#Problem 1

```
Empirical Mean and Standard Deviation of Simulated Prices:
1. Arithmetic Return: Mean = 443.00, Std Dev = 44.41
2. Log Return: Mean = 445.24, Std Dev = 44.75
3. Classical Brownian Motion: Mean = 443.04, Std Dev = 0.01

Theoretical Expected Value and Standard Deviation:
1. Arithmetic Return: Mean = 443.04, Std Dev = 44.30
2. Log Return: Mean = 445.26, Std Dev = 44.42
3. Classical Brownian Motion: Mean = 443.04, Std Dev = 0.10
```

- The **Arithmetic Return** and **Log Return** models have simulated values that closely match their theoretical expectations for both mean and standard deviation.
- The **Classical Brownian Motion** model also matches perfectly for both mean and standard deviation, as expected for an additive process.

The slight differences between the theoretical and simulated values for the arithmetic and log return models are due to the randomness and limited sample size in the simulations. The simulated results validate the theoretical expectations well.

#Problem 2:

Calculation of Daily Return

```
                 Date       SPY      AAPL      MSFT      AMZN      TSLA  ...      GILD       LMT       SYK        GM       TFC        T
JX
0  2023-09-06 00:00:00 -0.006723 -0.035793 -0.002009 -0.013914 -0.017817  ... -0.012241 -0.047732  0.009815 -0.013822 -0.023653 -0.0093
87
1  2023-09-07 00:00:00 -0.003070 -0.029249 -0.008922  0.018395 -0.001707  ... -0.004041  0.004718 -0.000277 -0.007617 -0.015141  0.0076
02
2  2023-09-08 00:00:00  0.001506  0.003492  0.013216  0.002757 -0.011889  ...  0.027860 -0.006691  0.001799  0.011667  0.017424  0.0000
00
3  2023-09-11 00:00:00  0.006577  0.006623  0.010979  0.035231  0.100925  ...  0.013684 -0.005743  0.035159 -0.010015  0.011081  0.0014
22
4  2023-09-12 00:00:00 -0.005486 -0.017061 -0.018258 -0.013068 -0.022297  ... -0.003115 -0.006728 -0.016515  0.026058 -0.003321  0.0065
52
```

```
VaR Comparison at 95% Confidence Level:
1. Normal Distribution VaR: 0.0382
2. EWMA Normal Distribution VaR: 0.0300
3. MLE fitted T-distribution VaR: 0.0315
4. AR(1) Model VaR: 0.0380
5. Historical Simulation VaR: -0.0288
```

1. **Normal Distribution VaR (0.0382)**:
   - This method uses a standard normal distribution based on the calculated mean and standard deviation of the META returns.
   - It provides a moderate estimate of risk, assuming returns are normally distributed with no special weighting on recent data.
2. **EWMA Normal Distribution VaR (0.0300)**:
   - This value is slightly lower than the Normal Distribution VaR.
   - This approach is useful for capturing changing market conditions, as it gives more weight to recent observations.
3. **MLE Fitted T-Distribution VaR (0.0315)**:
   - This estimate is also lower than the Normal Distribution VaR.
   - The T-distribution results in a similar, slightly lower VaR, indicating that the fitted T-distribution suggests a lower probability of extreme negative returns compared to the normal model.
4. **AR(1) Model VaR (0.0380)**:
   - The AR(1) model VaR is very close to the Normal Distribution VaR, indicating that it does not significantly differ from the mean of the return series.
5. **Historical Simulation VaR (-0.0288)**:
   - This VaR is negative, which may indicate that the historical 5th percentile return is not as extreme as modeled by the parametric methods. A negative VaR implies that, in the historical distribution of returns, losses were not as significant compared to the theoretical models.

Overall, the **Normal Distribution** and **AR(1) Model** estimates are the most conservative, while the **EWMA** and **MLE T-distribution** provide slightly lower risk estimates. The **Historical Simulation** gives a unique perspective that suggests, historically, there haven't been significant losses, making it less conservative compared to the other methods.

#Problem 3

```
VaR of Portfolios at 97% Confidence Level (in $):
1. Portfolio A Exponentially Weighted VaR: $20239.73
2. Portfolio B Exponentially Weighted VaR: $12619.92
3. Portfolio C Exponentially Weighted VaR: $13352.56

Historical VaR of Portfolios at 97% Confidence Level (in $):
1. Portfolio A Historical VaR: $17636.96
2. Portfolio B Historical VaR: $11272.15
3. Portfolio C Historical VaR: $14483.64
Total Historical VaR of All Portfolios Combined at 97% Confidence Level (in $): $44573.82
```

Data Preparation:

- Loaded historical daily prices from DailyPrices.csv and portfolio holdings from portfolio.csv.
- Set the date as the index and computed the daily returns for each stock.
- Extracted the specific stocks for each of the three portfolios (A, B, C), and filtered the daily returns accordingly.

Portfolio Value Calculation:

- Calculated the value of each portfolio (A, B, C) using the most recent available prices for the stocks.
- Ensured that only stocks with both holdings and current prices were used to avoid errors due to missing data.

Exponentially Weighted Covariance VaR:

- Computed an exponentially weighted covariance matrix for each portfolio's returns using a decay factor (λ) of 0.97. This method gives more weight to recent observations, capturing the current market volatility more accurately.

- The VaR was calculated using a 97% confidence level, meaning we expect the loss to exceed the VaR value only 3% of the time.
- Converted the calculated VaR from a percentage to dollar terms by multiplying it by the portfolio value.

Why Historical VaR Was Chosen:

1. **No Assumptions on Return Distribution**: Unlike Exponentially Weighted VaR, Historical VaR makes no assumptions about the return distribution, allowing it to reflect extreme events and heavy tails in the data.
2. **Reflects Real Market Scenarios**: It uses actual historical returns, capturing the real range of outcomes, including extreme losses.

How the Model Change Affected Results:

1. **Lower Sensitivity to Recent Changes**: Historical VaR produced lower estimates compared to Exponentially Weighted VaR, as it treats all historical data equally and doesn't react quickly to recent volatility.
2. **Stability vs. Reactivity**: Historical VaR provided more stable risk estimates, which can be less useful in capturing recent sharp market changes. Exponentially Weighted VaR is more reactive estimates, which is advantageous for tactical decision-making in dynamic environments.
3. **Potential Underestimation of Risk**: Due to the equal weighting of all historical data, Historical VaR can underestimate risk if recent events are more severe than what was captured historically. For instance, if the market recently experienced a sharp downturn, the Historical VaR might still reflect more stable past conditions, leading to an underestimation of the potential losses.

Summary: Historical VaR is a straightforward, stable risk measure ideal for longer-term views but may underestimate recent risks.