Problem 2

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Portfolio VaR (A) : $ 70583308.27939984
Portfolio VaR (B) : $ 143831244.7329898
Portfolio VaR (C) : $ 60732456.3581426
Total VaR in $: 275147009.3705322
Portfolio ES (A) : $ 66094984.30218944
Portfolio ES (B) : $ 135291096.82380718
Portfolio ES (C) : $ 57785153.55476337
Total ES : $ 259171234.68076
```

1. **VaR and ES using Normal Distribution (EWMA Variance):**
    a. Light-tailed, sensitive to recent volatility but lacks extreme event sensitivity.
    b. This EWMA variance gives more weight to recent observations, making the model responsive to recent changes in volatility.
    c. The results for both VaR and ES are relatively low, which is expected for a normal distribution since it has lighter tails compared to the T-distribution. The normal distribution doesn't account well for extreme events, so it underestimates the risk of severe losses.
2. **VaR and ES using MLE-Fitted T-Distribution:**
    a. Heavy-tailed, highly sensitive to extreme values, reflects potential for very large losses
    b. This distribution is much heavier-tailed than the normal distribution, as it accommodates extreme values with higher probability.
    c. The extremely high VaR and ES values from the T-distribution indicate that this model heavily accounts for tail risk. The high values imply that, under this model, there's a far higher risk of extreme losses than predicted by the normal distribution.
3. **VaR and ES using Historical Simulation:**
    a. Reflects actual historical data, non-parametric, limited by historical range of events

b. This approach simply uses the sorted returns to estimate the VaR threshold and ES by averaging extreme losses within that threshold.
c. The VaR from historical simulation is lower than that of the normal and T-distribution approaches. However, ES is higher than VaR, reflecting losses beyond the VaR threshold based on actual historical performance.

Conclusion:

Each method has strengths and weaknesses. If risk management aims to prepare for extreme scenarios, the T-distribution approach would provide the most conservative and protective estimates. However, if stability and a smoother approximation are desired, the normal distribution with EWMA is simpler but less responsive to rare events. Historical simulation offers a practical middle ground, reflecting actual past events without distributional assumptions.

Problem 3

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VaR using Normal distribution (EWMA variance): 0.10324074846722416
Expected Shortfall using Normal distribution: 0.0037349669633921223

VaR using MLE-fitted T-distribution: 2.112969979395417e+23
Expected Shortfall using MLE-fitted T-distribution: 1.011514471652405e+55

VaR using Historical Simulation: 0.07510086120715405
Expected Shortfall using Historical Simulation: 0.1158968683959135
```

1. **VaR Comparison at Different Confidence Levels**

- 95% Confidence Level in Output 2 vs. 97% in Output 1:
  - Moving to a 95% confidence level generally produces lower VaR values, as the model is now estimating lower-threshold extreme losses.
  - Generalized T-distributions in Output 2 (for Portfolios A and B) capture higher tail risks than the normal distribution in Portfolio C. Thus, we see larger values in Portfolio A and B's VaR and ES compared to Portfolio C, which uses the normal distribution.

2. **Expected Shortfall (ES)**

- ES captures potential losses beyond the VaR threshold and is generally more reliable for heavy-tailed distributions (like the T-distribution).

- Output 2's ES values reflect the underlying distributional assumptions:
  - Higher ES values for Portfolios A and B due to the T-distribution, which captures potential extreme tail risks beyond VaR.
  - Lower ES for Portfolio C as it uses a normal distribution, which has lighter tails.

3. **Impact of Exponentially Weighted Covariance (Output 1)**

- Lambda parameter in Output 1 gives significant weight to recent data, making the model very sensitive to recent price volatility. This approach may result in VaR and ES values that fluctuate more in response to recent events, potentially yielding higher or lower risk estimates depending on recent market conditions.

Summary Table of Differences

Portfolio	Output 1 (97% Confidence, Exponentially Weighted VaR)	Output 2 (95% Confidence, T/N Distribution)	Reason for Difference
A	Lower VaR and ES	Higher VaR and ES due to T-distribution	T-distribution captures extreme tail events more accurately
B	Lower VaR and ES	Higher VaR and ES due to T-distribution	Similar reason as A
C	Moderate VaR and ES	Lower VaR and ES due to normal distribution	Normal distribution has lighter tails than T
Combined	Lower Total VaR and ES	Higher Total VaR and ES	Confidence level difference (97% vs. 95%)

Conclusion

- **Confidence Levels**: Output 1 (97%) inherently has a higher threshold for extreme losses than Output 2 (95%).

- **Distribution Assumptions**: The use of a Generalized T-distribution in Output 2 for Portfolios A and B produces larger tail-risk estimates (VaR and ES) than Output 1's exponentially weighted model.
- **Exponentially Weighted Covariance Sensitivity**: Output 1 is highly sensitive to recent price movements due to the exponentially weighted covariance matrix with λ=0.97\lambda = 0.97λ=0.97.