

# Assignment #6

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## Problem 1

### Part i

$$p(\beta|y, \lambda_1, \lambda_2, k) \propto p(y|\lambda_1, \lambda_2, k, \beta) \times p(\lambda_1|y, \lambda_2, k, \beta) \times p(\lambda_2|y, \lambda_1, k, \beta) \times p(k|y, \lambda_1, \lambda_2, \beta) \times p(\beta)$$

Since  $p(y|\lambda_1, \lambda_2, k, \beta)$  and  $p(k|y, \lambda_1, \lambda_2, \beta)$  don't depend on  $\beta$ , we may ignore them and our equation becomes:

$$p(\beta|y, \lambda_1, \lambda_2, k) \propto p(\lambda_1|y, \lambda_2, k, \beta) \times p(\lambda_2|y, \lambda_1, k, \beta) \times p(\beta)$$

Where:

$$\begin{aligned} p(\lambda_1|y, \lambda_2, k, \beta) &= p(y|\lambda_1, \lambda_2, k)p(\lambda_1) \\ &\sim \text{Gamma}\left(\alpha + \sum_{t=1}^k y_t, \beta + k\right) \end{aligned}$$

$$\begin{aligned} p(\lambda_2|y, \lambda_1, k, \beta) &= p(y|\lambda_1, \lambda_2, k)p(\lambda_2) \\ &\sim \text{Gamma}\left(\alpha + \sum_{t=k+1}^n y_t, \beta + (n - k)\right) \end{aligned}$$

So:

$$\begin{aligned} p(\beta|y, \lambda_1, \lambda_2, k) &\propto p(\lambda_1|y, \lambda_2, k, \beta) \times p(\lambda_2|y, \lambda_1, k, \beta) \times p(\beta) \\ &\propto \lambda_1^{\alpha-1+\sum_{t=1}^k y_t} e^{-(\beta+k)\lambda_1} \lambda_2^{\alpha-1+\sum_{t=k+1}^n y_t} e^{-(\beta+(n-k))\lambda_2} \beta^{a_\beta-1} e^{-b_\beta\beta} \end{aligned}$$

Part ii

Part iii

Part iv

## Problem 2

Part i

$$\begin{aligned} p(y|\mu, \alpha, \beta) &= \int \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\beta\tau} \left( \frac{\tau}{2\pi} \right)^{\frac{1}{2}} e^{-\frac{\tau}{2}(y-\mu)^2} d\tau \\ &= \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{(2\pi)^{\frac{1}{2}}} \int \tau^{\alpha-\frac{1}{2}} e^{-\beta\tau} e^{-\frac{\tau}{2}(y-\mu)^2} d\tau \\ &= \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{(2\pi)^{\frac{1}{2}}} \int \tau^{\alpha-\frac{1}{2}} e^{-(\beta+\frac{1}{2}(y-\mu)^2)\tau} d\tau \\ &= \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{(2\pi)^{\frac{1}{2}}} \frac{\Gamma(\alpha+\frac{1}{2})}{(\beta+\frac{1}{2}(y-\mu)^2)^{\alpha+\frac{1}{2}}} \\ &= \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha)} \frac{1}{(2\pi\beta)^{\frac{1}{2}}} \frac{1}{(1+\frac{1}{2\beta}(y-\mu)^2)^{\alpha+\frac{1}{2}}} \end{aligned}$$

And now substituting in for  $z_i$  and our Gamma parameters for  $\alpha$  and  $\beta$  we get the following result:

$$p(z_i) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}\Gamma(\frac{\nu}{2})} \frac{1}{\left(1+\frac{z_i^2}{\nu}\right)^{\frac{\nu+1}{2}}}$$

Part ii

Part iii

## Problem 3

Part i

Part ii

Part iii

Part iv

Part v