Assignment #7

Elliot Smith

10/3/2018

Problem 1

Prior Distribution:

$$\begin{aligned} p(\theta) &= p(\mu)p(\phi)p(\tau) \\ &= \frac{1}{\sqrt{200\pi}} e^{-\frac{1}{2\times 100}\mu^2} \times \frac{1}{\sqrt{200\pi}} e^{-\frac{1}{2\times 100}\phi^2} \times \frac{0.01^0.01}{\Gamma(0.01)} \tau^{0.01-1} e^{-0.01\tau} \\ &\propto e^{\mu^2} \times e^{\phi^2} \times \tau^{-0.99} e^{-0.01\tau} \\ &= e^{\mu^2+\phi^2-0.01\tau} \tau^{-0.99} \end{aligned}$$

Likelihood:

$$\begin{aligned} p(y|\theta) &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}y^2} \\ &\propto e^{-\frac{y^2}{2\sigma^2}} \end{aligned}$$

Posterior Distribution:

$$\begin{aligned} p(\theta|y) &\propto p(y|\theta)p(\theta) \\ &\propto e^{-\frac{y^2}{2\sigma^2}} \times e^{\mu^2+\phi^2-0.01\tau} \tau^{-0.99} \\ &\propto e^{\mu^2+\phi^2-0.01\tau-\frac{y^2}{2\sigma^2}} \tau^{-0.99} \end{aligned}$$

Part i

Part ii

Part iii

Part iv

Part v

Problem 2

Part i

Part ii

Part i