Assignment #2

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8/30/2018

Problem 1

Part a

Here our goal will be to minimize a to show that $a = E[\theta|y]$ is the unique Bayes estimate of θ:

$$\begin{aligned}\frac{d}{da}E[L(a|y)] &= \frac{d}{da}\int L(\theta,a)p(\theta|y)d\theta \\ &= \frac{d}{da}\int (\theta-a)^2 p(\theta|y)d\theta \\ &= -2\int(\theta-a)p(\theta|y)d\theta \\ &= -2\left[\int \theta p(\theta|y)d\theta - a\int p(\theta|y)d\theta\right] \\ &= -2\big[E[\theta|y]-a\big]\end{aligned}$$

$-2\big[E[\theta|y]-a\big] = 0$ when $a = E[\theta|y]$

To prove that it is a unique minimizing statistic, we must look at the second derivative:

$\frac{d}{da}(-2\big[E[\theta|y]-a\big]) = 2$

As $2 > 0$, this shows that it is a unique minimzing statistic.