HW04

Elliot Smith

4/2/2018

Problem 1

Part a

ANOVA Table

```
## Analysis of Variance Table
##
## Response: value
##           Df Sum Sq Mean Sq F value    Pr(>F)
## doctor     2 496.33 248.163   139.1 < 2.2e-16 ***
## Residuals 42  74.93   1.784
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Estimates

- $\sigma_\epsilon^2 = 1.784$
- $\sigma_\alpha^2 = \frac{248.163 - 1.784}{15} = 16.42527$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2}{45}} = 0.1991091$
- Confidence Interval $= [129.6195, 130.4232]$

Part b

LM Summary

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: value ~ (1 | doctor)
##    Data: data_1a
##
##      AIC      BIC   logLik deviance df.resid
##    173.3    178.8    -83.7    167.3       42
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.01913 -0.53908 -0.04267  0.49641  2.50281
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  doctor   (Intercept) 10.911   3.303
##  Residual              1.784   1.336
## Number of obs: 45, groups:  doctor, 3
##
## Fixed effects:
```

```
##              Estimate Std. Error t value
## (Intercept)  130.021      1.917   67.81

## Computing profile confidence intervals ...
```

Estimates

- $\sigma_\epsilon^2 = 1.784$
- $\sigma_\alpha^2 = 10.911$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 1.917458$
- Confidence Interval $= [124.668177, 135.374484]$

Part c

LM Summary

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: value ~ (1 | doctor)
##    Data: data_1a
##
## REML criterion at convergence: 164
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.01922 -0.52809 -0.04894  0.49631  2.49191
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  doctor   (Intercept) 16.425    4.053
##  Residual              1.784    1.336
## Number of obs: 45, groups:  doctor, 3
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  130.021      2.348   55.37

## Computing profile confidence intervals ...
```

Estimates

- $\sigma_\epsilon^2 = 1.784$
- $\sigma_\alpha^2 = 16.425$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 2.348328$
- Confidence Interval $= [124.668177, 135.374484]$

Part d

- All of the methods have the same σ_ϵ^2
- ANOVA and REML have the same σ_α^2 of 16.425, while the ML method's is 10.911
- ANOVA has smallest $SE(\hat{\mu})$ at 0.1991, ML is next smallest at 1.917 and REML is the largest at 2.348

- The Confidence Interval for ANOVA is the tightest by far, where REML and ML are the same and much wider
- 1a and 1c are biased estimators, 1b estimators are unbiases
- I prefer 1b (the REML method) because the estimators are unbiased

Part e - ANOVA Method

ANOVA Table

```
## Analysis of Variance Table
##
## Response: value
##           Df Sum Sq Mean Sq F value Pr(>F)
## device     2    0.0   0.007   1e-04 0.9999
## Residuals 42 3699.4  88.082
```

Estimates

- $\sigma_\epsilon^2 = 88.082$
- $\sigma_\alpha^2 = \frac{0.007 - 88.082}{15} = -5.871667 = 0$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2}{45}} = 1.399063$
- Confidence Interval $= [125.243023, 130.889866]$

Part e - LM Method

LM Summary

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: value ~ (1 | device)
##    Data: data_1e
##
##      AIC      BIC   logLik deviance df.resid
##    332.1    337.5   -163.1    326.1       42
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.8988 -1.0363  0.3268  0.8827  1.2743
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  device   (Intercept)  0.00    0.000
##  Residual             82.21    9.067
## Number of obs: 45, groups:  device, 3
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  128.066      1.352   94.75

## Computing profile confidence intervals ...

## Warning in optwrap(optimizer, par = start, fn = function(x)
## dd(mkpar(npar1, : convergence code 3 from bobyqa: bobyqa -- a trust region
```

```
## step failed to reduce q
```

Estimates

- $\sigma_\epsilon^2 = 82.21$
- $\sigma_\alpha^2 = 0$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 1.351625$
- Confidence Interval $= [125.359749, 130.773140]$

Part e - REML Method

LM Summary

```
## Warning in optwrap(optimizer, devfun, getStart(start, rho$lower, rho$pp), :
## convergence code 3 from bobyqa: bobyqa -- a trust region step failed to
## reduce q

## Linear mixed model fit by REML ['lmerMod']
## Formula: value ~ (1 | device)
##    Data: data_1e
##
## REML criterion at convergence: 323.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.8776 -1.0248  0.3232  0.8729  1.2600
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  device   (Intercept)  0.00    0.000
##  Residual             84.08    9.169
## Number of obs: 45, groups:  device, 3
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  128.066      1.367   93.69
## convergence code: 3

## Computing profile confidence intervals ...

## Warning in optwrap(optimizer, par = start, fn = function(x)
## dd(mkpar(npar1, : convergence code 3 from bobyqa: bobyqa -- a trust region
## step failed to reduce q
```

Estimates

- $\sigma_\epsilon^2 = 84.08$
- $\sigma_\alpha^2 = 0$
- $SE(\hat{\mu}) = \sqrt{\frac{\sigma_\epsilon^2 + 15\sigma_\alpha^2}{45}} = 1.366911$
- Confidence Interval $= [125.359749, 130.773140]$

Part e - Results Comparison

- The largest σ_ϵ^2 is from ANOVA with 88.082, followed by REML with 84.08 and finally ML with 82.21
- ANOVA, ML and REML all have a σ_α^2 of 0
- ML has smallest $SE(\hat{\mu})$ at 1.352, REML is next smallest at 1.367 and ANOVA is the largest at 1.399
- All of the Confidence Intervals are about equally as tight, where REML and ML are the same again
- ANOVA and ML are biased estimators, REML estimators are unbiased
- I prefer the REML method because the estimators are unbiased