Assignment #8

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Problem 1

Sampling Distribution: $Binomial(n, \theta)$

Prior Distribution: $Beta(\alpha = 1, \beta = 1)$ vs $Uniform(\alpha = 0, \beta = 1)$

Model Fit Results

Beta Prior

```
## Inference for Stan model: bayes_binom.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##          mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
## theta    0.25    0.00 0.07    0.13    0.20    0.24    0.29    0.39   498    1
## lp__   -23.55    0.03 0.68  -25.40  -23.72  -23.29  -23.10  -23.05   727    1
##
## Samples were drawn using NUTS(diag_e) at Mon Oct 22 21:19:12 2018.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

Uniform Prior

```
## Inference for Stan model: bayes_binom_eps.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##          mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
## theta    0.25    0.00 0.06    0.14    0.20    0.25    0.29    0.38   554 1.01
## lp__   -23.52    0.02 0.65  -25.33  -23.71  -23.26  -23.10  -23.05   753 1.00
##
## Samples were drawn using NUTS(diag_e) at Mon Oct 22 21:19:12 2018.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```
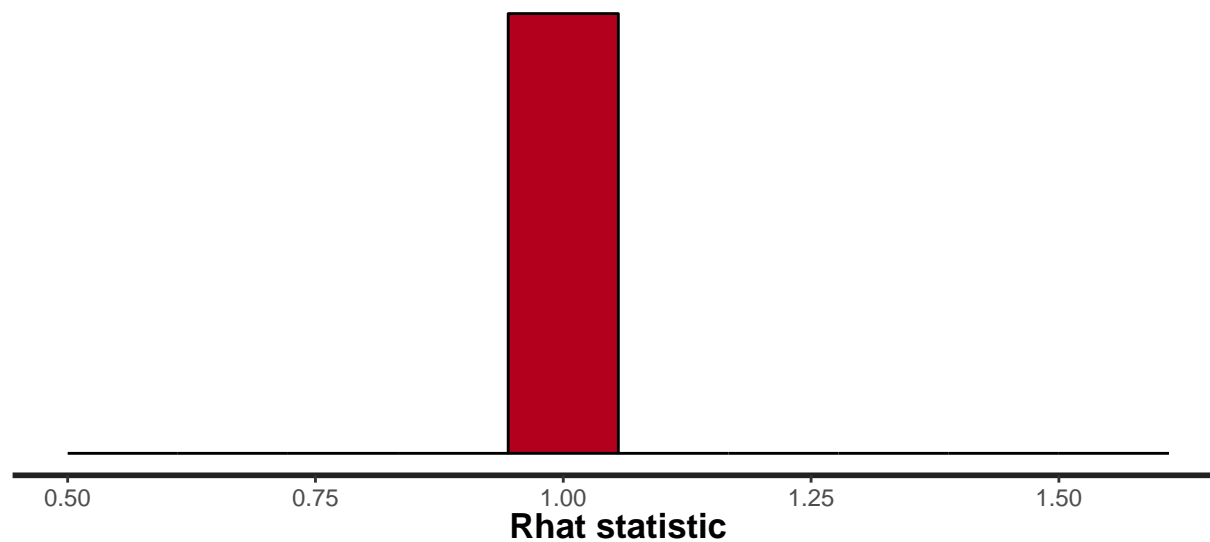
Comments

In regards to the Model Fit, the change in prior from $Beta(\alpha = 1, \beta = 1)$ to $Uniform(\alpha = 0, \beta = 1)$ had very little impact overall as we can see from the output. The one difference that does appear to manifest on some runs is a difference in n_{eff}, but aside from that the results are mostly consistent between the two choices for priors.
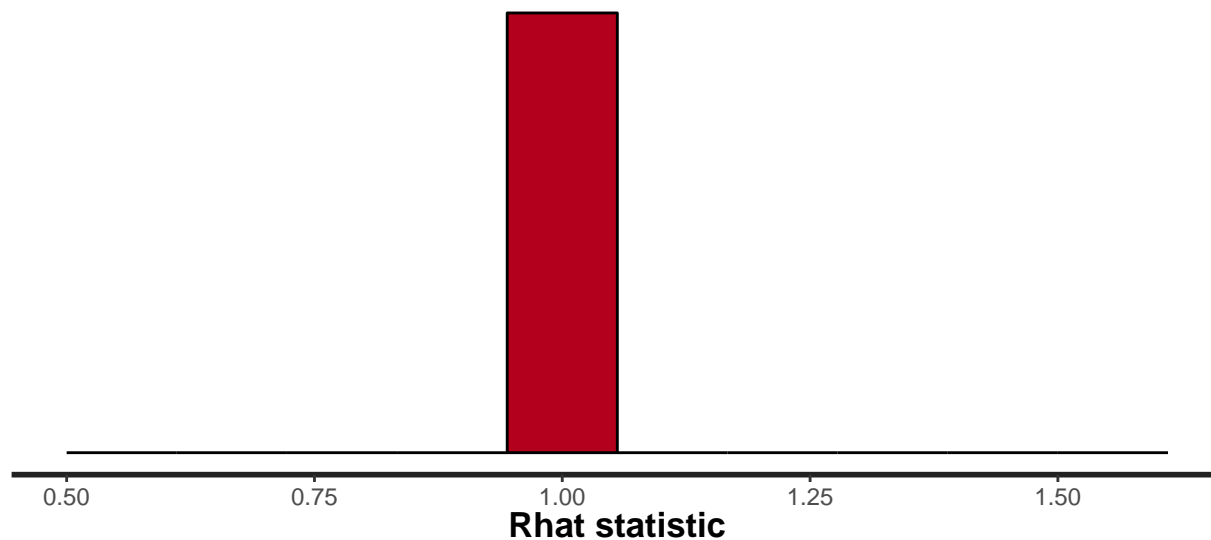
Diagnostic Results

Beta Prior

```
##
## Divergences:

## 1 of 2000 iterations ended with a divergence (0.05%).
## Try increasing 'adapt_delta' to remove the divergences.

##
## Tree depth:

## 0 of 2000 iterations saturated the maximum tree depth of 10.

##
## Energy:

## E-BFMI indicated no pathological behavior.
```


Rhat statistic

Uniform Prior

```
##
## Divergences:

## 0 of 2000 iterations ended with a divergence.

##
## Tree depth:

## 0 of 2000 iterations saturated the maximum tree depth of 10.

##
## Energy:

## E-BFMI indicated no pathological behavior.
```

Rhat statistic

Comments

As for the Diagnostic Results, again, we see little difference between the usage of the two different priors. Both of the posteriors using the distinct prior have very clean convergence in all cases and our Rhat value for each prior choice is always centered directly at 1. We can again say that our choice of prior has no material affect here.

RStan Traceplots

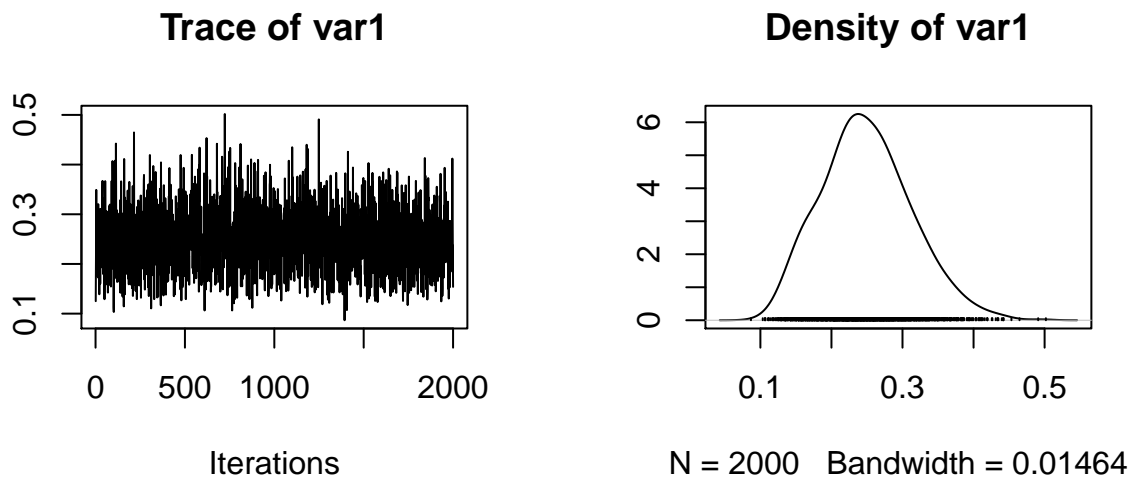**Beta Prior**
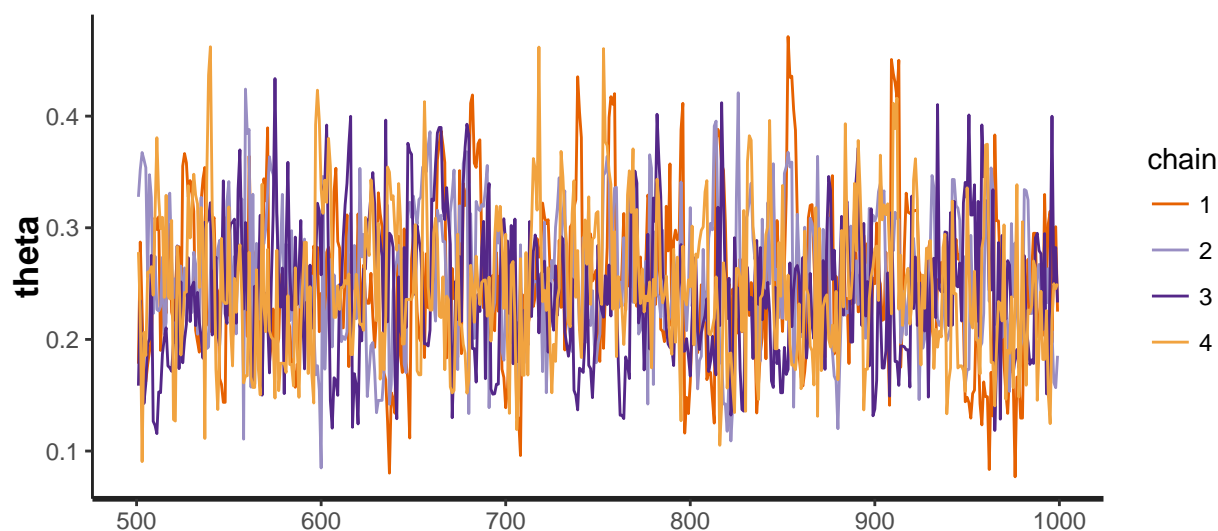
Uniform Prior
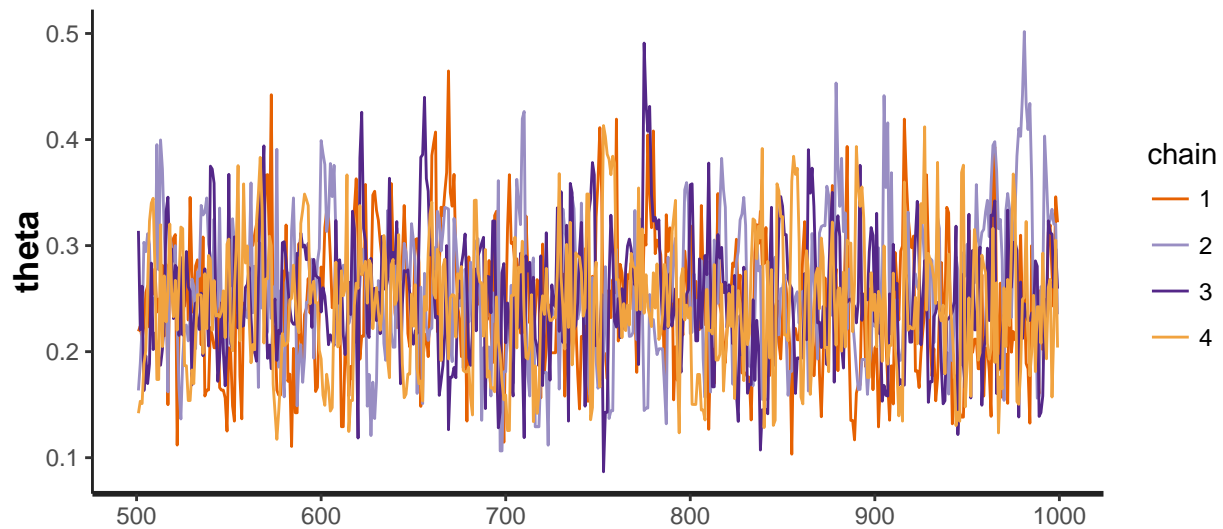
Comments

In regards to the RStan traceplots, we again see two very similar traceplots that converge at a reasonable value and appear to be very well formed. It would be almost impossible to be able to distinguish between the two groups of chains for each prior. Not only are we satisfied with the mixing and the convergence of our chains, we are again left with the realization that the choice of prior has no great impact on the posterior in this case.

R Traceplots and Posterior Densities

Beta Prior

Trace of var1

Density of var1

Iterations

N = 2000   Bandwidth = 0.01464
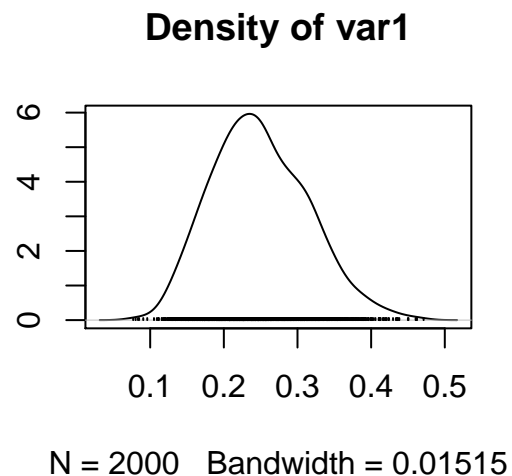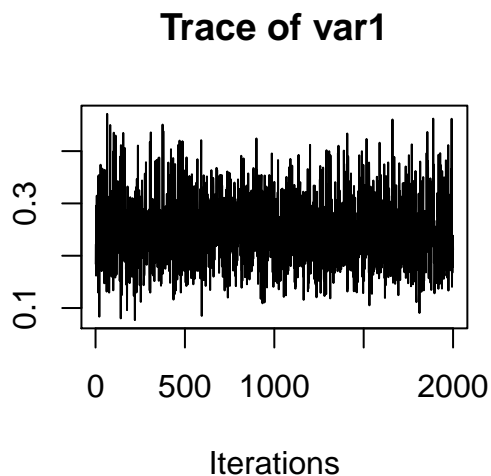
Comments

We again get similar results for the R Traceplots and Posterior Densities, the two plots are nearly identical and we can again conclude that the choice of either of these two prior has little affect on the posterior distribution. We would be hard-pressed to distinguish between our two traceplots or posterior densities.
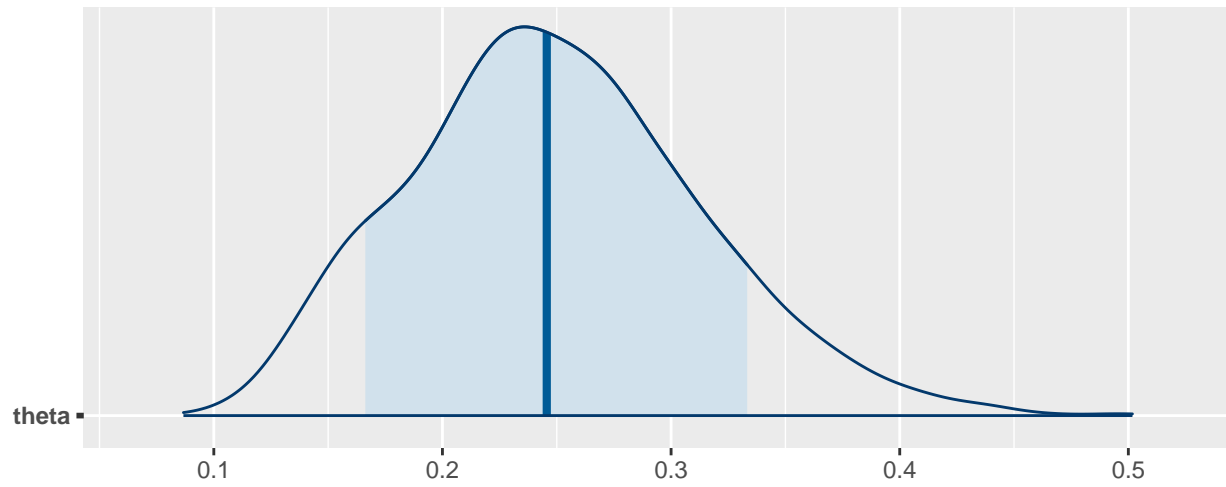
Posterior Distributions

Beta Prior

Posterior distribution
with median and 80% intervals

theta
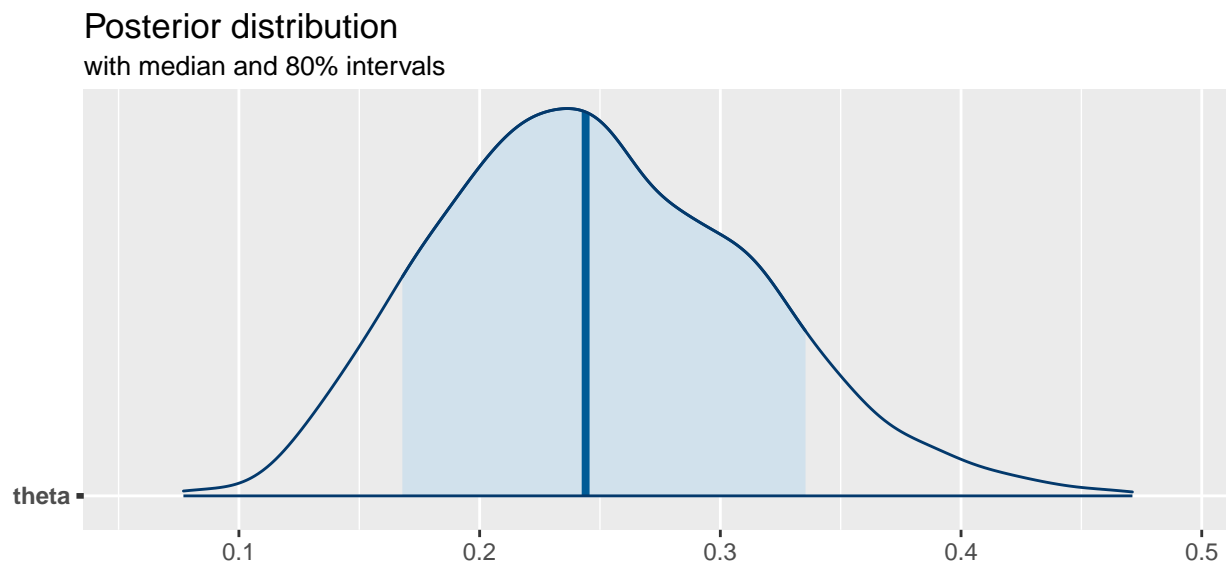
Uniform Prior

Posterior distribution
with median and 80% intervals

theta ▪

0.1	0.2	0.3	0.4	0.5

Comments

A rehash of the above, the two posterior densities are very similar, almost identical.

Conclusion

We may conclude that the choice of a $Beta(\alpha = 1, \beta = 1)$ prior or a $Uniform(\alpha = 0, \beta = 1)$ prior will make little difference on the posterior distribution. As a result, we may choose freely between them in this example without negatively affected our result.

Code Appendix

```
########## Prepare Workspace ##########



## Set the working directory

setwd('~/Documents/Rice_University/Fall_2018/STAT525/HW08')

## Load in the necessary packages

suppressMessages(
    suppressWarnings(
        library(rstan)
    )
)
suppressMessages(
    suppressWarnings(
        library(coda)
    )
)
suppressMessages(
    suppressWarnings(
        library(bayesplot)
    )
)

## Detect the number of core for parallel processing

options(mc.cores = parallel::detectCores())



########## Problem 1 ##########



## Set the known parameters

n <- 42
y <- 10

## Compile both of the stan models

mod_binom_beta <- stan_model('bayes_binom.stan')
mod_binom_unif <- stan_model('bayes_binom_eps.stan')

## Store the data in a list

dat_beta <- list(n = n, y = y, alpha = 1, beta = 1)
dat_unif <- list(n = n, y = y, alpha = 0, beta = 1)
```

```
## Perform the sampling from the beta prior posterior

fit_beta <- sampling(object = mod_binom_beta,
                     data = dat_beta,
                     iter = 1000, chains = 4)

## Perform the sampling from the uniform prior posterior

fit_unif <- sampling(object = mod_binom_unif,
                     data = dat_unif,
                     iter = 1000, chains = 4)

## Print out the results for beta prior and uniform prior

print(fit_beta)
print(fit_unif)

## Run diagnostic tests for beta prior and uniform prior

check_hmc_diagnostics(fit_beta)
check_hmc_diagnostics(fit_unif)

stan_rhat(fit_beta, 'theta', bins = 10)
stan_rhat(fit_unif, 'theta', bins = 10)

## Plot the traceplot for each prior

rstan::traceplot(fit_beta, pars = c("theta"))
rstan::traceplot(fit_unif, pars = c("theta"))

## Extract the posterior draws of each of the thetas

post_theta_beta <- rstan::extract(fit_beta, "theta", permuted = TRUE)
post_theta_unif = rstan::extract(fit_unif, "theta", permuted = TRUE)

## Plot each theta in the "conventional" way

plot(as.mcmc(as.matrix(post_theta_beta[[1]])))
plot(as.mcmc(as.matrix(post_theta_unif[[1]])))

## Plot the posterior distribution for each prior

mcmc_areas(as.matrix(fit_beta), pars = c('theta'), prob = 0.8) +
    ggtitle("Posterior distribution", "with median and 80% intervals")

mcmc_areas(as.matrix(fit_unif), pars = c('theta'), prob = 0.8) +
    ggtitle("Posterior distribution", "with median and 80% intervals")
```