HW04

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4/2/2018

Problem 1

Part a

Estimates

- $\sigma_\epsilon^2 = 1.784$
- $\sigma_\alpha^2 = \frac{248.163 - 1.784}{15} = 16.42527$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2}{45}} = 0.1991091$
- Confidence Interval $= [129.6195, 130.4232]$

Part b

Estimates

- $\sigma_\epsilon^2 = 1.784$
- $\sigma_\alpha^2 = 10.911$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 1.917458$
- Confidence Interval $= [124.668177, 135.374484]$

Part c

Estimates

- $\sigma_\epsilon^2 = 1.784$
- $\sigma_\alpha^2 = 16.425$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 2.348328$
- Confidence Interval $= [124.668177, 135.374484]$

Part d

- All of the methods have the same σ_ϵ^2
- ANOVA and REML have the same σ_α^2 of 16.425, while the ML method's is 10.911
- ANOVA has smallest $SE(\hat{\mu})$ at 0.1991, ML is next smallest at 1.917 and REML is the largest at 2.348
- The Confidence Interval for ANOVA is the tightest by far, where REML and ML are the same and much wider
- 1a and 1c are biased estimators, 1b estimators are unbiases
- I prefer 1b (the REML method) because the estimators are unbiased

Part e - ANOVA Method

Estimates

- $\sigma_\epsilon^2 = 88.082$
- $\sigma_\alpha^2 = \frac{0.007 - 88.082}{15} = -5.871667 = 0$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2}{45}} = 1.399063$
- Confidence Interval $= [125.243023, 130.889866]$

Part e - LM Method

Estimates

- $\sigma_\epsilon^2 = 82.21$
- $\sigma_\alpha^2 = 0$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 1.351625$
- Confidence Interval $= [125.359749, 130.773140]$

Part e - REML Method

Estimates

- $\sigma_\epsilon^2 = 84.08$
- $\sigma_\alpha^2 = 0$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 1.366911$
- Confidence Interval $= [125.359749, 130.773140]$

Part e - Results Comparison

- The largest σ_ϵ^2 is from ANOVA with 88.082, followed by REML with 84.08 and finally ML with 82.21
- ANOVA, ML and REML all have a σ_α^2 of 0
- ML has smallest $SE(\hat{\mu})$ at 1.352, REML is next smallest at 1.367 and ANOVA is the largest at 1.399
- All of the Confidence Intervals are about equally as tight, where REML and ML are the same again
- ANOVA and ML are biased estimators, REML estimators are unbiased
- I prefer the REML method because the estimators are unbiased

Problem 2

Interaction Effects

Diagnostics

Homoscedasticity of Residuals
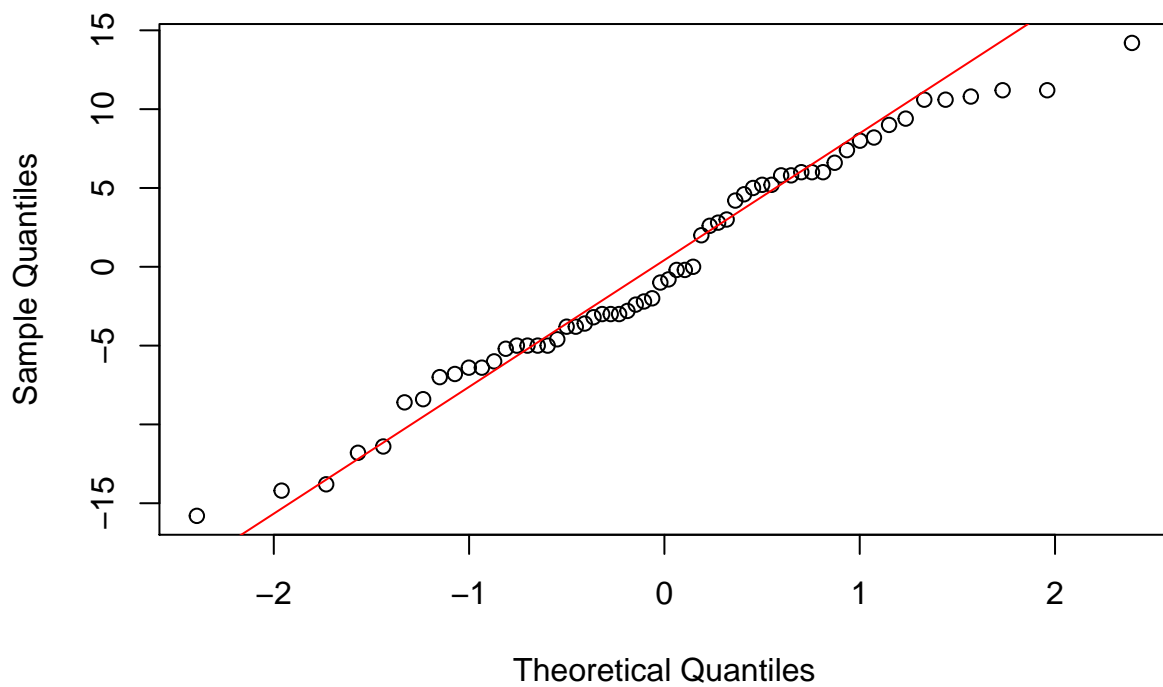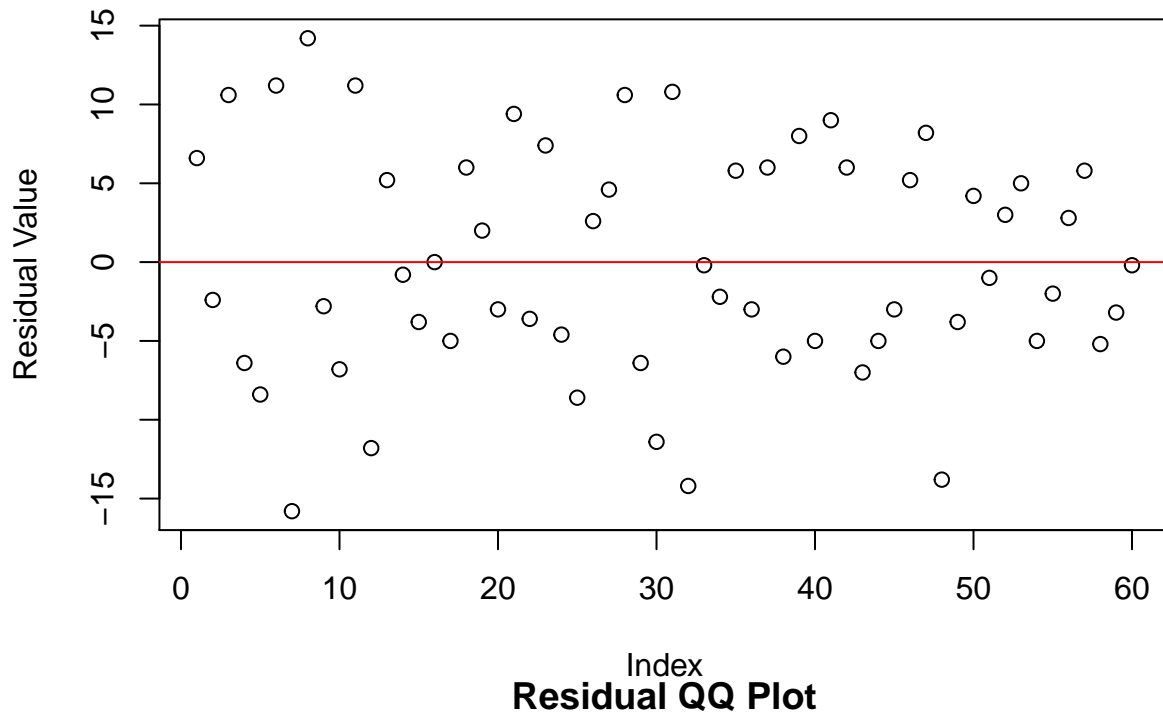
Residual QQ Plot

Summary

- Significant Location p-values: Ground, Lower, Middle
- Significant Trap p-values: None
- Significant Interaction p-values: None
- Based on ANOVA Table, Location is significant and a large portion of the variance is explained by Location
- Based on ANOVA Table, an even larger amount of the variance is explained by the error component
- The confidence interval is wider when incorporating interaction affects, accounting for increased uncertainty
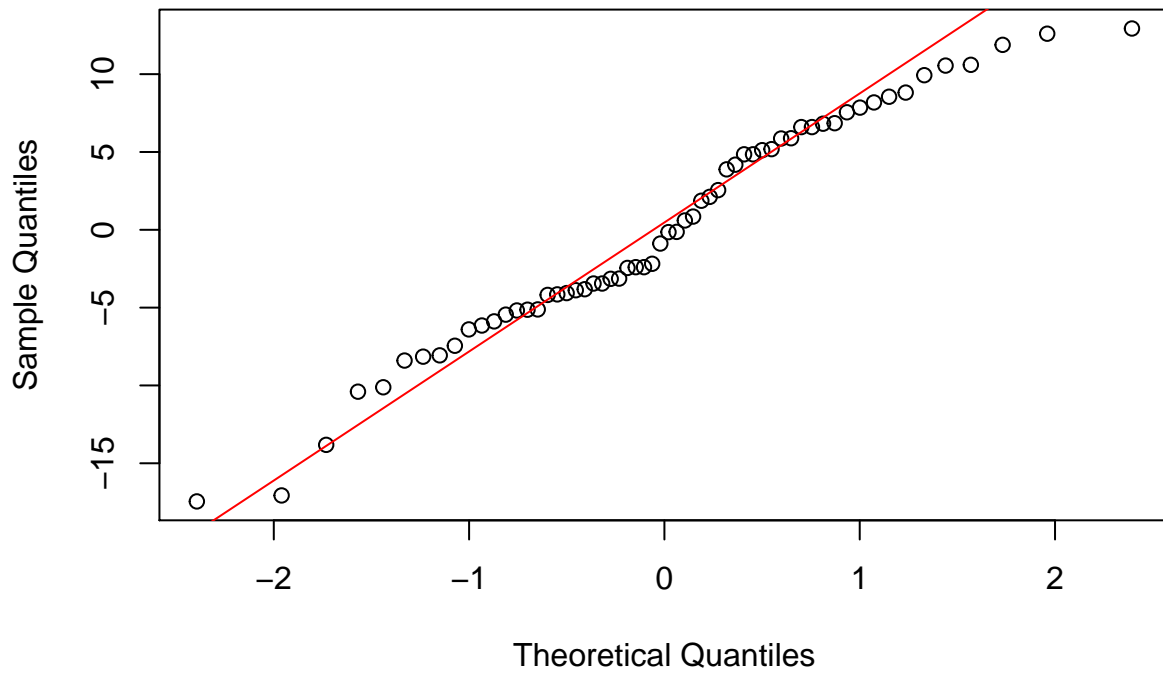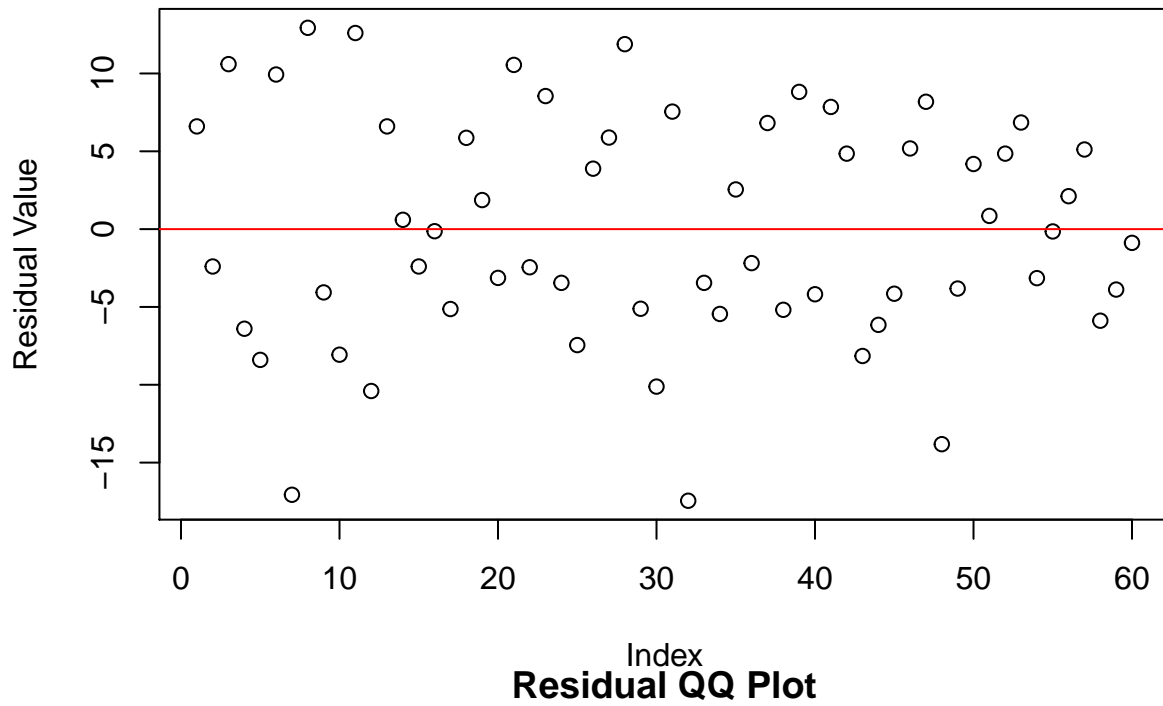- According to our diagnostic plots, residuals are approximately normal and have equal variance

ANOVA Table

```
## Analysis of Variance Table
##
## Response: count
##               Df  Sum Sq Mean Sq F value    Pr(>F)
## location       3 1981.38  660.46 10.4503 2.094e-05 ***
## trap           2  113.03   56.52  0.8943    0.4156
## location:trap  6  114.97   19.16  0.3032    0.9322
## Residuals     48 3033.60   63.20
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Diagnostics

Homoscedasticity of Residuals

Residual QQ Plot

Summary

- Significant Location p-values: Ground, Lower, Middle
- Significant Trap p-values: None
- Based on ANOVA Table, Location is significant and a large portion of the variance is explained by Location
- Based on ANOVA Table, an even larger amount of the variance is explained by the error component
- The confidence interval is tighter when ignoring interaction affects, accounting for decreased uncertainty
- According to our diagnostic plots, residuals are approximately normal and have equal variance

ANOVA Table

```
## Analysis of Variance Table
##
## Response: count
##           Df  Sum Sq Mean Sq F value    Pr(>F)
## location   3 1981.38  660.46 11.3273 7.167e-06 ***
## trap       2  113.03   56.52  0.9693    0.3859
## Residuals 54 3148.57   58.31
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Problem 3

Part a

Part b

Part c

Problem 4

Part a

Part b

Part c

Code Appendix