

Assignment #2

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Problem 1

Part a

Here our goal will be to minimize a to show that $a = E[\theta|y]$ is the unique Bayes estimate of θ :

$$\begin{aligned}\frac{d}{da}E[L(a|y)] &= \frac{d}{da} \int L(\theta, a)p(\theta|y)d\theta \\ &= \frac{d}{da} \int (\theta - a)^2 p(\theta|y)d\theta \\ &= -2 \int (\theta - a)p(\theta|y)d\theta \\ &= -2 \left[\int \theta p(\theta|y)d\theta - a \int p(\theta|y)d\theta \right] \\ &= -2[E[\theta|y] - a]\end{aligned}$$

$$-2[E[\theta|y] - a] = 0 \text{ when } a = E[\theta|y]$$

To prove that it is a unique minimizing statistic, we must look at the second derivative:

$$\frac{d}{da}(-2[E[\theta|y] - a]) = 2$$

As $2 > 0$, this shows that it is a unique minimizing statistic.

Part b

Here our goal will be to show that for any median value of a , the derivative of $L(\theta, a)$ will evaluate to 0.

$$\begin{aligned}\frac{d}{da}[E[L(a|y)]] &= \frac{d}{da} \left[\int_{-\infty}^a (a - \theta)p(\theta|y)d\theta + \int_a^{\infty} (\theta - a)p(\theta|y)d\theta \right] \\ &= \int_{-\infty}^a \frac{d}{da}(a - \theta)p(\theta|y)d\theta + \int_a^{\infty} \frac{d}{da}(\theta - a)p(\theta|y)d\theta \\ &= \int_{-\infty}^a p(\theta|y)d\theta + \int_a^{\infty} (-1)p(\theta|y)d\theta \\ &= \int_{-\infty}^a p(\theta|y)d\theta - \int_a^{\infty} p(\theta|y)d\theta \\ &= \frac{1}{2} - \frac{1}{2} \\ &= 0\end{aligned}$$

As a result, it has been shown that any posterior median of θ is a Bayes estimate of θ .

Part c