

0.1 Fourier Spectrums

0.1.1 Varying Tolerance

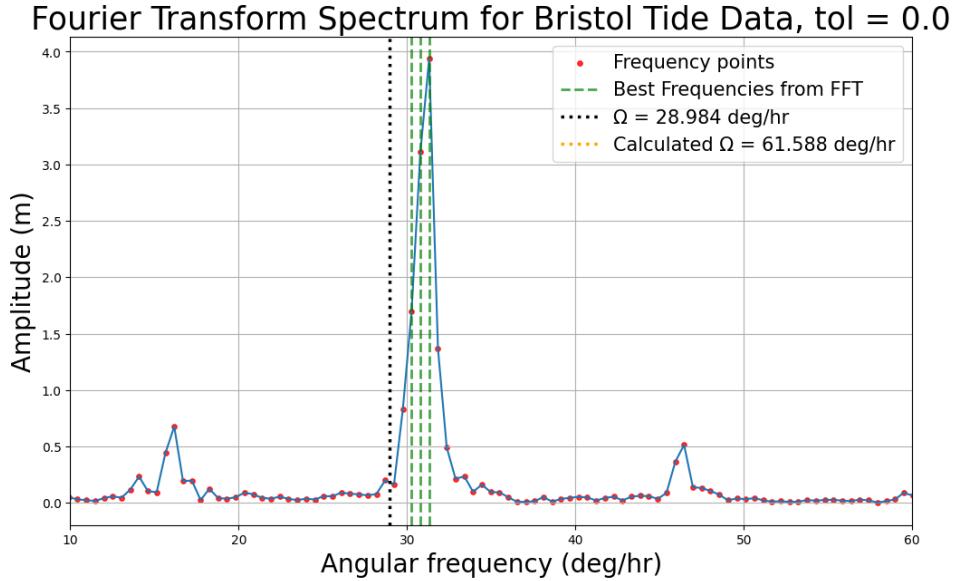


Figure 1: Spectrum of frequencies for a Fourier Transform on the tidal data for Bristol, against their amplitudes. The strongest frequencies are highlighted in green, and there is no tolerance. The known $\Omega = 28.984 \text{ deg hr}^{-1}$ is shown for reference (black dotted line), as well as the angular frequency calculated from the FFT components (orange dotted line).

As illustrated in Figure (1), when the tolerance is absent, frequencies are selected solely based on their spectral amplitudes. This leads to the selection of multiple closely spaced frequencies. Since these neighbouring frequencies represent the same underlying oscillation, the resulting sine and cosine terms are nearly identical and therefore redundant. Consequently, the model behaves similarly to a simple single-frequency model. Figure (1) also displays that the calculated $\Omega = 61.588 \text{ deg/hr}$, using equation (1) where $\omega_1, \omega_2, \omega_3$ represent the best 3 frequencies, is far higher than expected.

$$\Omega = 2(\omega_1 - \omega_2 + \omega_3) = 28.984 \text{ deg/hr}. \quad (1)$$

Figure (2) shows the spectrum of frequencies when a tolerance of 0.02 is imposed. As we can see, if the tolerance is set too high, frequency components are incorrectly grouped together and removed, leading to the loss of important harmonic information. Selected frequencies are picked for smaller, less significant peaks and chosen primarily on distance from each other rather than the contribution they have to the tides' structure.

Fourier Transform Spectrum for Bristol Tide Data, tol = 0.03

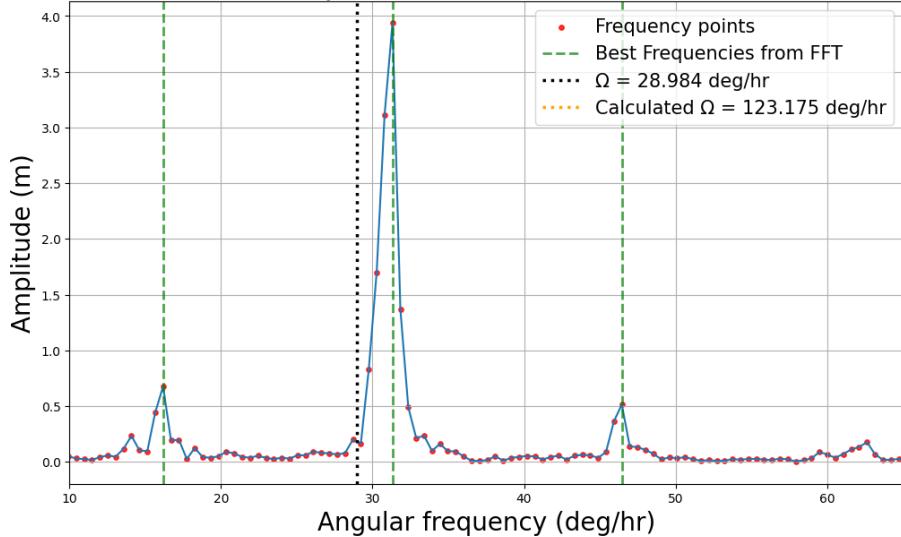


Figure 2: Spectrum of frequencies for a Fourier Transform on the tidal data for Bristol, against their amplitudes. The strongest clustered frequencies are highlighted in green, and the tolerance is 0.03. The known $\Omega = 28.984 \text{ deg hr}^{-1}$ is shown for reference (black dotted line), as well as the angular frequency calculated from the clustered FFT components (orange dotted line).

Together, these results highlight the importance of a controlled tolerance value, preventing the selection of redundant neighbouring frequencies whilst retaining distinct harmonic components.

0.1.2 Varying Location

As there was limited recorded data for Donaghadee, we investigated Bangor, as it is the nearest flood monitoring station to Donaghadee, and shares a similar geographical environment. We collected data from Bangor (Station ID E73839), across 4 weeks, 7th January - 4th February

As displayed in Figure (3), the frequency spectrum following a Fourier transform on the collected data shows very little change to our original spectrum plot for Bristol, Figures (1, 2). There is some minor change in the amplitude of some of the peaks; however, the dominant frequencies remain consistent.

We extended the Fourier frequency analysis to a third UK location, Barmouth (Station ID E73239), in order to examine how the tidal spectrum varies across different coastal sites. As shown in Figure (4), the resulting frequency spectrum is very similar to those observed previously, with the

Fourier Transform Spectrum for Bangor Tide Data, tol = 0.01

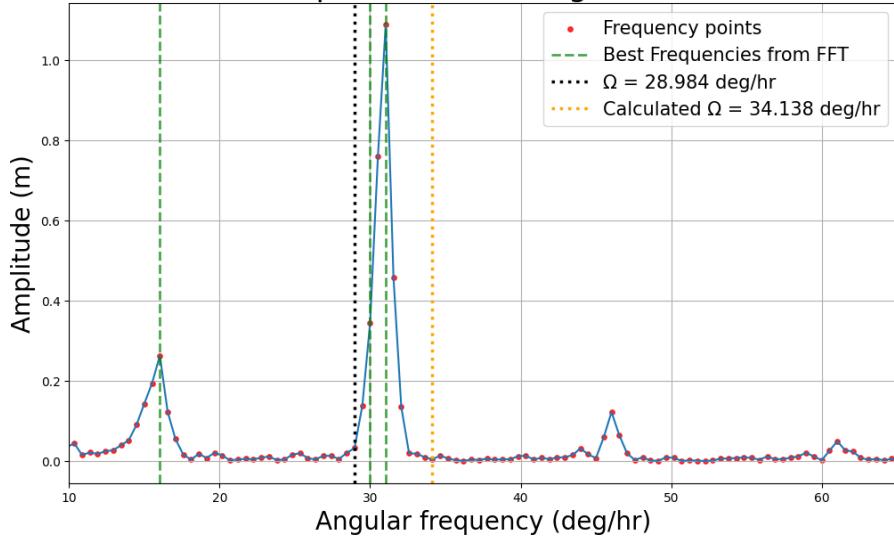


Figure 3: Spectrum of frequencies for a Fourier Transform on the tidal data for Bangor, against their amplitudes. The strongest clustered frequencies are highlighted in green, and the tolerance is 0.01. The known $\Omega = 28.984 \text{ deg hr}^{-1}$ is shown for reference (black dotted line), as well as the angular frequency calculated from the clustered FFT components (orange dotted line).

same dominant frequencies clearly present. This indicates that tidal frequencies are largely independent of location and are instead governed by astronomical forces, primarily the Earth's rotation, the Moon's orbit, and the Earth's motion around the Sun.

Using equation (1) a near identical value for Ω is calculated at each of these locations, 34.447 deg/hr for Bristol, 34.138 deg/hr for Bangor and 34.547 deg/hr for Barmouth. This could be computed and averaged across more locations within the UK to achieve a more accurate result.

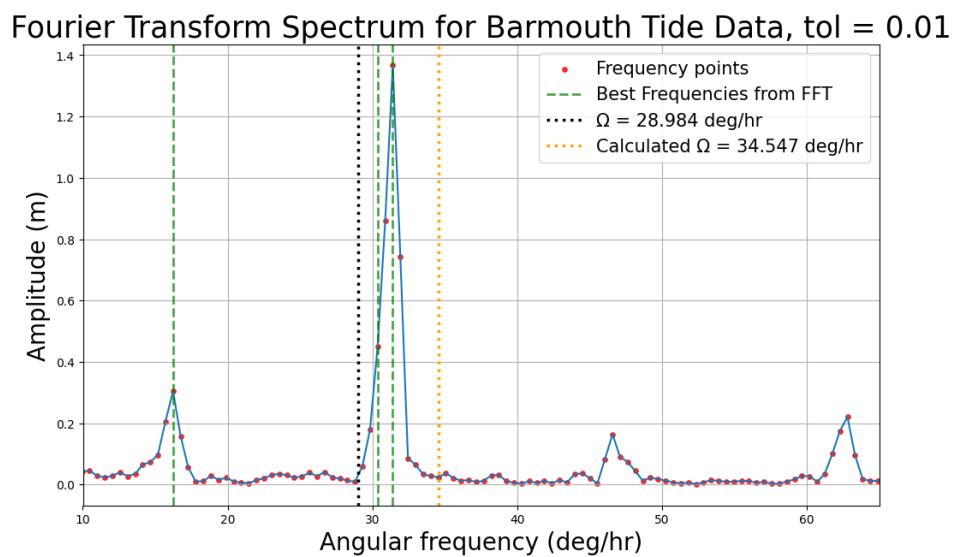


Figure 4: Spectrum of frequencies for a Fourier Transform on the tidal data for Barmouth, against their amplitudes. The strongest clustered frequencies are highlighted in green, and the tolerance is 0.01. The known $\Omega = 28.984 \text{ deg hr}^{-1}$ is shown for reference (black dotted line), as well as the angular frequency calculated from the clustered FFT components (orange dotted line).