Assignment #6

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Problem 1

Part i

$p(\beta|y, \lambda_1, \lambda_2, k) \propto p(y|\lambda_1, \lambda_2, k, \beta) \times p(\lambda_1|y, \lambda_2, k, \beta) \times p(\lambda_2|y, \lambda_1, k, \beta) \times p(k|y, \lambda_1, \lambda_2, \beta) \times p(\beta)$

Since $p(y|\lambda_1, \lambda_2, k, \beta)$ and $p(k|y, \lambda_1, \lambda_2, \beta)$ don't depend on β, we may ignore them and our equation becomes:

$p(\beta|y, \lambda_1, \lambda_2, k) \propto p(\lambda_1|y, \lambda_2, k, \beta) \times p(\lambda_2|y, \lambda_1, k, \beta) \times p(\beta)$

Where:

$$\begin{aligned} p(\lambda_1|y, \lambda_2, k, \beta) &= p(y|\lambda_1, \lambda_2, k)p(\lambda_1) \\ &\sim Gamma\left(\alpha + \sum_{t=1}^{k} y_t, \beta + k\right) \end{aligned}$$

$$\begin{aligned} p(\lambda_2|y, \lambda_1, k, \beta) &= p(y|\lambda_1, \lambda_2, k)p(\lambda_2) \\ &\sim Gamma\left(\alpha + \sum_{t=k+1}^{n} y_t, \beta + (n-k)\right) \end{aligned}$$

So:

$$\begin{aligned} p(\beta|y, \lambda_1, \lambda_2, k) &\propto p(\lambda_1|y, \lambda_2, k, \beta) \times p(\lambda_2|y, \lambda_1, k, \beta) \times p(\beta) \\ &\propto \lambda_1^{\alpha - 1 + \sum_{t=1}^{k} y_t} e^{-(\beta+k)\lambda_1} \lambda_2^{\alpha - 1 + \sum_{t=k+1}^{n} y_t} e^{-(\beta+(n-k))\lambda_2} \beta^{a_\beta - 1} e^{-b_\beta \beta} \end{aligned}$$

Part ii

Part iii

Part iv

Problem 2

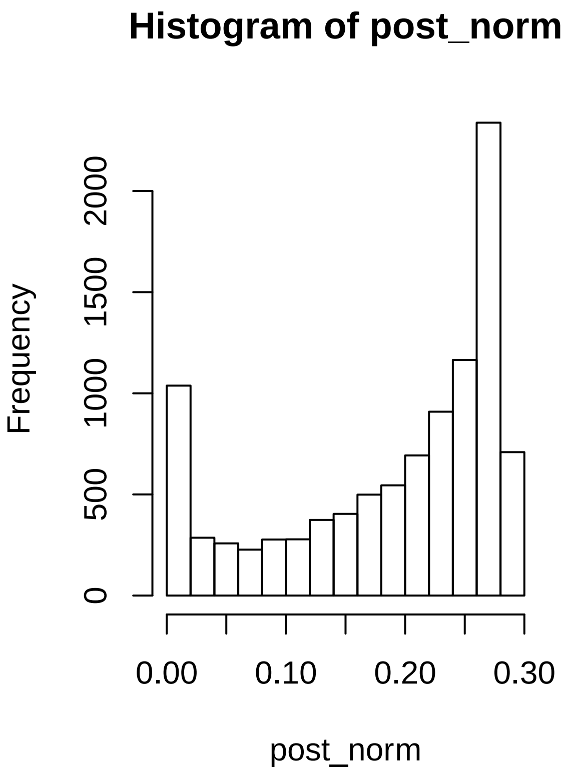
Part i

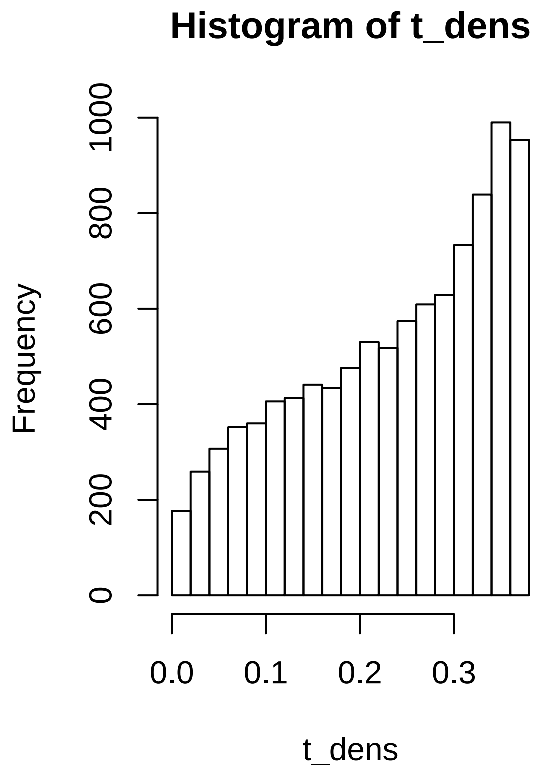
$$
\begin{aligned}
p(y|\mu,\alpha,\beta) &= \int \frac{\beta^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{-\beta\tau}\left(\frac{\tau}{2\pi}\right)^{\frac{1}{2}}e^{-\frac{\tau}{2}(y-\mu)^2}d\tau \\
&= \frac{\beta^\alpha}{\Gamma(\alpha)}\frac{1}{(2\pi)^{\frac{1}{2}}}\int \tau^{\alpha-\frac{1}{2}}e^{-\beta\tau}e^{-\frac{\tau}{2}(y-\mu)^2}d\tau \\
&= \frac{\beta^\alpha}{\Gamma(\alpha)}\frac{1}{(2\pi)^{\frac{1}{2}}}\int \tau^{\alpha-\frac{1}{2}}e^{-(\beta+\frac{1}{2}(y-\mu)^2)\tau}d\tau \\
&= \frac{\beta^\alpha}{\Gamma(\alpha)}\frac{1}{(2\pi)^{\frac{1}{2}}}\frac{\Gamma(\alpha+\frac{1}{2})}{(\beta+\frac{1}{2}(y-\mu)^2)^{\alpha+\frac{1}{2}}} \\
&= \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha)}\frac{1}{(2\pi\beta)^{\frac{1}{2}}}\frac{1}{(1+\frac{1}{2\beta}(y-\mu)^2)^{\alpha+\frac{1}{2}}}
\end{aligned}
$$

And now substituting in for z_i and our Gamma parameters for α and β we get the following result:

$p(z_i) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}\Gamma(\frac{\nu}{2})}\frac{1}{\left(1+\frac{z_i^2}{\nu}\right)^{\frac{\nu+1}{2}}}$

Part ii





As we can see from our result, we get similar density when comparing our results from Part i with the $t_\nu(0, 1)$ distribution.

Part iii

Yes, this result does make sense, because as $\nu \to \infty$ the $Gamma(\frac{\nu}{2}, \frac{\nu}{2})$ will approach the Normal distribution.

Problem 3

Part i

Part ii

Part iii

Part iv

Part v