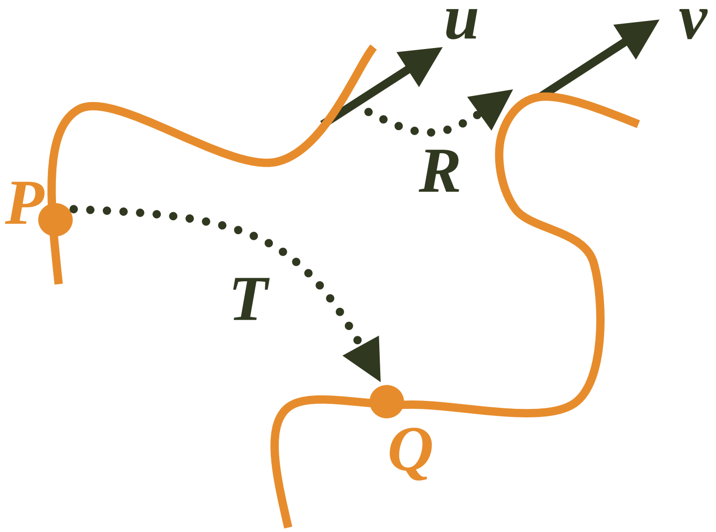
Transformations (1)

5TH WEEK, 2021

Transformations

- Take a point (or vector) and map that point (or vector) into another point (or vector)



4D Column Matrices

$$Q = T(P) \longrightarrow \mathbf{q} = f(\mathbf{p})$$

Homogeneous Coordinate

$$v = R(u) \longrightarrow \mathbf{v} = f(\mathbf{u})$$

Transformation function

Affine Transformations (1)

- Linearity – linear function

$$f(\alpha p + \beta q) = \alpha f(p) + \beta f(q)$$

- Linear transformation
  - Transforming the representation of a point (or vector) into another representation of a point (or vector)

$$\mathbf{v} = \mathbf{A}\mathbf{u}$$

$$\mathbf{A} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ 1 \end{bmatrix}$$

4×4 Matrix Vector Point

Affine Transformations (2)

- Linear transformation (cont')
  - Preserving lines – transforming a line into another line
  - ➔ Only transforming the endpoints of a line segment
- Most transformations in CG are affine
  - Rotation, translation, scaling, and shearing

$$P(\alpha) = P_0 + \alpha d$$

$\downarrow$ Homogeneous Coordinate

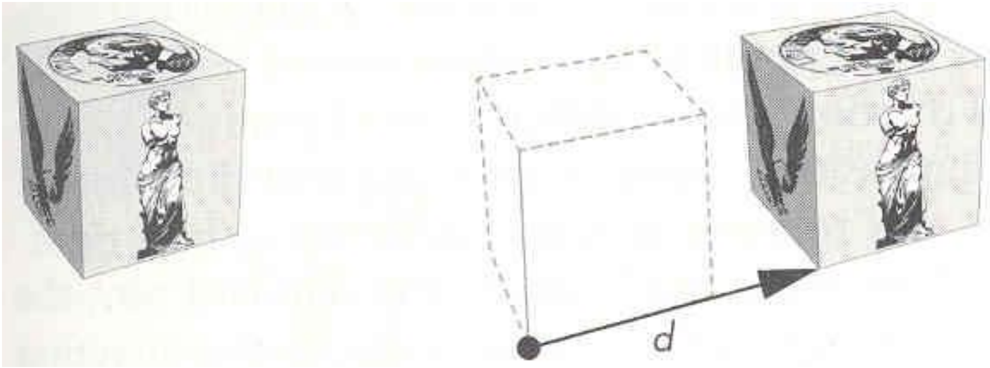
$$\mathbf{p}(\alpha) = \mathbf{p}_0 + \alpha \mathbf{d}$$

$\downarrow$ Affine Transformation

$$\mathbf{Ap}(\alpha) = \mathbf{Ap}_0 + \alpha \mathbf{Ad}$$

Translation

- Operation that displace points by a fixed distance in a given direction
  - Displacement vector $\boldsymbol{d}$



(a) Object in original position

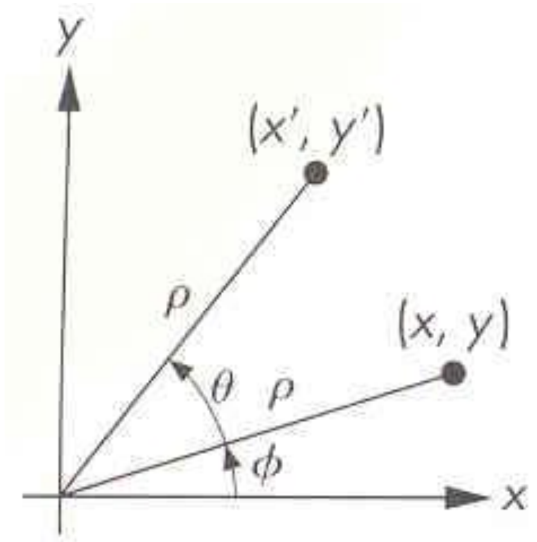
(b) Object translated

$$P' = P + d$$

Rotation (1)

- Simple example of 2D rotation

$$x = \rho \cos \phi$$
$$y = \rho \sin \phi$$
$$x' = \rho \cos(\theta + \phi)$$
$$y' = \rho \sin(\theta + \phi)$$



Rotation (1)

- Simple example of 2D rotation

$$x = \rho \cos\phi$$
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$$x' = \rho \cos(\theta + \phi)$$
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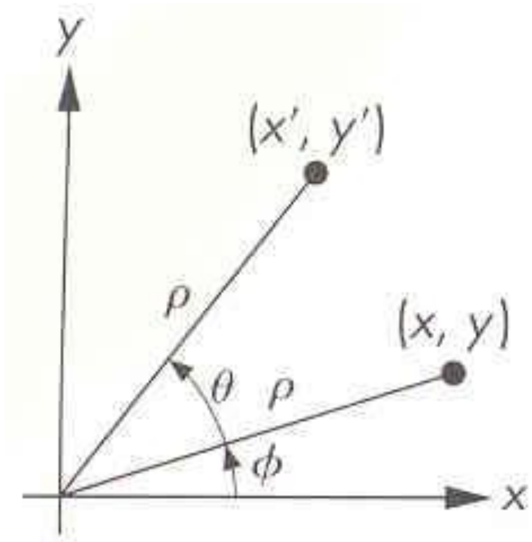
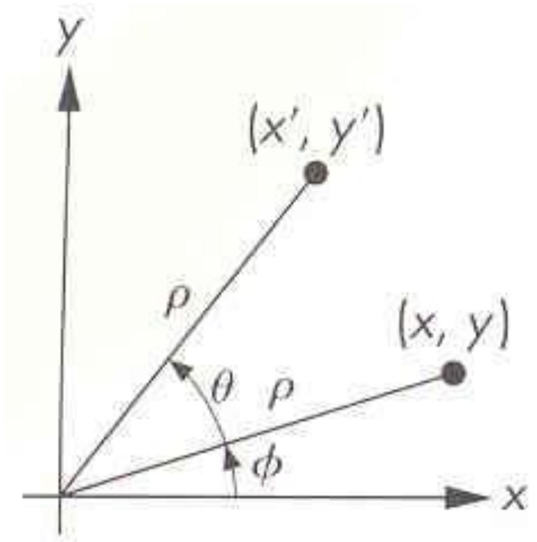


$$x' = \rho \cos\phi \cos\theta - \rho \sin\phi \sin\theta = x\cos\theta - y\sin\theta$$
$$y' = \rho \cos\phi \sin\theta + \rho \sin\phi \cos\theta = x\sin\theta + y\cos\theta$$

Rotation (1)

- Simple example of 2D rotation

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$x' = \rho\cos(\theta+\phi)$$
$$y' = \rho\sin(\theta+\phi)$$

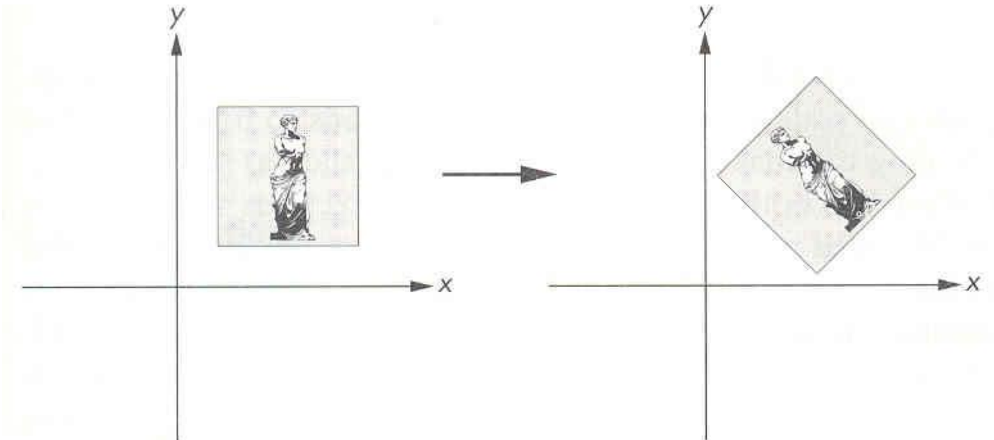


$$x' = \rho\cos\phi\cos\theta - \rho\sin\phi\sin\theta = x\cos\theta - y\sin\theta$$
$$y' = \rho\cos\phi\sin\theta + \rho\sin\phi\cos\theta = x\sin\theta + y\cos\theta$$

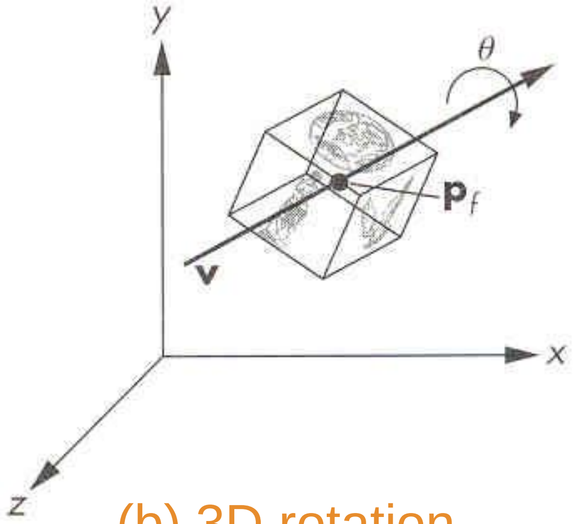
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation (2)

- Needs
  - Fixed point – a point is unchanged by the rotation
  - Rotation angle – positive rotation (counterclockwise in right hand system)
  - Rotation axis in 3D – values on axis are unchanged by the rotation



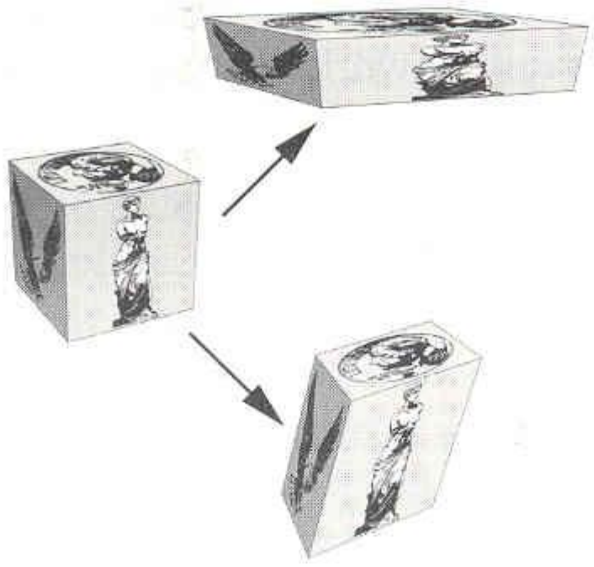
(a) Rotation about a fixed point



(b) 3D rotation

Rigid-Body Transformations

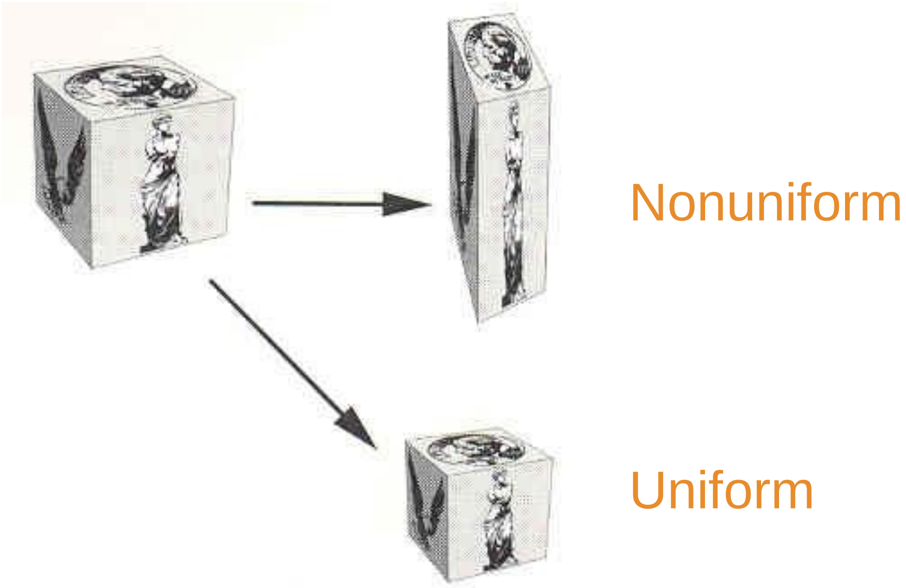
- Rotation and translation
- No combination of rotations and translations can alter the shape of object
  - ➔ Altering only the object's location and orientation



Affine transformations, but non-rigid body transformations

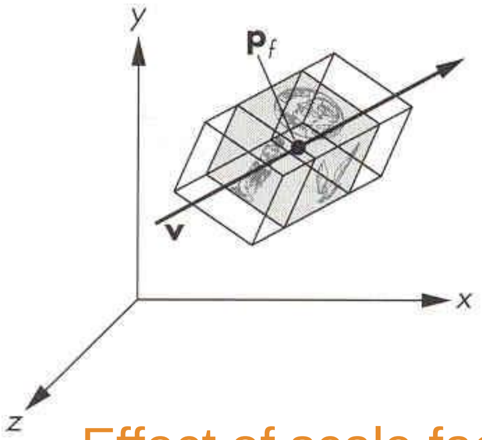
Scaling (1)

- Making an object bigger or smaller
  - Uniform – scaling in all directions
- Affine non-rigid-body transformations
  - Scaling and shearing
  - Cf) rigid-body : translation and rotation

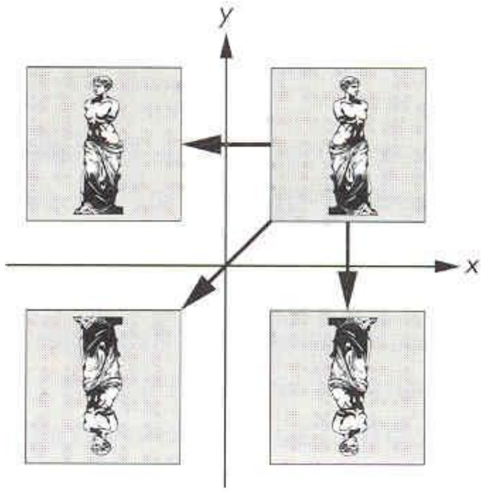


Scaling (2)

- Needs
  - <u>Fixed point</u>
  - <u>Direction to scale</u>
  - <u>Scale factor</u>
    - Longer (α>1) or smaller (0≤α<1)
- Reflection – negative scale factor



Effect of scale factor



Reflection

Transformations in Homogeneous Coordinates

- Representations in homogeneous coordinates

$$Q = P + \alpha v \longrightarrow \mathbf{q} = \mathbf{p} + \alpha \mathbf{v} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ 0 \end{bmatrix}$$

- Affine transformation – 4×4 matrix

$$\mathbf{M} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation

- Point $\mathbf{p}$ to $\mathbf{p}'$ by displacing by a distance d

$$\mathbf{p}' = \mathbf{p} + \mathbf{d} \longrightarrow \mathbf{p}' = T\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, \quad \mathbf{p}' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ 0 \end{bmatrix}$$

$$x' = x + \alpha_x$$
$$y' = y + \alpha_y$$
$$z' = z + \alpha_z$$

?

Translation matrix

Translation

- Point $\mathbf{p}$ to $\mathbf{p'}$ by displacing by a distance d

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$$x' = x + \alpha_x$$
$$y' = y + \alpha_y$$
$$z' = z + \alpha_z$$

$$T = \begin{bmatrix} 1 & 0 & 0 & \alpha_x \\ 0 & 1 & 0 & \alpha_y \\ 0 & 0 & 1 & \alpha_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation

- Point $\mathbf{p}$ to $\mathbf{p'}$ by displacing by a distance d

$$\mathbf{p}' = \mathbf{p} + \mathbf{d} \longrightarrow \mathbf{p}' = T\mathbf{p}$$

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- Inverse of a translation matrix

$$T^{-1}(\alpha_x, \alpha_y, \alpha_z) = T(-\alpha_x, -\alpha_y, -\alpha_z) = \quad ?$$

Translation

- Point $\mathbf{p}$ to $\mathbf{p}'$ by displacing by a distance d

$$\boxed{\mathbf{p}' = \mathbf{p} + \mathbf{d}} \longrightarrow \boxed{\mathbf{p}' = T\mathbf{p}}$$

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$$T^{-1}(\alpha_x, \alpha_y, \alpha_z) = T(-\alpha_x, -\alpha_y, -\alpha_z) = \begin{bmatrix} 1 & 0 & 0 & -\alpha_x \\ 0 & 1 & 0 & -\alpha_y \\ 0 & 0 & 1 & -\alpha_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

- Scaling matrix with a fixed point of the origin

$$x' = \beta_x x$$
$$y' = \beta_y y$$
$$z' = \beta_z z$$

$$\xrightarrow{S(\beta_x, \beta_y, \beta_z)}$$

$$\mathbf{p}' = S\mathbf{p}$$

?

Scaling matrix

Scaling

- Scaling matrix with a fixed point of the origin

$$x' = \beta_x x$$
$$y' = \beta_y y$$
$$z' = \beta_z z$$

$$s(\beta_x, \beta_y, \beta_z)$$

$$\mathbf{p}' = S\mathbf{p}$$

$$S = \begin{bmatrix} \beta_x & 0 & 0 & 0 \\ 0 & \beta_y & 0 & 0 \\ 0 & 0 & \beta_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

- Scaling matrix with a fixed point of the origin

$$x' = \beta_x x$$
$$y' = \beta_y y$$
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$$\xrightarrow{S(\beta_x, \beta_y, \beta_z)} \quad \mathbf{p}' = S\mathbf{p}$$

$$S = \begin{bmatrix} \beta_x & 0 & 0 & 0 \\ 0 & \beta_y & 0 & 0 \\ 0 & 0 & \beta_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Inverse of a scaling matrix

$$S^{-1}(\beta_x, \beta_y, \beta_z) = S\left(\frac{1}{\beta_x}, \frac{1}{\beta_y}, \frac{1}{\beta_z}\right) =$$

?

Scaling

- Scaling matrix with a fixed point of the origin

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$$S^{-1}(\beta_x, \beta_y, \beta_z) = S\left(\frac{1}{\beta_x}, \frac{1}{\beta_y}, \frac{1}{\beta_z}\right) = \begin{bmatrix} 1/\beta_x & 0 & 0 & 0 \\ 0 & 1/\beta_y & 0 & 0 \\ 0 & 0 & 1/\beta_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (1)

- Rotation with a fixed point at the origin

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$\xrightarrow{R_z(\theta)}$ $\mathbf{p}' = R_z\mathbf{p}$

?

Rotation matrix

Rotation (1)

- Rotation with a fixed point at the origin

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\xrightarrow{R_z(\theta)} \quad \mathbf{p}' = R_z\mathbf{p}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x = R_x(\theta) = \quad ?$$

Rotation (1)

- Rotation with a fixed point at the origin

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\xrightarrow{R_z(\theta)}$$

$$\mathbf{p}' = R_z\mathbf{p}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x = R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y = R_y(\theta) = \text{?}$$

Rotation (1)

- Rotation with a fixed point at the origin

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\xrightarrow{R_z(\theta)} \quad \mathbf{p}' = R_z\mathbf{p}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x = R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y = R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (2)

- Inverse of a rotation matrix

$$R^{-1}(\theta) = R(-\theta)$$

$$\downarrow$$

$$\cos(-\theta) = \cos\theta, \quad \sin(-\theta) = -\sin\theta$$

$$\downarrow$$

$$R_z^{-1}(\theta) = R_z(-\theta) = \quad ?$$

Rotation (2)

- Inverse of a rotation matrix

$$R^{-1}(\theta) = R(-\theta)$$

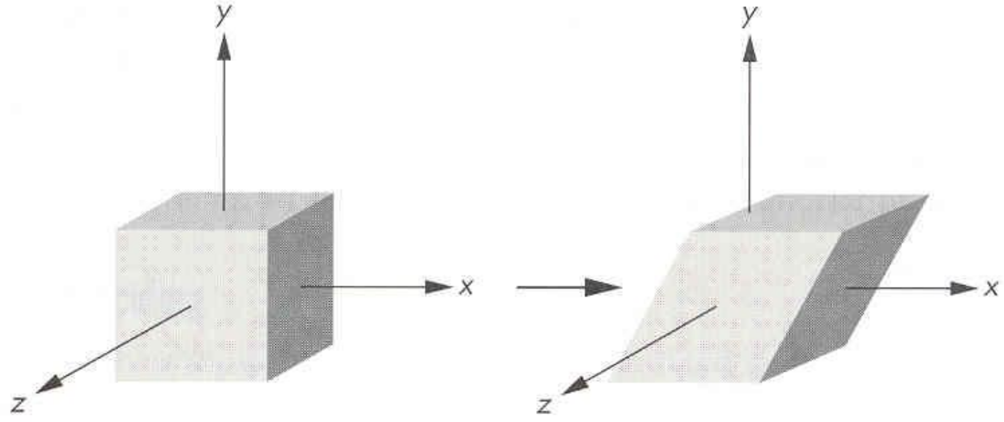
$$\cos(-\theta) = \cos\theta, \quad \sin(-\theta) = -\sin\theta$$

$$R_z^{-1}(\theta) = R_z(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

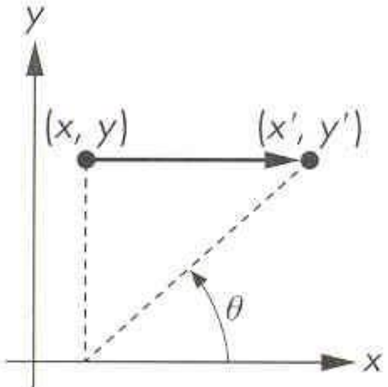
$$\boxed{R^{-1} = R^T}$$: Orthogonal matrix

Shearing (1)

- One more affine transformation



Shearing the object in the x direction



$$x' =$$

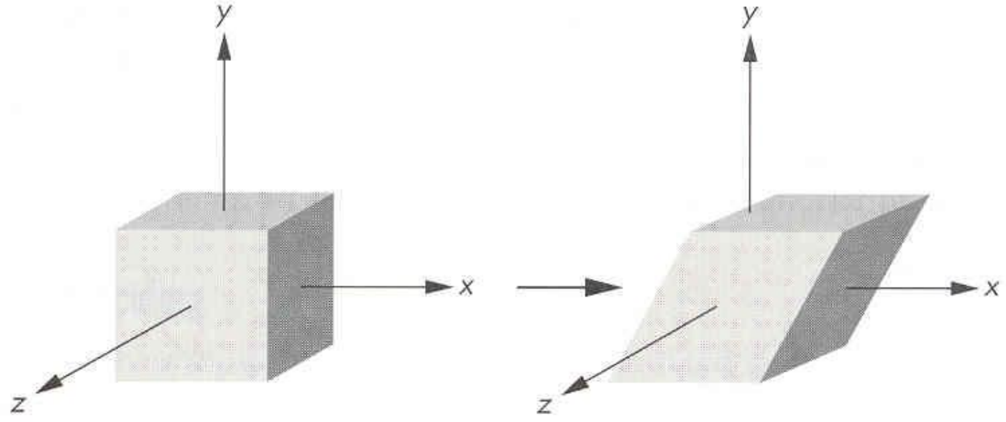
$$y' =$$

$$z' =$$

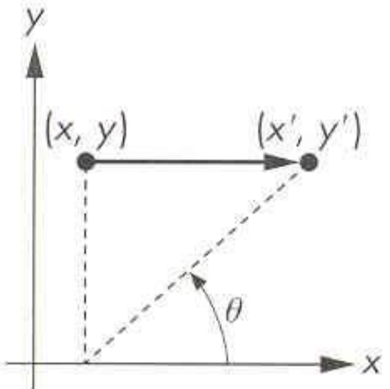
?

Shearing (1)

- One more affine transformation



Shearing the object in the x direction



$$x' = x + y\cot\theta$$
$$y' = y$$
$$z' = z$$

Shearing (2)

- Shearing in the x direction

$$x' = x + y\cot\theta$$
$$y' = y$$
$$z' = z$$

$$\xrightarrow{H_x(\theta)}$$

$$\mathbf{p}' = H_x\mathbf{p}$$

?

Shearing matrix

Shearing (2)

- Shearing in the x direction

$$x' = x + y\cot\theta$$
$$y' = y$$
$$z' = z$$

$$\xrightarrow{H_x(\theta)}$$

$$\mathbf{p}' = H_x\mathbf{p}$$

$$H_x = \begin{bmatrix} 1 & \cot\theta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Shearing (2)

- Shearing in the x direction

$$x' = x + y\cot\theta$$
$$y' = y$$
$$z' = z$$

$$\xrightarrow{H_x(\theta)} \quad \boxed{\mathbf{p}' = H_x\mathbf{p}}$$

$$H_x = \begin{bmatrix} 1 & \cot\theta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Inverse of a shearing matrix

$$H_x^{-1}(\theta) = H_x(-\theta) = \quad ?$$

Shearing (2)

- Shearing in the x direction

$$x' = x + y\cot\theta$$
$$y' = y$$
$$z' = z$$

$$H_x(\theta)$$

$$\mathbf{p}' = H_x\mathbf{p}$$

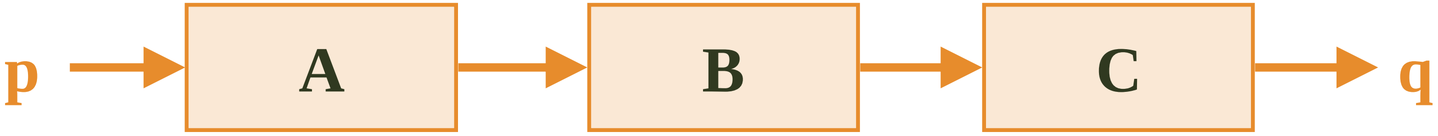
$$H_x = \begin{bmatrix} 1 & \cot\theta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Inverse of a shearing matrix

$$H_x^{-1}(\theta) = H_x(-\theta) = \begin{bmatrix} 1 & -\cot\theta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Concatenation of Transformations

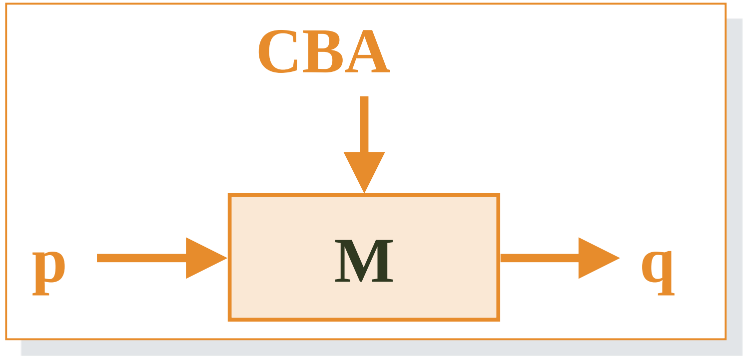
- Concatenating
  - Affine transformations by multiplying together
  - Sequences of the basic transformations
    ➔ Defining an arbitrary transformation directly
  - Ex) three successive transformations



$$\mathbf{q} = (\mathbf{C}(\mathbf{B}(\mathbf{A}\mathbf{p}))) = \mathbf{CBAp}$$

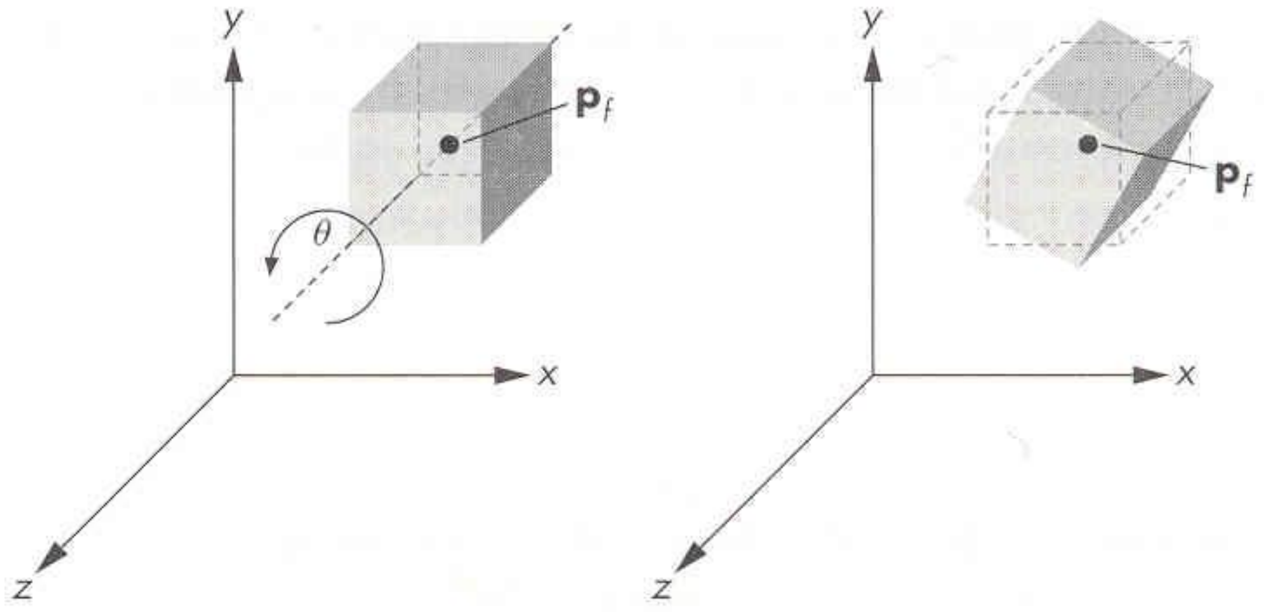
$$\mathbf{M} = \mathbf{CBA}$$

$$\mathbf{q} = \mathbf{Mp}$$



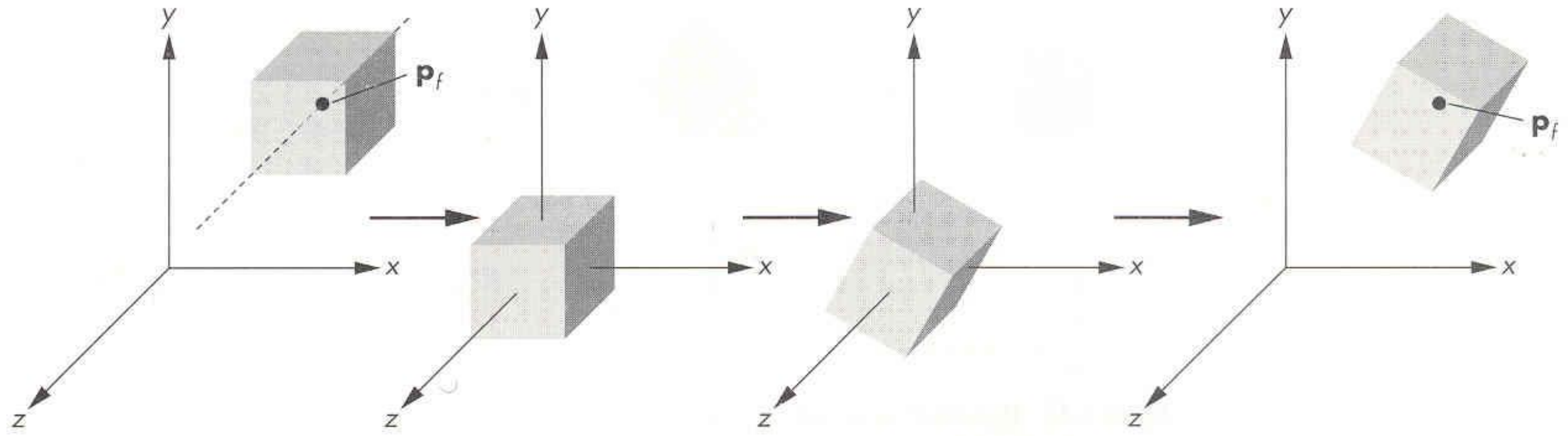
Rotation about a Fixed Point (1)

- Fixed point: $\mathbf{p}_f$
  - Applying $R_z(\theta)$ to rotation about a fixed point



Rotation of a cube about its center

Rotation about a Fixed Point (2)



Sequence of transformations

$$\mathbf{M} = T(p_f)R_z(\theta)T(-p_f)$$

Rotation about a Fixed Point (3)

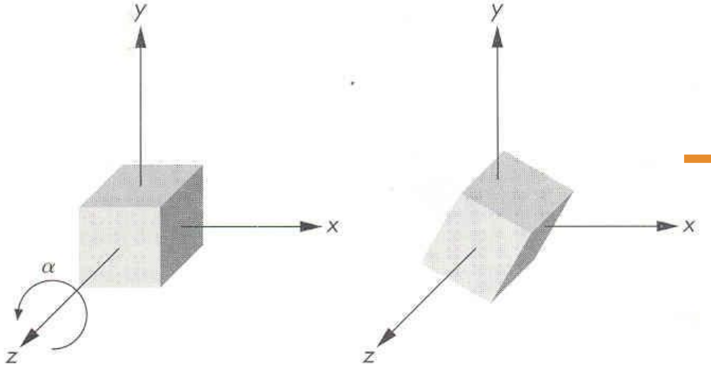
$$\mathbf{M} = T(p_f)R_z(\theta)T(-p_f)$$

$$\begin{bmatrix} 1 & 0 & 0 & x_f \\ 0 & 1 & 0 & y_f \\ 0 & 0 & 1 & z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_f \\ 0 & 1 & 0 & -y_f \\ 0 & 0 & 1 & -z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

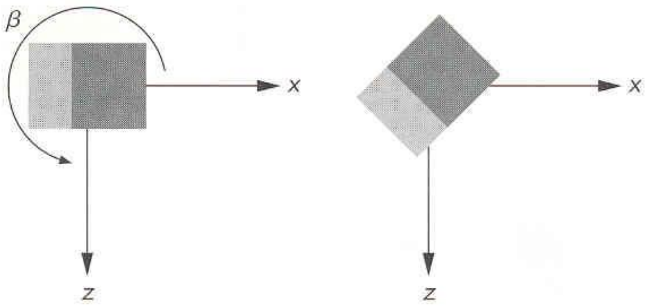
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & x_f - x_f\cos\theta + y_f\sin\theta \\ \sin\theta & \cos\theta & 0 & y_f - x_f\sin\theta - y_f\cos\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

General Rotation (1)

- Three successive rotations about the three axes

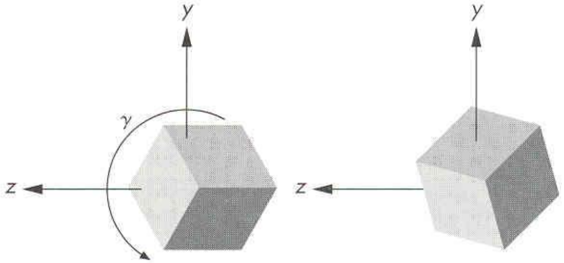


Rotation of a cube about the z axis



Rotation of a cube about the y axis

?



Rotation of a cube about the x axis

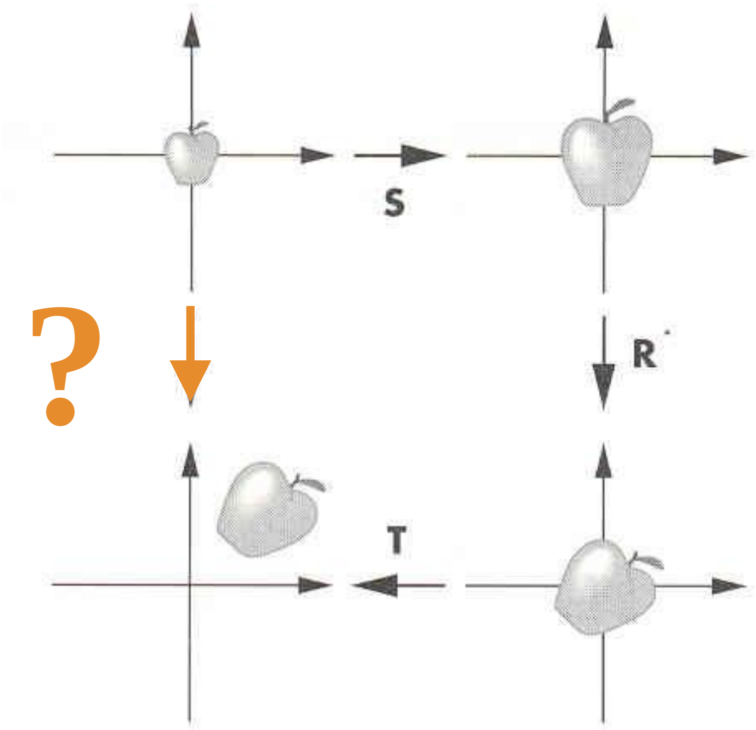
General Rotation (2)

$$\mathbf{R} = R_x R_y R_z$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma & 0 \\ 0 & \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Instance Transformation (1)

- Instance of an object's prototype
  - Occurrence of that object in the scene
- Instance transformation
  - Applying an affine transformation to the prototype to obtain desired size, orientation, and location



Instance transformation

Instance Transformation (2)

$$\mathbf{M} = \mathbf{TRS}$$

$$\begin{bmatrix} 1 & 0 & 0 & \gamma_x \\ 0 & 1 & 0 & \gamma_y \\ 0 & 0 & 1 & \gamma_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_x & 0 & 0 & 0 \\ 0 & \alpha_y & 0 & 0 \\ 0 & 0 & \alpha_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

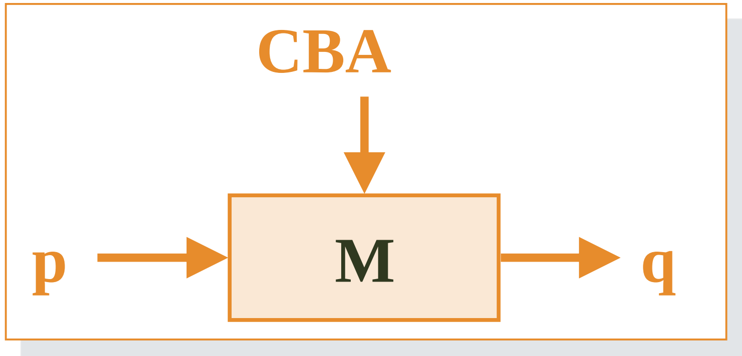
Summary

- Transformations
  - Translation, rotation, scaling, shearing

$$T=\begin{bmatrix}1 & 0 & 0 & \alpha_x\\ 0 & 1 & 0 & \alpha_y\\ 0 & 0 & 1 & \alpha_z\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_z=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad S=\begin{bmatrix}\beta_x & 0 & 0 & 0\\ 0 & \beta_y & 0 & 0\\ 0 & 0 & \beta_z & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

- Concatenation of transformations

$$\mathbf{p} \rightarrow \boxed{\mathbf{A}} \rightarrow \boxed{\mathbf{B}} \rightarrow \boxed{\mathbf{C}} \rightarrow \mathbf{q}$$



$$\mathbf{q}=\left(\mathbf{C}\left(\mathbf{B}\left(\mathbf{A}\mathbf{p}\right)\right)\right)=\mathbf{CBAp}$$

$$\mathbf{q}=\mathbf{Mp}$$

  - Ex) $\mathbf{M}=T\left(p_f\right)R_z(\theta)T\left(-p_f\right)$ $\quad$ $\mathbf{R}=R_x R_y R_z$ $\quad$ $\mathbf{M}=\mathbf{TRS}$

수고하셨습니다