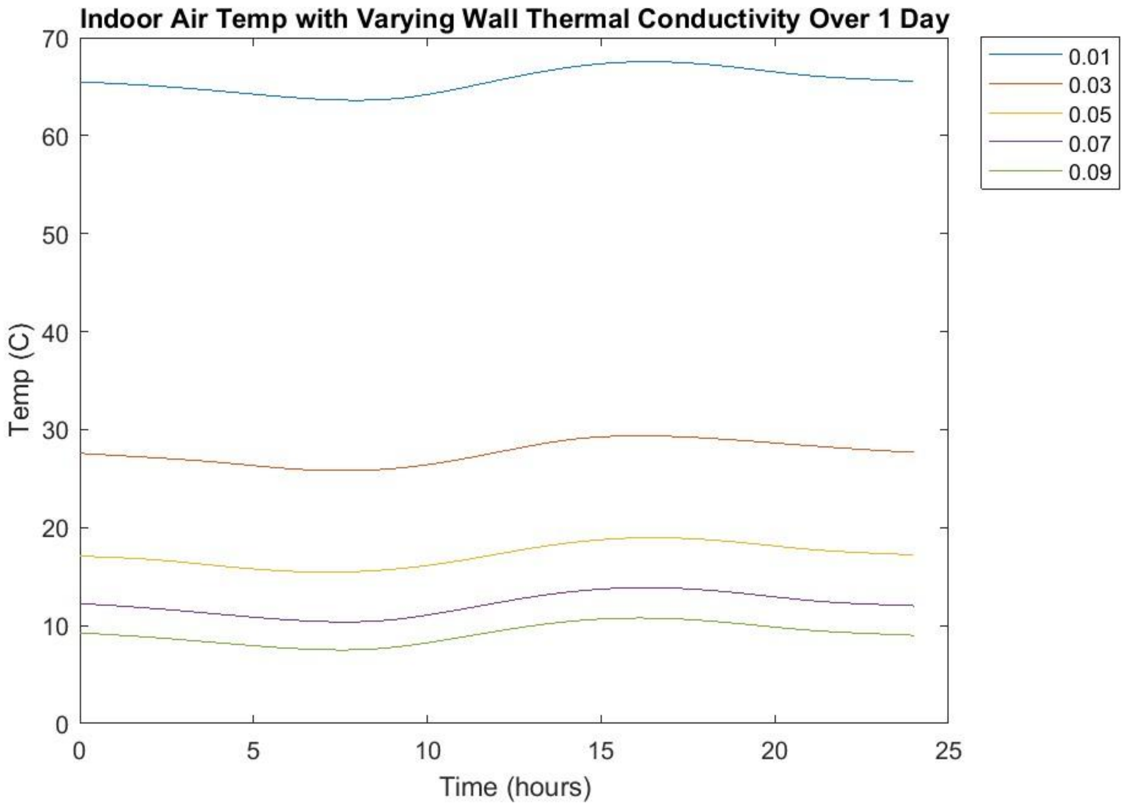

Figure 3b. Plot of the Indoor Air Temperature for Varying Wall Thermal Conductivity Over the Course of 1 Day after Reaching Equilibrium

After doing our parameter sweeps, we found that a wall thickness of 0.2 m and a heat storage unit thickness of 0.3 m keeps the temperature within the house between 17 and 25 degrees Celsius, the temperature range comfortable for the average person.

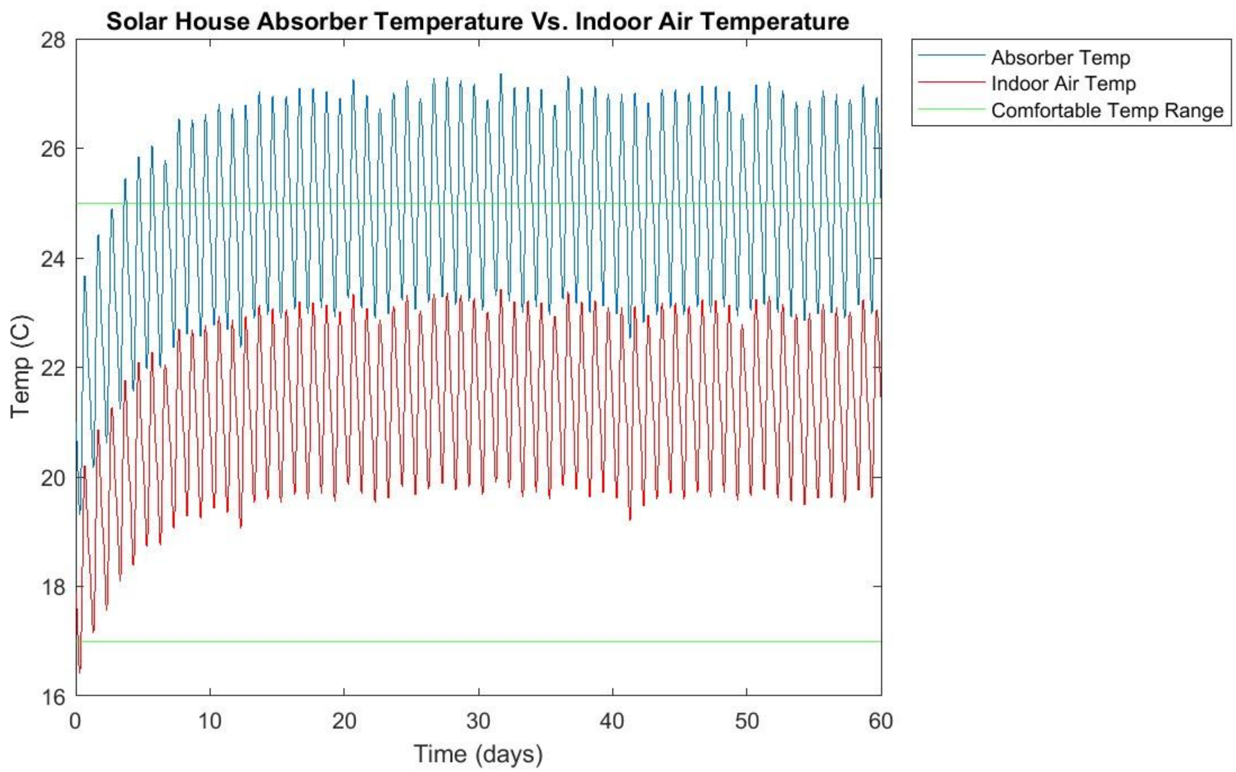

Figure 3a. Plot of the Temperature of the Heat Storage Unit (the Absorber) to the Indoor Air Temperature for Optimal Wall and Absorber Thickness

Initially, we were concerned that the absorber temperature would be too hot to stand on since it is a few degrees above the comfortable air temperature, but human feet feel cold at 25C and warm at 33C (Murphy). Our passive solar house model would remain a comfortable air temperature for your body and your feet, as long as you have some socks.

All code and graphics can be found on our GitHub: [EllieRamos/QEA-3-Solar-House (github.com)]

Sources

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