Assignment #4

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Problem 1

Problem 2

Problem 3

Solving for a in equation (b) yields:

$$\begin{aligned} La &= \ell_\theta \\ a &= \frac{\ell_\theta}{L} \end{aligned}$$

Plugging in our result for a into equation (c) yields:

$$\begin{aligned} L^T\theta^* &= a + e \\ &= \frac{\ell_\theta}{L} + e \\ LL^T\theta^* &= \ell_\theta + Le \\ Q_\theta\theta^* &= \ell_\theta + Le \\ \theta^* &= \frac{\ell_\theta + Le}{Q_\theta} \\ &= Q_\theta^{-1}(\ell_\theta + Le) \\ &= Q_\theta^{-1}\ell_\theta + Q_\theta^{-1}Le \\ &= Q_\theta^{-1}\ell_\theta + N\big[0, (Q_\theta^{-1}L)^T I_d (Q_\theta^{-1}L)\big] \\ &= Q_\theta^{-1}\ell_\theta + N\big[0, L^T {Q_\theta^{-1}}^T Q_\theta^{-1}L\big] \\ &= Q_\theta^{-1}\ell_\theta + N\big[0, \frac{1}{Q_\theta}\big] \\ &= Q_\theta^{-1}\ell_\theta + N\big[0, Q_\theta^{-1}\big] \\ &\sim N(Q_\theta^{-1}\ell_\theta, Q_\theta^{-1}) \end{aligned}$$

Thus we see that $\theta^* \sim N(Q_\theta^{-1}\ell_\theta, Q_\theta^{-1})$.