Assignment #5

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Problem 1

Part i

Prior Distribution: $p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}$

Likelihood: $p(y|\theta) = \theta e^{-\theta y}$

Posterior Distribution:

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

$$\propto \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta} \times \theta^n e^{-\sum_{i=1}^n \theta y_i}$$

$$\propto \theta^{(\alpha+n)-1} e^{-(\beta+\sum_{i=1}^n y_i)\theta}$$

$$\sim Gamma(\alpha + n, \beta + \sum_{i=1}^n y_i)$$

Please refer to the Code Appendix section for the sampling technique used.

Part ii

Please refer to the Code Appendix section for the technique used to construct the replicate data set.

Part iii

Please refer to the Code Appendix section for how the test statistics were computed.

Histogram of y_rep_min

Histogram of y_rep_q25

Histogram of y_rep_med

Histogram of y_rep_sd
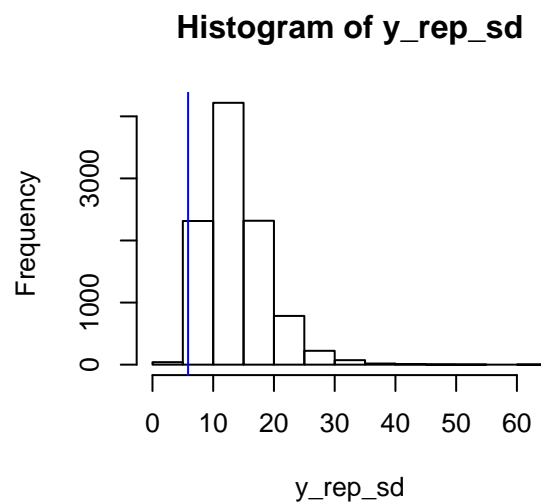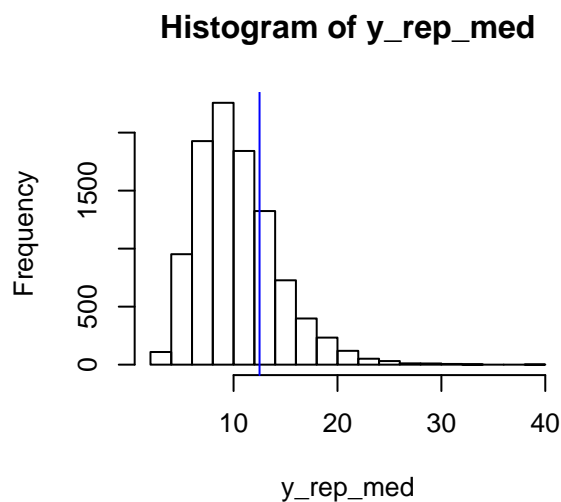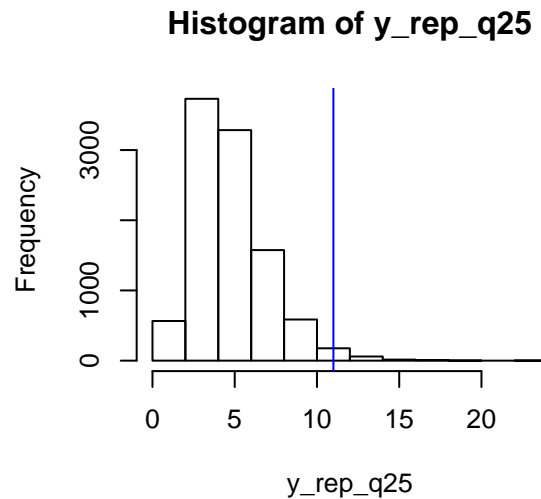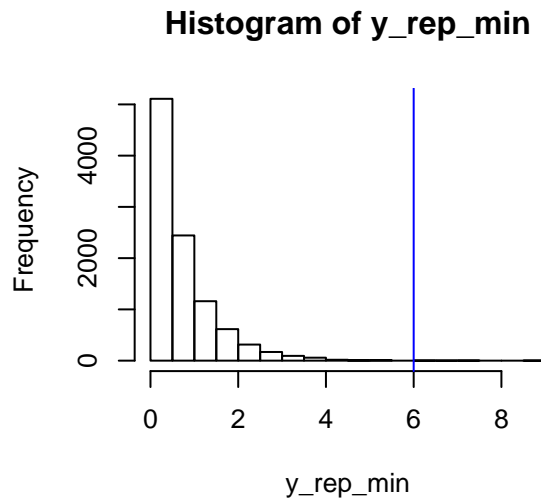
Part iv

```
## Minimum p-value:  0.0006

## 25% Quantile p-value:  0.0154

## Median p-value:  0.2508

## Standard Deviation p-value:  0.9845

## Probability 10 Minute Wait:  0.5101
```

Our results imply that our model not be correct since many of our p-values are quite different from our observed value; this tells us that it is very unlikely that the observed value of each statistic lies in our posterior predictive distributions. This suggests that the quantity we computed in class may be incorrect since the median observed values is larger than 10 and our calculated distribution shows a similar result. Since the probability of a 10 minute wait according to our posterior predictive distribution is about 50%, our model may not be accurate.

Problem 2

Part i

Please refer to the Code Appendix section for the sampling technique, the technique used to construct the replicate data set and how test statistics were computed.

Histogram of z_rep_min

Histogram of z_rep_q25

Histogram of z_rep_med

Histogram of z_rep_sd
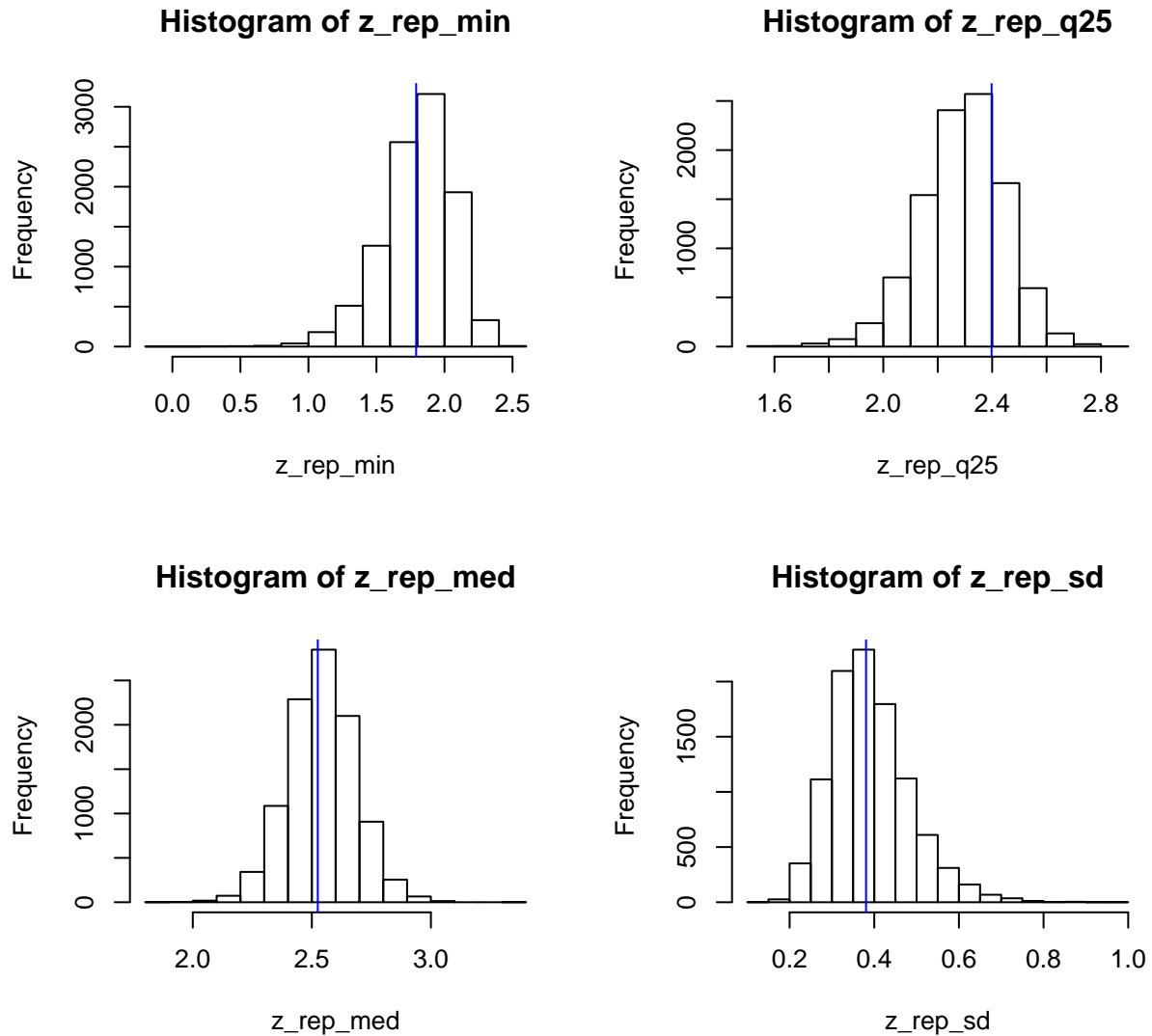
```
## Minimum p-value:  0.5554
## 25% Quantile p-value:  0.2465
## Median p-value:  0.5487
## Standard Deviation p-value:  0.4987
```

Part ii

```
## Probability 10 Minute Wait:  1
```

Yes this estimate seems to be much more reasonable, our result implies that we would certainly expect our result due to the result of the probability of a 10 minute wait according to our posterior predictive

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distribution.