Labor Productivity Forecast Report

Forecasting the Annual Percentage Change in
Labor Productivity

by

Elliot Barta

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Econ 460 - Economic Forecasting

Introduction

How productive are American workers? The Bureau of Labor Statistics defines labor productivity as the economic output per hour: if Americans are more productive, they will produce more given the same number of hours. Labor productivity data informs the private and public sectors on the current and future status of the economy, particularly the labor market. The private sector uses the data to determine how many workers they may need in the future, and the public sector, particularly the Federal Reserve, may use this to gauge the status of its Dual Mandate. Moreover, forecasting labor productivity is important for workers, as an increase or decrease in labor productivity could lead to wage changes. This report aims to forecast the revision of the annual percentage change of labor productivity in the third quarter of 2024 using econometric methods based on macroeconomic theory.

Background

The Bureau of Labor Statistics (BLS) uses the following formula to calculate labor productivity (Bureau of Labor Statistics 2024a):

$$\text{Labor Productivity} = \frac{\text{Output Index}}{\text{Hours Worked}}$$

The BLS seasonally adjusts its labor productivity data. The output index used in the equation is the real business sector output, which is the gross domestic product minus general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. Hours Worked is calculated by the following formula:

$$\text{Hours Worked} = \text{AWH} * \text{hwhp} * \text{Emp} * 52$$

AWH is the average weekly hours of production workers, hwhp is the hours worked to hours paid ratio, and Emp is the production worker employment. While there is little Intermediate Macroeconomic theory relating to the number of hours worked, we can use Growth Theory to dissect what causes business sector output to change over time. The BLS releases its labor productivity data on its *Productivity and Costs* news release (Bureau of Labor Statistics 2024b). The BLS first releases the preliminary results and its revisions 30 days later. Importantly, we are most interested in the revisions. We will forecast the yearly percentage change in labor productivity for non-farm businesses in the revised third quarter of 2024, released on December 10th, 2024. The following graph details the forecast variable from January 1947 to July 2024, with the final result being the preliminary for the 3rd quarter.

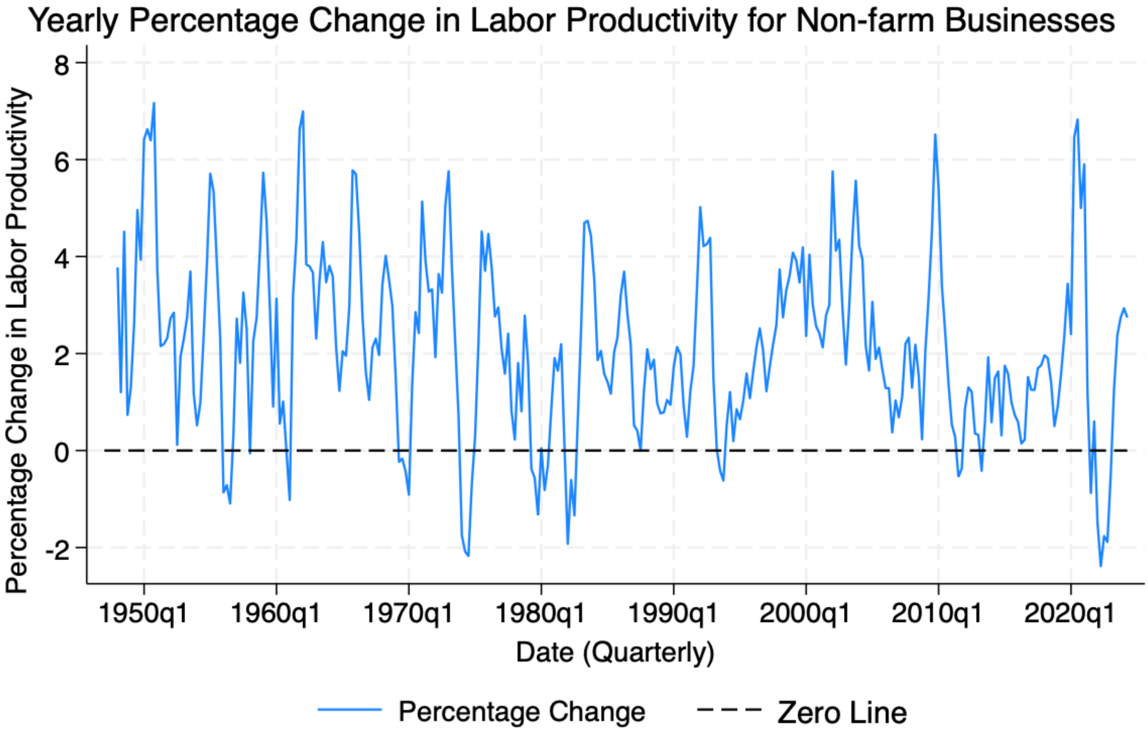

Figure 1: Historical labor productivity changes from 1947-01-01 to 2024-07-01.

Theory

We know from macroeconomics, particularly growth theory, that various factors influence labor productivity. Human capital is known to increase labor productivity. The more human capital a worker accumulates, the more productive they are. We can consider gross domestic output per capita as an instrument for labor productivity, as they are similar with it and has applicable formulas. Consider a simple version of the *Romer model* (Alder 2024a):

$$Y_t = A_t * L_{yt}$$

At time t, Y_t is output, A_t is the total number of ideas in the economy, and L_{yt} is the share of labor not in research. If we divide both sides by L_t, total share of labor, we would have:

$$\frac{Y_t}{L_t} = y_t = A_t * \frac{L_{yt}}{\bar{L}_t} = A_t * (1 - \frac{L_{at}}{\bar{L}_t})$$

L_{at} is the share of researchers in labor and $\bar{L}_t = L_{yt} + L_{at}$ is the total share of labor. From this equation, we can see that, holding the labor-related factors constant, an increase in the output per worker must be derived from the growth of ideas. We can also consider the allocation of labor equation and idea growth equation:

$$L_{at} = \gamma * \bar{L}_t$$

In the allocation of labor equation, γ is the share of labor allocated to researchers.

$$\Delta A_{t+1} = \bar{z} * A_t * L_{at}$$

In the idea growth equation, $\bar{z}$ represents researcher productivity. We can take the difference in A_t to find the algebraic growth rate.

$$\frac{\Delta A_{t+1}}{A_{t+1}} = \bar{z} * L_{at}$$

We can rewrite this growth rate in terms of the initial value of A and its growth rate (Alder 2024a):

$$A_t = A_0(1 + \bar{z} * \gamma * L_t)^t$$

Here, A_0 represents the initial set of ideas and can see the set of ideas grow based on the derived growth rate.

It can be shown that we achieve the following equation by combining A_t in the production and the idea growth equation:

$$y_t = A_0(1-\gamma)(1 + \gamma * \bar{z} * L)^t$$

From this model, we can see that a population increase leads to a positive growth effect. When increasing the share of researchers, initially there is a negative level effect but an increase in the growth of factor. Importantly, we can see that given our exogenous variables, our growth rate remains constant in the model. When creating our forecast model, the *Romer Model* implies that the trend does not change. However, since the exogenous parameters change in different periods, the trend may not be constant outside of theory. Moreover, we can see that the growth of ideas, pivotal for total factor productivity, leads to long-term growth and could be a significant regressor for the forecast model . We use this model in constructing our forecast model by noting the potential for a constant trend, implying that the time component may be insignificant.

We can additionally consider the *Solow-Swan Model*, which highlights how capital investment impacts output per worker. In this version, we will consider technological and population growth. Remark that from the simple neoclassical model, we have (Alder 2024b):

$$Y = A * K^{\alpha} * L^{1-\alpha}$$

Where Y is output, K is capital, L is labor, and A is total factor productivity. Moreover, we can consider the capital accumulation equation:

$$K_{t+1} = s * Y + \delta * K_t$$

where s is the savings rate and δ is the depreciation rate. It can be shown that to increase output per worker in the steady state, we can consider increasing the savings rate or decreasing either the population growth rate or the depreciation rate. Because we are looking at the United States, which can be presumed to be comparatively near its steady state compared to other developing countries, the growth rate of capital is likely to be comparatively low.

To approximate the growth rates of each component, we can apply logarithm rules to both sides of the equation.

$$g_{Yt} = g_{At} + \alpha * g_{Kt} + (1-\alpha) * g_{Lt}$$

Where g_{Yt} is the growth rate of output, g_{At} is the growth rate of total factor productivity, g_{Kt} is the growth rate of capital, and g_{Lt} is the growth rate of labor. Then, we can subtract g_{Lt} on both sides to get the growth rate of output per person.

$$g_{Yt} - g_{Lt} = g_{At} + \alpha * (g_{Kt} - g_{Lt}) + (1-\alpha) * (g_{Lt} - g_{Lt})$$

$$g_{Yt} - g_{Lt} = g_{At} + \alpha * (g_{Kt} - g_{Lt})$$

Remark that $g_{Yt} - g_{Lt}$ is the growth rate of output per person. Here, we see that the growth rate of output per person is impacted by the growth rate of technology and the growth rate of capital per person. Because the growth rate of capital per person is zero on the balance growth path, technology drives the growth of output per person in the long term. This details how important the growth of total factor productivity is for output per person, as well as how important the savings rate is for the growth of capital per person, which impacts output per person. Since total factor productivity and savings rates are important, we will add them as regressors in our forecast model. We would expect that total factor productivity is more significant than the personal savings rate. Moreover, it justs that the trend may not be constant, contrary to the *Romer Model*, given the dynamic fluctuations of the growth of capital per person.

While we cannot fully substitute output per capita for labor productivity, we can use the relevant theory of output per capita to best predict it.

Initial Variable Selection

When selecting regressor variables, we must acknowledge that many potential variables correlate. We want to minimize the number of variables necessary in our model while maximizing our model's explanatory potential. Moreover, we must contend that economic variables are often dynamic, which poses hurdles when creating the linear model. To approximate the growth of total factor productivity, we will consider total factor productivity at constant national prices for the United States. This variable should encompass the technological and human capital changes within the United States. To approximate the growth of capital, we will consider looking at the United States' savings rate, particularly the personal savings rate. Finally, we will use hours worked by full and part-time employees, given its use in the labor productivity equation and possible correlation with the population of the United States. Labor productivity, total factor productivity, hours worked, and the personal savings rate were all sourced from the Federal Reserve Economic Data (FRED) database (Federal Reserve Bank of St. Louis 2024a).
An important variable that will be ignored in this investigation is the preliminary results of the forecast variable. From the data, we can see that the preliminary results often align with the revised results from the Archival Federal Reserve Economic Data (ALFRED) database (Federal Reserve Bank of St. Louis 2024b). All results before the first quarter of 2018 have the same preliminary and revision results. If we use the preliminary results in the regression, there would likely be a unit root, messing up standard errors and the forecast. We will also not consider adding gross domestic product per capita to the model because we can model select specific components (total factor productivity and savings rate) instead of only output per person.

Basic Time Series Model

The typical time series model can be decomposed into the following:

$$Y_t = T_t + C_t + S_t$$

T_t represents the trend component, C_t represents the cyclical component, and S_t represents the seasonal component.

Trend

Because we are taking the percentage change in labor productivity, the trend's impact on the response variable will likely be weak. We will argue that there is a non-constant trend in this model. Consider that in the *Romer model*, we essentially assume that the growth rate of technology is a constant factor. However, we can see that if the parameters of the *Solow-Swan Model* change, then the growth rate of capital per capita will not be zero, which could make the trend non-constant.

Cycle

There will be a cyclical component in the model. Visually we can see this as there seem to be fluctuations in the data that last more than a year: for instance, consider the time period between 2000 and 2020. We can use an auto-regressive model to capture the cyclical component.

Seasonal

We will not include a seasonal component in the model because the labor productivity data from the BLS already is seasonally adjusted. This implies that any deterministic seasonal adjustment would be unnecessary and likely statistically insignificant.

Data Manipulation

The labor productivity data ranges from 1947-01-01 to 2024-07-01. Remark that the data records the revised results of labor productivity for all time entries except for 2024-07-01, where it records the preliminary results. The personal savings rate data range from 1959-01-01 to 2022-10-01 monthly. The total factor productivity data ranges from 1954-01-01 to 2019-01-01 annually. The hours worked data range from 1948-01-01 to 2022-01-01 annually. To effectively conduct econometric analysis, we need quarterly values for all variables up to 2024-04-01—we will not include 2024-07-01 as we will forecast the revision for that date. We will start at 1959-01-01 as the personal savings rate data begins there. For the personal savings rate, we will only consider data recorded in the 1st, 4th, 7th, and 10th month. For the total factor productivity and hours worked data, we will assume the data entry for the 1st month remains the same for the 4th month, 7th month, and 10th month. For entries past 2022-10-01 for personal savings, 2019-01-01 for total factor productivity, and 2022-01-01 for hours worked, the sample mean is used for the missing values. The impact of these assumptions will be further explored in the forecast analysis. The potential regressors we will consider are the year-over-year percentage change in total factor productivity, hours worked, and personal savings rate, while the regressand is the year-over-year percentage change in labor productivity.

Model Exploration

Trend

We will initially test whether there is a trend present in the model. We will estimate the following model $y_t = \beta_0 + \beta_1 * t + \varepsilon_t$ and test whether $\beta_1 = 0$. Hence, $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$

. reg pct_change_L2_prod yq

Source	SS	df	MS
Model	.001192114	1	.001192114
Residual	.081102628	257	.000315574
Total	.082294742	258	.000318972

Number of obs	=	259
F(1, 257)	=	3.78
Prob > F	=	0.0530
R-squared	=	0.0145
Adj R-squared	=	0.0107
Root MSE	=	.01776

pct_change~d	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
yq	-.0000287	.0000148	-1.94	0.053	-.0000578	3.78e-07
_cons	.0239884	.0022013	10.90	0.000	.0196536	.0283233

Figure 2: Trend Regression Table

From the table, we see that the linear trend is not statistically significant at a 5% level of significance. The p-value for the t-test is 0.053, which is greater than 0.05. Our uncertainty in including a linear trend was valid given the p-value is close to 5%. Ultimately, the trend will remain in the model as although it is not statistically significant, it may provide predictive power and improve model fit. Moreover, the theory suggests that there should not be a constant trend given the *Solow-Swan* dynamic properties, thus having a variable to highlight this property remains pertinent.

Autocorrelation

We will look at the Autocorrelation of the annual percentage change in labor productivity.

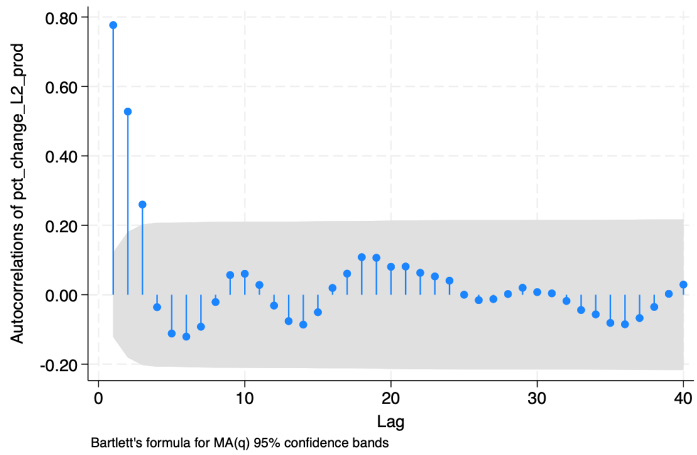

Figure 3: Labor Productivity Autocorrelation

From the graph, we can see that at a 5% level, we can infer that the annual percentage change in labor productivity is auto-correlated with its first, second, and third lag. The forecast variable then does not seem highly persistent. The autocorrelation seems to be oscillatory, particularly after the 3rd lag. It hovers around 0, potentially suggesting the autocorrelation is ergodic. Importantly, the autocorrelation with its 1st lag is around 0.78, which provides unit root concerns. We will verify this with an augmented Dickey-Fuller Test with an AR(3). The model we are testing is the following: $\Delta y_t = \beta_0 + \beta_1 * L * y_t + \beta_2 * L * \Delta y_t + \beta_3 * L^2 * \Delta y_t + \beta_4 * L^3 * \Delta y_t + \varepsilon_t$ where H_0 =there is a unit root and H_a =there is not a unit root.

```
. test L.pct_change_L2_prod

 ( 1)  L.pct_change_L2_prod = 0

       F(  1,   249) =   81.24
            Prob > F =    0.0000
```

Figure 4: Dickey-Fuller Test Result

From the test, we see the 1st lag is statistically significant at a 5% level of significance. The p-value for the t-test is 0.000, which is less than 0.05. This suggests that there is not a unit root, implying that the standard error regression reports are appropriate to forecast with. This is unsurprising, given that we take the fourth difference in the numerator by taking the yearly percentage change; taking differences is seen as a common solution to unit roots. The lack of a unit root, in tandem with a high number of observations (254), suggests our standard errors are appropriate.

Cyclical Component

The presence of autocorrelation and theory suggest there may be auto-regressive models to fit the forecast variable. This can further be explored by looking at the AIC, BIC, and Predictive Least Squares of various models. We will analyze model-fit outputs of AR(1) to AR(15) models with a trend variable. Explicitly, we will estimate from $y = \beta_0 + \beta_1 * t + \beta_2 * Ly_t + \varepsilon_t$ to $y = \beta_0 + \beta_1 * t + \sum_{i=1}^{15}(\beta_{i+1} * L^i y_t) + \varepsilon_t$. Here, 19 observations were dropped to equally compare regressions model fit on the same number of observations on all models: 15 were dropped for going up to an AR(15) and 4 were dropped by taking the fourth lag when calculating the forecast variable.

Model	N	ll(null)	ll(model)	df	AIC	BIC
AR1	243	639.7497	755.5687	3	-1505.137	-1494.658
AR2	243	639.7497	760.567	4	-1513.134	-1499.162
AR3	243	639.7497	766.5306	5	-1523.061	-1505.596
AR4	243	639.7497	777.4022	6	-1542.804	-1521.846
AR5	243	639.7497	789.6119	7	-1565.224	-1540.772
AR6	243	639.7497	789.704	8	-1563.408	-1535.463
AR7	243	639.7497	790.0029	9	-1562.006	-1530.568
AR8	243	639.7497	791.2803	10	-1562.561	-1527.63
AR9	243	639.7497	799.16	11	-1576.32	-1537.896
AR10	243	639.7497	802.4738	12	-1580.948	-1539.031
AR11	243	639.7497	804.5077	13	-1583.015	-1537.606
AR12	243	639.7497	805.5019	14	-1583.004	-1534.101
AR13	243	639.7497	816.2506	15	-1602.501	-1550.105
AR14	243	639.7497	817.5872	16	-1603.174	-1547.285
AR15	243	639.7497	817.5911	17	-1601.182	-1541.8

Figure 5: AIC and BIC results for Auto-regressive Models

By looking at the lowest values, the table suggests the AR(13) model produces the lowest BIC value while the AR(14) model has the lowest AIC value. This implies that the AR(13) is comparatively the most consistent while the AR(14) is the most efficient. Typically we select the most efficient model for forecasting as it is more tightly related. We can best decide by looking at the Predictive Least Squares of the models. We will start with B = 100, where B is the number of observations for the first model. B = 100 was selected as 100 is roughly $\frac{1}{3}$ of the observations. In the output table, remark *oos_errork* corresponds to the model with an AR(k).

Variable	Obs	Mean	Std. dev.	Min	Max
oos_error1	162	.0001055	.0002195	4.64e-08	.0018546
oos_error2	162	.000106	.0002358	8.32e-10	.0021304
oos_error3	162	.0001065	.000211	3.68e-08	.0019397
oos_error4	162	.0001018	.0001958	1.04e-09	.0018384
oos_error5	162	.0000909	.000189	5.65e-09	.0019457
oos_error6	162	.0000916	.000188	5.47e-11	.0019356
oos_error7	162	.0000919	.0001889	2.91e-11	.0019577
oos_error8	162	.0000921	.0001884	7.15e-09	.0019697
oos_error9	162	.0000839	.0001836	1.59e-09	.0020591
oos_error10	162	.000083	.0001798	1.21e-12	.0020292
oos_error11	162	.0000843	.0001809	2.73e-08	.0020243
oos_error12	162	.0000858	.0001811	4.81e-09	.0020085
oos_error13	162	.0000769	.0001732	6.42e-10	.002
oos_error14	162	.0000774	.0001773	3.54e-09	.0020534
oos_error15	162	.000078	.0001809	3.82e-10	.0021113

Figure 6: Predictive Least Squares Results for Auto-regressive Models

Looking at the lowest predictive least squares means, we can see that AR(13) is the best model, followed by AR(14). This is believable given that these two models performed the best in terms of BIC and AIC respectively. Our final model will then use the AR(13) model given that it gives the lowest predictive least squares and is more consistent. Since we will add more variables to the model, we must be diligent about over-fitting.

Grainger Causality

We want to determine whether the annual percentage change in total factor productivity, the annual percentage change in personal savings rate, and the annual percentage change in the hours worked by full and part-time employees have any statistically significant predictive power. We will first test the Grainger Causality of these three variables to determine whether they are leading variables to the forecast variable. We will test each potential regressor with four lags. We will test with the following model, where the specific variable is represented by x_t: $y = \beta_0 + \beta_1 * t + \sum_{i=1}^{13}(\beta_{i+1} * L^i y_t) + \sum_{i=1}^{4}(\beta_{i+14} * L^i x_t) + \varepsilon_t$ with $H_0 : \beta_{15} = \beta_{16} = \beta_{17} = \beta_{18} = 0$ and H_a : one of the coefficients is significant.

```
test L.TFPannualchange L2.TFPannualchange L3.TFPannualchange L4.TFPannualchange

( 1)  L.TFPannualchange = 0
( 2)  L2.TFPannualchange = 0
( 3)  L3.TFPannualchange = 0
( 4)  L4.TFPannualchange = 0

      F(  4,   226) =    2.13
           Prob > F =    0.0782
```

Figure 7: Grainger Results for TFP

```
test L.HoursWorkAnnualChange L2.HoursWorkAnnualChange L3.HoursWorkAnnualChange L4.HoursWorkAnnualChange

( 1)  L.HoursWorkAnnualChange = 0
( 2)  L2.HoursWorkAnnualChange = 0
( 3)  L3.HoursWorkAnnualChange = 0
( 4)  L4.HoursWorkAnnualChange = 0

      F(  4,   226) =    6.95
           Prob > F =    0.0000
```

Figure 8: Grainger Results for Hours Worker

```
test L.SavingRateAnnualChange L2.SavingRateAnnualChange L3.SavingRateAnnualChange L4.SavingRateAnnualChange

( 1)  L.SavingRateAnnualChange = 0
( 2)  L2.SavingRateAnnualChange = 0
( 3)  L3.SavingRateAnnualChange = 0
( 4)  L4.SavingRateAnnualChange = 0

      F(  4,   226) =    2.58
           Prob > F =    0.0384
```

Figure 9: Grainger Results for Personal Savings Rate

At a 5% level of test, there is sufficient evidence to suggest that the annual percentage change in hours worked, and personal savings rate Grainger-causes the annual percentage change in labor productivity since their respective p-values (0.0000 and 0.0384) are less than 0.05. However, at a 5% level of test, there is not sufficient evidence to suggest that the annual percentage change in total factor productivity Grainger-causes the annual percentage change in Labor Productivity since its p-value (0.0782) is greater than 0.05. This potentially means that the annual percentage change in hours worked and personal savings are leading indicators for the forecast variable; however, the annual percentage change in total factor productivity is not. This investigation will still use the latter variable when selecting models as its p-value in the Grainger test is subjectively close to 0.05, and based on theory should be important for predicting the forecast variable.

Autoregressive Distributed Lag Model Selection

We will first look at the 8 possible combinations of the variables and compare AIC and BIC values to select a model. We will add one lag of each independent variable into the model. We will add the variable to the following model: $y = \beta_0 + \beta_1 * t + \sum_{i=1}^{13}(\beta_{i+1} * L^i y_t) + \varepsilon_t$. The model name corresponds to which variable was added: NA means none, TFP is for the annual percentage change in total factor productivity, SR is for the annual percentage change in personal savings rate, and HW is the annual percentage change in hours worked.

Model	N	ll(null)	ll(model)	df	AIC	BIC
NA	245	644.7359	822.355	15	-1614.71	-1562.191
TFP	245	644.7359	823.525	16	-1615.05	-1559.03
SR	245	644.7359	824.1426	16	-1616.285	-1560.265
HW	245	644.7359	832.4433	16	-1632.887	-1576.866
TFP_SR	245	644.7359	826.7669	17	-1619.534	-1560.013
TFP_HW	245	644.7359	843.9403	17	-1653.881	-1594.359
SR_HW	245	644.7359	832.5448	17	-1631.09	-1571.568
TFP_SR_HW	245	644.7359	833.5401	18	-1631.08	-1568.057

Figure 10: AIC and BIC results for Auto-regressive Models with Regressors

The model with total factor productivity and hours worked is the best-fit model as it has the lowest AIC and BIC values. This makes sense, given that total factor productivity growth was the main growth in output per person and hours worked is in the labor productivity formula. Interestingly, it is seen that including personal savings does not significantly improve the model. Despite the annual percentage change in total factor productivity not being statistically significant in the Grainger test, it does improve model fit. Notably, the annual percentage change in the personal savings rate is not in the final model: the amount it explains is offset by the over-fitting penalties of AIC and BIC. It may have lower predictive power because the United States, as a developed country, is close to its steady state, and according to the *Solow-Swan model*, it would not have a significant impact.

We will further investigate whether including more lags for annual percentage change in total factor productivity and hours worked would improve model fit. We will be fitting $y = \beta_0 + \beta_1 * t + \sum_{i=1}^{13}(\beta_{i+1} * L^i y_t) + \varepsilon_t$ with combinations of 1, 2, and 4 lags for the annual percentage change in total factor productivity and hours worked. In the table below, *TFPq_HWp* corresponds to q lags for the annual percentage change in total factor productivity and p lags for the annual percentage change of hours worked.

Model	N	ll(null)	ll(model)	df	AIC	BIC
TFP1_HW1	245	644.7359	843.9403	17	-1653.881	-1594.359
TFP2_HW1	245	644.7359	843.9422	18	-1651.884	-1588.862
TFP4_HW1	245	644.7359	847.3803	20	-1654.761	-1584.735
TFP1_HW2	245	644.7359	844.3897	18	-1652.779	-1589.757
TFP2_HW2	245	644.7359	844.5207	19	-1651.041	-1584.517
TFP4_HW2	245	644.7359	848.0334	21	-1654.067	-1580.54
TFP1_HW4	245	644.7359	845.705	20	-1651.41	-1581.385
TFP2_HW4	245	644.7359	845.8325	21	-1649.665	-1576.139
TFP4_HW4	245	644.7359	851.8093	23	-1657.619	-1577.09

Figure 11: AIC and BIC Results for Auto-regressive Models with Combinations of TFP and Hours Worked

The AIC and BIC model selection differs significantly. The BIC selects the consistent *TFP1_HW1* while the AIC selects the efficient *TFP4_HW4*. We can perform predictive least squares for reassurance. Remark that for the table below, the variable name corresponds to *oos_errorqp* with q and p retaining the same meaning.

Variable	Obs	Mean	Std. dev.	Min	Max
oos_error11	162	.000066	.0001187	5.72e-09	.0012368
oos_error12	162	.000068	.0001296	2.66e-09	.0014275
oos_error14	162	.0000696	.0001301	5.80e-09	.0014397
oos_error21	162	.0000666	.0001201	4.07e-09	.0012558
oos_error22	162	.0000696	.0001292	6.73e-09	.0014048
oos_error24	162	.0000712	.00013	6.96e-09	.0014179
oos_error41	162	.0000649	.0001225	4.27e-09	.0013073
oos_error42	162	.0000679	.000132	2.98e-09	.0014597
oos_error44	162	.0000678	.0001316	1.26e-09	.0014413

Figure 12: Predictive Least Squares Results for Auto-regressive Models with Combinations of TFP and Hours Worked

The predictive least squares results indicate that the model with *TFP4_HW1* is the most optimal. This is interesting as all 3 different model selection techniques provide different answers. Since the predictive least squares method is the best at mimicking the actual forecast process, we will select its model. Hence, the model we have constructed is $y = \beta_0 + \beta_1 * t + \sum_{i=1}^{13}(\beta_{i+1} * L^i y_t) + \sum_{i=1}^{4}(\beta_{i+14} * L^i \mathrm{TFP}_t) + \beta_{19} * L\mathrm{HW}t + \varepsilon_t$

Trend Shift

We will now test for a trend shift. Depending on whether the United States is near its steady state, the growth rate of output per capita may change as it gets closer. Thus, break dates may show this change in labor productivity. We will conduct a QLR test to determine whether there is a trend shift in the model $y = \beta_0 + \beta_1 * t + \sum_{i=1}^{13}(\beta_{i+1} * L^i y_t) + \sum_{i=1}^{4}(\beta_{i+14} * L^i \mathrm{TFP}_t) + \beta_{19} * L\mathrm{HW}t + \varepsilon_t$

```
Test for a structural break: Unknown break date
Full sample:    1963q2 thru 2024q2
Trimmed sample: 1972q3 thru 2015q2
Estimated break date: 1994q4

H0: No structural break

                       Number of obs = 245
------------------------------------------
             Test    Statistic    p-value
------------------------------------------
    Supremum Wald       9.0990     0.0399
------------------------------------------
Exogenous variables: L.pct_change_L2_prod L2.pct_change_L2_prod L3.pct_change_L2_prod L4.pct_change_L2_prod
                     L5.pct_change_L2_prod L6.pct_change_L2_prod L7.pct_change_L2_prod L8.pct_change_L2_prod
                     L9.pct_change_L2_prod L10.pct_change_L2_prod L11.pct_change_L2_prod L12.pct_change_L2_prod
                     L13.pct_change_L2_prod L.HoursWorkAnnualChange L.TFPannualchange L2.TFPannualchange
                     L3.TFPannualchange L4.TFPannualchange yq
Coefficients included in test: yq
```

Figure 13: QLR Test for Trend Shift

There is evidence to suggest that there is a trend shift in the model. The QLR STATA output suggests that the break date is the fourth quarter of 1994. This result is not surprising given the possible changes in the personal savings rate captured by the trend. Historically, total factor productivity grew in the 1990s (Alder 2024a). This growth, potentially from the advent of information technologies, may have impacted the trend to the point where a trend shift is considered. The final model, which we will use to forecast, is

$$y = \beta_0 + \beta_1 * t + \beta_2 * d * t + \sum_{i=1}^{13} (L^i * \beta_{i+2} * y_t) + \sum_{i=1}^{4} (L^i * \beta_{i+15} * \mathrm{TFP}_t) + \beta_{20} * L\mathrm{HW}t + \varepsilon_t$$

```
reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.HoursWorkAnnualChange L(1/4).TFPannualchange yq dyq

      Source |       SS           df       MS      Number of obs   =       245
-------------+----------------------------------   F(20, 224)      =     49.74
       Model |  .060638129        20  .003031906   Prob > F        =    0.0000
    Residual |  .013652589       224  .000060949   R-squared       =    0.8162
-------------+----------------------------------   Adj R-squared   =    0.7998
       Total |  .074290718       244   .00030447   Root MSE        =    .00781

-------------------------------------------------------------------------------
pct_change_L2_prod | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------------+-----------------------------------------------------------
pct_change_L2_prod |
               L1. |   .7584512   .0651084    11.65   0.000     .6301479    .8867544
               L2. |  -.0299118   .0791811    -0.38   0.706     -.185947    .1261234
               L3. |   .0926252   .0790728     1.17   0.243    -.0631965    .2484468
               L4. |  -.7804897   .0785869    -9.93   0.000    -.9353538   -.6256256
               L5. |   .5073556   .0850146     5.97   0.000      .339825    .6748863
               L6. |   .0203774   .0856899     0.24   0.812    -.1484841    .1892388
               L7. |   .0522098   .0857791     0.61   0.543    -.1168274     .221247
               L8. |  -.4962672   .0857641    -5.79   0.000     -.665275   -.3272595
               L9. |   .5103616   .0827797     6.17   0.000     .3472349    .6734882
              L10. |  -.0816149   .0747855    -1.09   0.276    -.2289881    .0657584
              L11. |   .0256681   .0754273     0.34   0.734    -.1229698     .174306
              L12. |  -.3483774   .0757195    -4.60   0.000    -.4975911   -.1991637
              L13. |   .2413603   .0558792     4.32   0.000     .1312441    .3514764
                   |
HoursWorkAnnualChange |
               L1. |  -.2190139   .0321948    -6.80   0.000    -.2824572   -.1555705
                   |
   TFPannualchange |
               L1. |   .4466166   .1003029     4.45   0.000     .2489586    .6442747
               L2. |   .0776263   .1116974     0.69   0.488    -.1424858    .2977385
               L3. |  -.2446936   .1116325    -2.19   0.029    -.4646779   -.0247094
               L4. |   .2268089   .0951397     2.38   0.018     .0393256    .4142921
                   |
                yq |  -.0000802   .0000211    -3.79   0.000    -.0001218   -.0000385
               dyq |   .0000424   .0000141     3.02   0.003     .0000147    .0000702
             _cons |   .0170451   .0028488     5.98   0.000     .0114313    .0226589
-------------------------------------------------------------------------------
```

Figure 14: Complete Model

Residual Analysis

Because we have a finalized model, we can verify that this model validates the white noise necessity for forecast modes. We will look at a residual plot and an autocorrelation plot to verify that the residuals have a mean of 0, are homoskedastic, and have 0 autocorrelation.

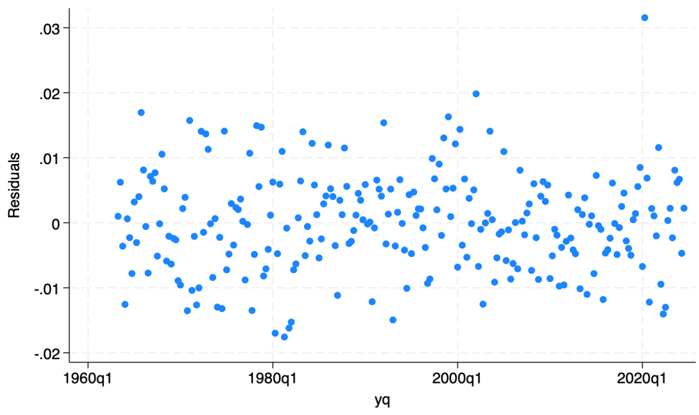

Figure 15: Residual Plot

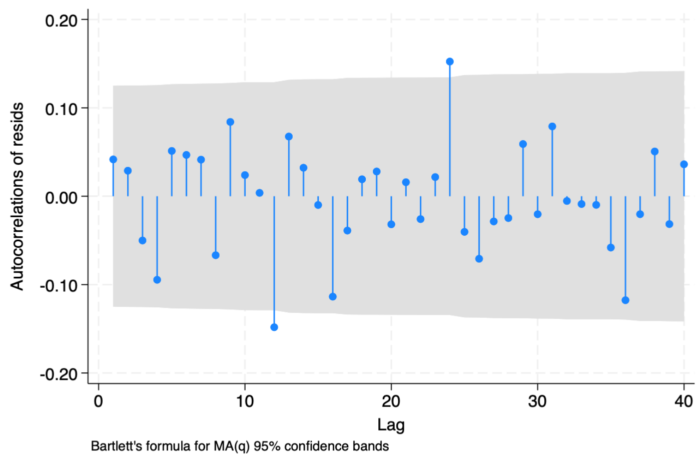

Figure 16: Autocorrelation Plot

The residuals seem to fulfill the White Noise criteria. The residuals do not funnel, which suggests that there is constant variance in the residual plot. Moreover, the residuals hover around 0 and distributed symmetrically, suggesting the residuals have a mean of 0. Finally, it seems there is no autocorrelation between the forecast errors, as most of the autocorrelation is within the confidence band. Interestingly, there seems to potentially be an outlying residual in the residual plot around the first quarter of 2020, which corresponds with the COVID pandemic. Ultimately, this is an appropriate forecast model because the residuals satisfy the White Noise criteria.

Forecasting

Given the final model, we can forecast the revision of the annual percentage change in labor productivity for the 4th quarter. For clarity, the forecast will be up to a year in advance —there will be four forecast values.

We will conduct three forecasts: one with the direct method and two using the iterative method. The first iterative forecast will simulate the future independent variable forecasts using simulations from ARMA(1,1) models. This exploration will use ARMA(1,1) models without in-depth model selection as the model is simple and will nonetheless have some predictive power: it is unlikely that unit-roots will be present as all the independent variables have differences. The second iterative forecast will forecast the independent variables based on their sample means as an approximation to their expectation. Moreover, they suggest that the regression does not need any robust standard errors.

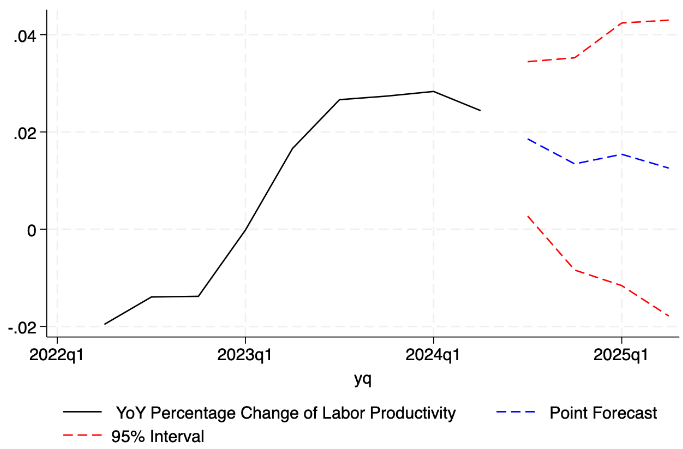

Figure 17: Direct Forecast

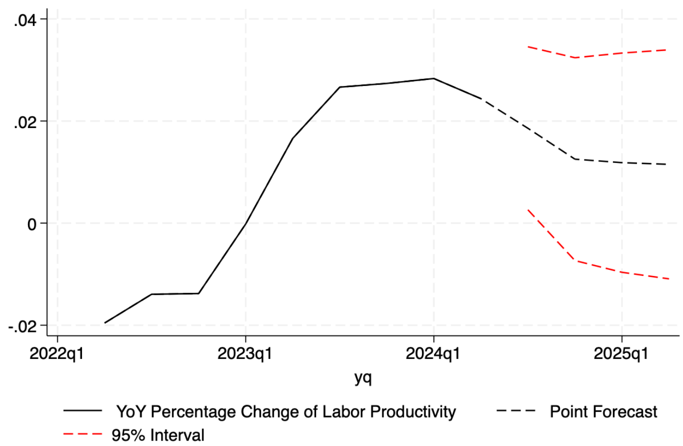

Figure 18: Iterative Forecast with ARMA(1,1) Utilization

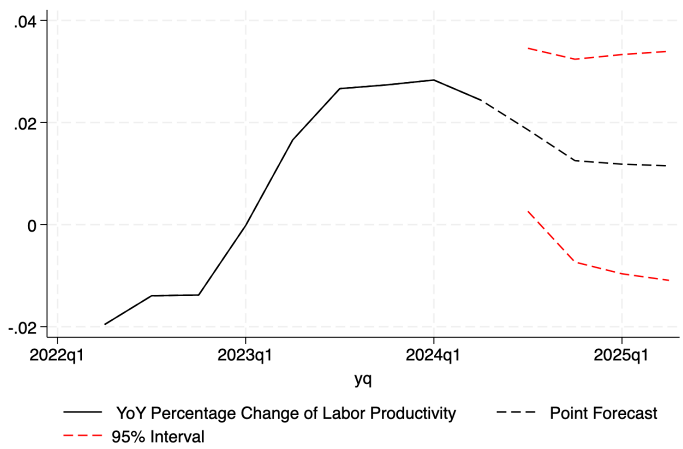

Figure 19: Iterative Forecast with Sample Mean Utilization

	Direct	Iterative (Arima)	Iterative (Sample Mean)
2024Q3	0.018579	0.018579	0.018579
2024Q4	0.0134335	0.0125322	0.0125322
2025Q1	0.0154085	0.0118445	0.0118445
2025Q2	0.012565	0.0115195	0.0115195

Table 1: Forecast Comparison for Different Quarters

All three forecasts generally look similar, as they predict that the annual percentage change in labor productivity will decrease for the first quarter. The direct forecast has larger standard errors compared to its iterative counterparts and has a zig-zag formation: its prediction for 2025Q1 is larger than its prediction for 2024Q4. The iterative methods point forecasts seem to moderately decrease from 2024Q2 to 2024Q4, and then gradually decrease for the last 2 forecasts. The forecast confidence interval for the iterative forecast with ARMA(1,1) utilization is less than the other iterative forecast. With the objective of minimizing the forecast standard errors, we would choose the iterative forecast that uses the ARMA(1,1) given the tighter confidence interval. The point forecast for the 3rd quarter of 2024 for the annual percentage change in labor productivity is 1.8579% in all 3 forecasts.

Comparing Forecast Values

There are very few forecasts for the annual percentage change in labor productivity. Nonetheless, the preliminary value for the forecast variable is 2.0%. My model varies this estimate slightly, with there being approximately 0.14 percentage point difference. It is sensible to forecast solely based on the preliminary results when not considering unit root concerns. If the true revision results are near the preliminary results, it is likely the forecast output is less than the true value. Nonetheless, the revision forecast being near the preliminary result may imply the model is close.

References

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```
import fred OPHNFB B4701C0A222NBEA PSAVERT RTFPNAUSA632NRUG B230RC0Q173SBEA, clear
set seed 2003

*Setting dates for merging

gen yq = yq(year(daten), quarter(daten))
gen ym = ym(year(daten), month(daten))
gen year = year(daten)
gen quarter = quarter(dofq(yq))

*Data manipulation for yearly, monthly, and quarterly variables

foreach q of numlist 1/4 {
 gen m`q' = quarter(dofq(yq)) == `q'
}

drop if mod(ym, 3) != 0

drop if ym < -12

bysort year: replace RTFPNAUSA632NRUG = RTFPNAUSA632NRUG[1] if quarter > 1 & missing(RTFPNAUSA632NRUG)
bysort year: replace B4701C0A222NBEA = B4701C0A222NBEA[1] if quarter > 1 & missing(B4701C0A222NBEA)


tsset yq
format yq %tq

*Generating YoY percentage change variables

gen TFPannualchange = (RTFPNAUSA632NRUG - L4.RTFPNAUSA632NRUG)/(L4.RTFPNAUSA632NRUG)
gen HoursWorkAnnualChange = (B4701C0A222NBEA - L4.B4701C0A222NBEA)/(L4.B4701C0A222NBEA)
gen SavingRateAnnualChange = (PSAVERT - L4.PSAVERT)/(L4.PSAVERT)
gen pct_change_L2_prod = (OPHNFB - L4.OPHNFB)/(L4.OPHNFB)



quietly summarize TFPannualchange
replace TFPannualchange = r(mean) if missing(TFPannualchange)
quietly summarize HoursWorkAnnualChange
replace HoursWorkAnnualChange = r(mean) if missing(HoursWorkAnnualChange)

drop if yq > yq(2024,2)

*Looking at autocorrelation

ac pct_change_L2_prod

corrgram pct_change_L2_prod

*Testing for Unit Root

reg D.pct_change_L2_prod L.pct_change_L2_prod L(1/3).D.pct_change_L2_prod
test L.pct_change_L2_prod


***Testing AIC, BIC, and PLS Model for Cyclical component:

quietly reg pct_change_L2_prod L(1/1).pct_change_L2_prod yq if _n > 19
estimates store AR1
quietly reg pct_change_L2_prod L(1/2).pct_change_L2_prod yq if _n > 19
estimates store AR2
quietly reg pct_change_L2_prod L(1/3).pct_change_L2_prod yq if _n > 19
estimates store AR3
quietly reg pct_change_L2_prod L(1/4).pct_change_L2_prod yq  if _n > 19
estimates store AR4
quietly reg pct_change_L2_prod L(1/5).pct_change_L2_prod yq if _n > 19
estimates store AR5
quietly reg pct_change_L2_prod L(1/6).pct_change_L2_prod yq if _n > 19
estimates store AR6
quietly reg pct_change_L2_prod L(1/7).pct_change_L2_prod yq if _n > 19
estimates store AR7
quietly reg pct_change_L2_prod L(1/8).pct_change_L2_prod yq if _n > 19
estimates store AR8
quietly reg pct_change_L2_prod L(1/9).pct_change_L2_prod yq if _n > 19
estimates store AR9
quietly reg pct_change_L2_prod L(1/10).pct_change_L2_prod yq if _n > 19
estimates store AR10
quietly reg pct_change_L2_prod L(1/11).pct_change_L2_prod yq if _n > 19
estimates store AR11
quietly reg pct_change_L2_prod L(1/12).pct_change_L2_prod yq if _n > 19
estimates store AR12
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod yq if _n > 19
estimates store AR13
quietly reg pct_change_L2_prod L(1/14).pct_change_L2_prod yq if _n > 19
estimates store AR14
quietly reg pct_change_L2_prod L(1/15).pct_change_L2_prod yq if _n > 19
estimates store AR15

estimates stats AR1 AR2 AR3 AR4 AR5 AR6 AR7 AR8 AR9 AR10 AR11 AR12 AR13 AR14 AR15

*Predictive Least Squares for Model

local Tminus1 = _N - 1

local B = 100

foreach L of numlist 1/15{

    gen oos_error`L' = .
    foreach t of numlist `B'/`Tminus1' {

        quietly: reg pct_change_L2_prod L(1/`L').pct_change_L2_prod yq if _n <= `t' & _n > 19

        quietly predict forecast, xb

        quietly replace oos_error`L' = (pct_change_L2_prod-forecast)^2 if _n == `t'+1
```

```
        quietly drop forecast
    }
}
sum oos*
quietly drop oos*
*Testing for Grainger
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L(1/4).TFPannualchange yq
test L.TFPannualchange L2.TFPannualchange L3.TFPannualchange L4.TFPannualchange
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L(1/4).HoursWorkAnnualChange yq
test L.HoursWorkAnnualChange L2.HoursWorkAnnualChange L3.HoursWorkAnnualChange L4.HoursWorkAnnualChange
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L(1/4).SavingRateAnnualChange yq
test L.SavingRateAnnualChange L2.SavingRateAnnualChange L3.SavingRateAnnualChange L4.SavingRateAnnualChange
*Comparing lags for independent variables
quietly  reg pct_change_L2_prod L(1/13).pct_change_L2_prod yq if _n > 17
estimates store NA
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.TFPannualchange yq if _n > 17
estimates store TFP
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.SavingRateAnnualChange yq if _n > 17
estimates store SR
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.HoursWorkAnnualChange yq if _n > 17
estimates store HW
quietly  reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.TFPannualchange L.SavingRateAnnualChange yq if _n > 17
estimates store TFP_SR
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.TFPannualchange L.HoursWorkAnnualChange yq if _n > 17
estimates store TFP_HW
quietly  reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.SavingRateAnnualChange L.HoursWorkAnnualChange yq if _n > 17
estimates store SR_HW
quietly  reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.TFPannualchange L.SavingRateAnnualChange HoursWorkAnnualChange yq if _n > 17
estimates store TFP_SR_HW
estimates stats NA TFP SR HW TFP_SR TFP_HW SR_HW TFP_SR_HW
foreach L of numlist 1 2 4{
        foreach J of numlist 1 2 4{
            quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L(1/`L').TFPannualchange L(1/`J').HoursWorkAnnualChange yq if _n > 17
            estimates store TFP`L'_HW`J'
    }
}
estimates stats TFP1_HW1 TFP2_HW1 TFP4_HW1 TFP1_HW2 TFP2_HW2 TFP4_HW2 TFP1_HW4 TFP2_HW4 TFP4_HW4
*Predictive Least Squares for Independent Variables
local Tminus1 = _N - 1
local B = 100
foreach L of numlist 1 2 4{
    foreach J of numlist 1 2 4{
    gen oos_error`L'`J' = .
        foreach t of numlist `B'/`Tminus1' {
            quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L(1/`L').TFPannualchange L(1/`J').HoursWorkAnnualChange yq if _n <= `t' & _n > 17
            quietly predict forecast, xb
            quietly replace oos_error`L'`J' = (pct_change_L2_prod-forecast)^2 if _n == `t'+1
            quietly drop forecast
        }
    }
}
sum oos_*
*Checking whether there is a time shift
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.HoursWorkAnnualChange L(1/4).TFPannualchange yq
estat sbsingle, swald trim(15) breakvars(yq)
*Implementing trend shift
gen d = yq >= yq(1994, 4)
gen dyq = d*yq
*Looking at residual analysis
quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.HoursWorkAnnualChange L(1/4).TFPannualchange yq dyq
predict resids, residuals
scatter resids yq
ac resids
*Forecasting (Direct Method)
```

```
replace d = yq >= yq(1994, 4)

replace dyq = d*yq


gen point = .
gen sf = .

foreach h of numlist 1/4 {

local l = `h'
local L = `h'+ 12
local G = `h'+ 3
reg pct_change_L2_prod L(`l'/`L').pct_change_L2_prod L(`l'/`G').TFPannualchange L(`l'/`l').HoursWorkAnnualChange yq dyq

predict y`h'
predict sf`h', stdf

replace point = y`h' if yq == yq(2024,2)+`h'
replace sf = sf`h' if yq == yq(2024,2)+`h'
}

gen L_d = point + invnorm(0.025) * sf
gen U_d = point + invnorm(0.975) * sf



tsline  pct_change_L2_prod point L_d U_d if yq > yq(2022 ,1), legend(label(1 YoY Percentage Change of Labor Productivity) label(2 Point Forecast) label(3 "95%
Interval") order(1 2 3)) lcolor(black blue red red) lpattern(solid dash dash dash )
graph export forecast3.png, replace

list point in 263/266

*Forecasting (Iterative with ARIMA)

arima TFPannualchange, arima(1,0,1)

estimates store TFP_results

arima HoursWorkAnnualChange, arima(1,0,1)

estimates store HW_results

forecast create TFP_model, replace

forecast estimates TFP_results

forecast solve

forecast create AR_model, replace

forecast estimates HW_results

forecast solve


quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.f_HoursWorkAnnualChange L(1/4).f_TFPannualchange yq dyq

estimates store for_results

forecast create forecast_model, replace

forecast estimates for_results

forecast solve, simulate(errors betas, statistic(stddev, prefix(sd_)) reps(1000))
gen L = f_pct_change_L2_prod + invnormal(0.025) * sd_pct_change_L2_prod
gen U = f_pct_change_L2_prod + invnormal(0.975) * sd_pct_change_L2_prod


tsline  pct_change_L2_prod f_pct_change_L2_prod L U if yq > yq(2022 ,1), legend(label(1 YoY Percentage Change of Labor Productivity) label(2 Point Forecast)
label(3 "95% Interval") order(1 2 3)) lcolor(black black red red) lpattern(solid dash dash dash )
graph export forecast1.png, replace

list f_pct_change_L2_prod in 263/266

*Forecasting (Iterative with sample mean)

drop f_pct_change_L2_prod
drop sd_pct_change_L2_prod

quietly summarize TFPannualchange
replace TFPannualchange = r(mean) if yq > yq(2024, 2)
quietly summarize HoursWorkAnnualChange
replace HoursWorkAnnualChange = r(mean) if yq > yq(2024, 2)


quietly reg pct_change_L2_prod L(1/13).pct_change_L2_prod L.HoursWorkAnnualChange L(1/4).TFPannualchange yq dyq

estimates store for_results_2

forecast create forecast_model_2, replace

forecast estimates for_results_2

forecast solve, simulate(errors betas, statistic(stddev, prefix(sd_)) reps(1000))
gen L_2 = f_pct_change_L2_prod + invnormal(0.025) * sd_pct_change_L2_prod
gen U_2 = f_pct_change_L2_prod + invnormal(0.975) * sd_pct_change_L2_prod


tsline pct_change_L2_prod f_pct_change_L2_prod L_2 U_2 if yq > yq(2022 ,1), legend(label(1 YoY Percentage Change of Labor Productivity) label(2 Point Forecast)
 label(3 "95% Interval") order(1 2 3)) lcolor(black black red red) lpattern(solid dash dash dash )
graph export forecast2.png, replace

list f_pct_change_L2_prod in 263/266
```