Proof that the French and English ways of calculating Variance are the same

$$V(X) = \sum_i p_i(x_i - \bar{x})^2 = \overline{x^2} - \bar{x}^2$$

We can start by defining $\sum_i p_i x_i = \bar{x} = E(X)$:

$$\begin{aligned}
V(x) &= \sum_i p_i(x_i - \bar{x})^2 \\
&= \sum_i p_i \left((\bar{x}^2 - 2\bar{x}x_i + (x_i)^2\right) \\
&= \sum_i p_i(\bar{x})^2 - 2\bar{x}p_i x_i + p_i(x_i)^2 \\
&= (\bar{x})^2 \left(\sum_i p_i\right) - 2\bar{x}\left(\sum_i x_i p_i\right) + \sum_i p_i(x_i)^2 \\
&= (\bar{x})^2 \cdot 1 - 2\bar{x}\bar{x} + \sum_i p_i(x_i)^2 \\
&= (\bar{x})^2 - 2(\bar{x})^2 + \overline{x^2} \\
&= \overline{x^2} - \bar{x}^2
\end{aligned}$$

Donc, $V(x) = \sum_i p_i(x_i - \bar{x})^2 = \overline{x^2} - \bar{x}^2$ $\square$