

# Mark Scheme (Pre-Standardisation) Summer 2007

GCE

GCE Mathematics (6663/01)

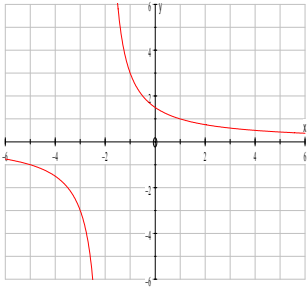
June 2007  
6663 Core Mathematics C1  
Mark Scheme

Question number	Scheme	Marks
1.	$9 - 5$ or $3^2 + 3\sqrt{5} - 3\sqrt{5} - \sqrt{5} \times \sqrt{5}$ or $3^2 - \sqrt{5} \times \sqrt{5}$ $= 4$	M1 A1 (2) <b>2</b>
	<p>M1 for any correct, unsimplified expression</p> <p><math>(\sqrt{5})^2</math> instead of 5 is OK</p> <p>Expansion of <math>(3 + \sqrt{5})(3 + \sqrt{5})</math> is M0A0</p>	

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2.	<p>(a) Attempt <math>\sqrt[3]{8}</math> or <math>\sqrt[3]{(8^4)}</math></p> <p>= <u>16</u></p> <p>(b) <u><math>5x^{\frac{1}{3}}</math></u></p>	<p>M1</p> <p>A1 (2)</p> <p>B1, B1 (2)</p> <p><b>4</b></p>
	<p>(a) M1 for: 2 or <math>\sqrt[3]{8}</math> or <math>(\sqrt[3]{8})^4</math> or <math>2^4</math> or <math>\sqrt[3]{8^4}</math> or <math>\sqrt[3]{4096}</math></p> <p>A1 for 16 only</p> <p>SC If they get <math>\sqrt[3]{8} = \alpha</math> and then attempt <math>\alpha^4</math>, <math>\alpha \neq 2</math>, this scores M1A0</p> <p>(b) 1<sup>st</sup> B1 for 5</p> <p>2<sup>nd</sup> B1 for <math>x^{\frac{1}{3}}</math></p>	

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3.	<p>(a) <math>\left(\frac{dy}{dx}\right) = 6x + \frac{4}{2}x^{-\frac{1}{2}}</math> or <math>\left(6x + 2x^{-\frac{1}{2}}\right)</math></p> <p>(b) <math>6 - x^{-\frac{3}{2}}</math></p> <p>(c) <math>x^3 + \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C</math></p>	<p>M1 A1 (2)</p> <p>M1 A1ft (2)</p> <p>M1 A1 A1 (3)</p>
		<b>7</b>
(a)	<p>M1 for <u>some</u> attempt to differentiate: <math>x^n \rightarrow x^{n-1}</math>  Condone missing <math>\frac{dy}{dx}</math> or <math>y = \dots</math></p> <p>A1 for both terms correct, as written or better.</p>	
(b)	<p>M1 for some attempt to differentiate again. Follow through their <math>\frac{dy}{dx}</math></p> <p>A1ft. as written or better, follow through must have 2 terms and simplify e.g. <math>\frac{4}{4} = 1</math></p>	
(c)	<p>M1 for some attempt to integrate: <math>x^n \rightarrow x^{n+1}</math></p> <p>1<sup>st</sup> A1 for either <math>x^3</math> or <math>\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}</math> (or better)</p> <p>2<sup>nd</sup> A1 for <u>both</u> <math>x^3</math> and <math>\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}</math> <u>and</u> <math>+C</math>.</p> <p>Obviously <math>\frac{8}{3}x^{\frac{3}{2}}</math> or <math>\frac{8}{3}x\sqrt{x}</math> etc is acceptable (but not <u>required</u>) for either A1.</p>	

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4.	<p>(a) Identify <math>a = 5</math> and <math>d = 2</math> (May be implied)</p> $(u_{200} =) a + 199d \quad (= 5 + 199 \times 2)$ $= \underline{403}(\text{p}) \quad \text{or } (\pounds) \underline{4.03}$ <p>(b) <math>(S_{200} =) \frac{200}{2}[2a + 199d]</math> or <math>\frac{200}{2}(a + \text{"their 403"})</math></p> $= \frac{200}{2}[10 + 199 \times 2] \quad \text{or} \quad \frac{200}{2}(5 + \text{"their 403"})$ $= \underline{40\,800} \quad \text{or} \quad \underline{\pounds 408}$	<p>B1</p> <p>M1</p> <p>A1 (3)</p> <p>M1</p> <p>A1</p> <p>A1 (3)</p> <p><b>6</b></p>
<p>(a)</p> <p>N.B.</p> <p>(b)</p>	<p>M1 for use of <math>n</math>th term formula with <math>n = 200</math>. Follow through their <math>a</math> and <math>d</math>. Must be 199 not 200.</p> <p>A1 for 403 or 4.03 (i.e. condone missing £ sign here)</p> <p><math>a = 3, d = 2</math> is B0 and <math>a + 200d</math> is M0 <u>BUT</u> <math>3 + 200 \times 2</math> is B1M1 and A1 if it leads to 403. Answer only of 403 (or 4.03) scores 3/3.</p> <p>M1 for use of correct sum formula with <math>n = 200</math>. Follow through their <math>a</math> and <math>d</math> and their 403.</p> <p>1<sup>st</sup> A1 for any correct expression (i.e. must have <math>a = 5</math> and <math>d = 2</math>) but can f.t. their 403 still.</p> <p>2<sup>nd</sup> A1 for 40800 or £408 (i.e. the £ sign is required before we accept 408 this time).</p>	

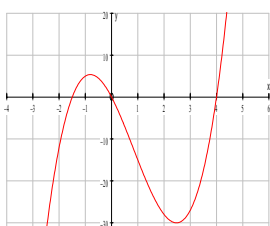
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5.	<p>(a)</p>  <p>Translation parallel to <math>x</math>-axis  Top branch intersects +ve <math>y</math>-axis  Lower branch has no intersections  No obvious overlap</p> <p><math>\left(0, \frac{3}{2}\right)</math> or <math>\frac{3}{2}</math> marked on <math>y</math>-axis</p> <p>(b) <math>x = -2, y = 0</math></p> <p>S.C. [Allow ft on first B1 for <math>x = 2</math> when translated “the wrong way” but must be compatible with their sketch.]</p>	<p>M1</p> <p>A1</p> <p>B1 (3)</p> <p>B1, B1 (2)</p> <p><b>5</b></p>
(a)	<p>M1 for a horizontal translation – one branch should cut <math>y</math> – axis</p> <p>A1 for a horizontal translation to left. Ignore any figures on axes for this mark.</p> <p>B1 for correct intersection on positive <math>y</math>-axis. More than 1 intersection is B0.</p>	
(b)	<p>1<sup>st</sup> B1 for <math>x = -2</math>. Can accept <math>x = +2</math> if this is compatible with their sketch.</p> <p>Usually they will have M1A0 in part (a) (and usually B0 too)</p> <p>2<sup>nd</sup> B1 for <math>y = 0</math>.</p>	

Question number	Scheme	Marks
6.	<p>(a) <math>2x^2 - x(x - 4) = 8</math></p> <p><math>x^2 + 4x - 8 = 0</math> (*)</p> <p>(b) <math>x = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times -8)}}{2}</math> or <math>(x + 2)^2 \pm 4 - 8 = 0</math></p> <p><math>x = -2 \pm \dots</math></p> <p><math>\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}</math></p> <p><math>y = (-2 \pm 2\sqrt{3}) - 4</math> M: Attempt at least one y value</p> <p><u><math>x = -2 + 2\sqrt{3}, y = -6 + 2\sqrt{3}</math></u> <u><math>x = -2 - 2\sqrt{3}, y = -6 - 2\sqrt{3}</math></u></p>	<p>M1</p> <p>A1cso (2)</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p> <p><b>7</b></p>
(a)	<p>M1 for correct attempt to form an equation in <math>x</math> only. Condone sign errors/slips</p> <p>A1cso for correctly simplifying to printed form. No incorrect working seen.</p>	
(b)	<p>1<sup>st</sup> M1 for use of correct formula. If formula is not quoted then a fully correct substitution is required. Condone missing <math>x =</math> for M1.</p> <p>For completing the square must have as printed or better.</p> <p>1<sup>st</sup> A1 for <math>x = -2 \pm</math> any correct expression</p> <p>B1 for simplifying the surd e.g. <math>\sqrt{48} = 4\sqrt{3}</math>. Must reduce to <math>b\sqrt{3}</math></p> <p>2<sup>nd</sup> M1 for attempting to find at least one y value.</p> <p>2<sup>nd</sup> A1 for both correct pairs of answers</p>	

Question number	Scheme	Marks
7.	<p>(a) Attempt to use discriminant <math>b^2 - 4ac</math></p> $k^2 - 4(k+3) > 0 \Rightarrow k^2 - 4k - 12 > 0 \quad (*)$ <p>(b) <math>k^2 - 4k - 12 = 0 \Rightarrow</math></p> $(k \pm a)(k \pm b), \text{ with } ab = 12 \text{ or } (k =) \frac{4 \pm \sqrt{4^2 - 4 \times 12}}{2} \text{ or } (k - 2)^2 \pm 2^2 - 12$ <p><math>k = -2 \text{ and } 6</math></p> <p><u><math>k &lt; -2, k &gt; 6</math></u> <span style="float: right;">M: choosing “outside”</span></p>	<p>M1</p> <p>A1cso (2)</p> <p>M1</p> <p>A1</p> <p>M1 A1ft (4)</p> <p><b>6</b></p>
(a)	<p>M1 for use of <math>b^2 - 4ac</math>, one of <math>b</math> or <math>c</math> must be correct. Or full attempt using completing the square that leads to a 3TQ in <math>k</math></p> <p>e.g. <math>\left( \left[ x + \frac{k}{2} \right]^2 = \right) \frac{k^2}{4} - (k+3)</math></p> <p>A1cso Correct argument to printed result. Need to state (or imply) that <math>b^2 - 4ac &gt; 0</math> and no incorrect working seen.</p>	
(b)	<p>1<sup>st</sup> M1 for attempting to find critical regions. Factors, formula or completing the square. 1<sup>st</sup> A1 for <math>k = 6</math> or <math>-2</math> only 2<sup>nd</sup> M1 for choosing the outside regions 2<sup>nd</sup> A1f.t. as printed or f.t. their (non identical) critical values</p> <p><math>6 &lt; k &lt; -2</math> is M1A0 but ignore if it follows a correct version <math>-2 &lt; k &lt; 6</math> is M0A0 whatever their diagram looks like</p>	



Question number	Scheme	Marks
8.	<p>(a) <math>(a_2 =) \underline{3k + 5}</math></p> <p>(b) <math>(a_3 =) 3(3k + 5) + 5</math>  <math>= \underline{9k + 20}</math> (*)</p> <p>(c)(i) <math>a_4 = 3(9k + 20) + 5 \quad (= 27k + 65)</math></p> $\sum_{r=1}^4 a_r = k + (3k + 5) + (9k + 20) + (27k + 65)$ <p>(ii) <math>= 40k + 90</math>  <math>= \underline{10(4k + 9)} \quad (\text{or explain why divisible by 10})</math></p>	<p>B1 (1)</p> <p>M1</p> <p>A1cso (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1ft (4)</p> <p>7</p>
(b)	<p>M1 for attempting to find <math>a_3</math>, follow through their <math>a_2 \neq k</math>.</p> <p>A1cso for simplifying to printed result with no incorrect working seen.</p>	
(c)	<p>1<sup>st</sup> M1 for attempting to find <math>a_4</math></p> <p>2<sup>nd</sup> M1 for attempting sum of 4 relevant terms</p> <p>1<sup>st</sup> A1 for simplifying to <math>40k + 90</math> or better</p> <p>2<sup>nd</sup> A1ft for taking out a factor of 10 or an explanation in words.</p> <p>Follow through their sum of 4 terms provided that both Ms are scored and their sum <u>is</u> divisible by 10.</p>	

Question number	Scheme	Marks
9.	<p>(a) <math>f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x (+C)</math></p> <p><math>x = 5: \quad 250 - 125 - 60 + C = 65 \quad C = 0</math></p> <p>(b) <math>x(2x^2 - 5x - 12)</math> or <math>(2x^2 + 3x)(x - 4)</math> or <math>(2x + 3)(x^2 - 4x)</math></p> <p><math>= x(2x + 3)(x - 4)</math> (*)</p> <p>(c) </p> <p style="text-align: right;">Shape Through origin <math>\left(-\frac{3}{2}, 0\right)</math> and <math>(4, 0)</math></p>	<p>M1 A1</p> <p>M1 A1 (4)</p> <p>M1</p> <p>A1cso (2)</p> <p>B1</p> <p>B1</p> <p>B1 (3)</p> <p><b>9</b></p>
(a)	<p>1<sup>st</sup> M1 for attempting to integrate, <math>x^n \rightarrow x^{n+1}</math></p> <p>1<sup>st</sup> A1 for all <math>x</math> terms correct, need not be simplified. Ignore <math>+C</math> here.</p> <p>2<sup>nd</sup> M1 for some use of <math>x = 5</math> and <math>f(5) = 65</math> to form an equation in <math>C</math>. No <math>+C</math> is M0.</p> <p>2<sup>nd</sup> A1 for <math>C = 0</math>. This mark cannot be scored unless a suitable equation is seen.</p>	
(b)	<p>M1 for attempting to take out a correct factor. Allow usual errors on signs.</p> <p>A1cso for proceeding to printed answer with no incorrect working seen.</p>	
(c)	<p>1<sup>st</sup> B1 for positive <math>x^3</math> shaped curve (with a max and a min) positioned anywhere.</p> <p>2<sup>nd</sup> B1 for any curve that passes through the origin</p> <p>3<sup>rd</sup> B1 for the two points given or values marked in appropriate places on <math>x</math> axis.</p>	

Question number	Scheme	Marks
10.	<p>(a) <math>x = 1: y = -5 + 4 = \underline{-1}</math>, <math>x = 2: y = -16 + 2 = \underline{-14}</math></p> <p><math>PQ = \sqrt{(2-1)^2 + (-14-(-1))^2} = \sqrt{170}</math> (*)</p> <p>(b) <math>y = x^3 - 6x^2 + 4x^{-1}</math></p> <p><math>\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}</math></p> <p><math>x = 1: \frac{dy}{dx} = 3 - 12 - 4 = -13</math> M: Evaluate at one of the points</p> <p><math>x = 2: \frac{dy}{dx} = 12 - 24 - 1 = -13</math> ∴ Parallel A: Both correct + conclusion</p> <p>(c) Finding gradient of normal <math>\left(m = \frac{1}{13}\right)</math></p> <p><math>y - -1 = \frac{1}{13}(x - 1)</math></p> <p><u><math>x - 13y - 14 = 0</math></u> o.e.</p>	<p>B1, B1</p> <p>M1 A1cso (4)</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p><b>13</b></p>
(a)	<p>M1 for attempting <math>PQ</math> or <math>PQ^2</math> using their <math>P</math> and their <math>Q</math>.</p> <p>A1cso for proceeding to the correct answer with no incorrect working seen.</p>	
(b)	<p>1<sup>st</sup> M1 for multiplying by <math>x^2</math>, the <math>x^3</math> or <math>-6x^2</math> must be correct.</p> <p>2<sup>nd</sup> M1 for some correct differentiation, at least one term must be correct as printed.</p> <p>1<sup>st</sup> A1 for a fully correct derivative.</p> <p>3<sup>rd</sup> M1 for attempting to substitute <math>x = 1</math> or <math>x = 2</math> in their derivative. Substituting in <math>y</math> is M0.</p> <p>2<sup>nd</sup> A1 for -13 from both substitutions <u>and</u> a brief comment.</p>	
(c)	<p>1<sup>st</sup> M1 for use of the perpendicular gradient rule. Follow through their <math>-13</math>.</p> <p>2<sup>nd</sup> M1 for full method to find the equation of the normal. If formula is quoted allow slips in substitution, otherwise a correct substitution is required.</p> <p>1<sup>st</sup> A1ft for a correct expression. Follow through their <math>-1</math> and their <math>m</math> if different from <math>-13</math>.</p> <p>2<sup>nd</sup> A1 for a correct equation with <math>= 0</math> and integer coefficients.</p>	

Question number	Scheme	Marks
11.	<p>(a) <math>y = -\frac{3}{2}x(+4)</math> Gradient = <math>-\frac{3}{2}</math></p> <p>(b) <math>3x + 2 = -\frac{3}{2}x + 4</math> <math>x = \dots, \frac{4}{9}</math></p> <p><math>y = 3\left(\frac{4}{9}\right) + 2 = \frac{10}{3} \left(= 3\frac{1}{3}\right)</math></p> <p>(c) Where <math>y = 1</math>, <math>l_1 : x_A = -\frac{1}{3}</math> <math>l_2 : x_B = 2</math> M: Attempt one of these</p> <p>Area = <math>\frac{1}{2}(x_B - x_A)(y_P - 1)</math></p> <p><math>= \frac{1}{2} \times \frac{7}{3} \times \frac{7}{3} = \frac{49}{18} = 2\frac{13}{18}</math></p>	<p>M1 A1 (2)</p> <p>M1, A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p><b>9</b></p>
(a)	<p>M1 for an attempt to write in the form <math>y = mx + c</math> or a full method that leads to <math>m =</math>, e.g. find 2 points, and attempt gradient using <math>\frac{y_2 - y_1}{x_2 - x_1}</math></p> <p>A1 for <math>m = -\frac{3}{2}</math> (can ignore the <math>+c</math>)</p>	
(b)	<p>M1 for forming a suitable equation in one variable and attempting to solve leading to <math>x = \dots</math></p> <p>1<sup>st</sup> A1 for any exact correct value for <math>x</math></p> <p>2<sup>nd</sup> A1 for any exact correct value for <math>y</math></p>	
(c)	<p>1<sup>st</sup> M1 for attempting the <math>x</math> coordinate of <math>A</math> or <math>B</math></p> <p>1<sup>st</sup> A1 for <math>x_A = -\frac{1}{3}</math> and <math>x_B = 2</math></p> <p>2<sup>nd</sup> M1 for a full method for the area of the triangle e.g. determinant approach</p> <p><math>\frac{1}{2} \begin{vmatrix} 2 &amp; -\frac{1}{3} &amp; \frac{4}{9} &amp; 2 \\ 1 &amp; 1 &amp; \frac{10}{3} &amp; 1 \end{vmatrix} = \frac{1}{2} \left  2 - \dots - \left(-\frac{1}{3}\dots\right) \right </math></p> <p>2<sup>nd</sup> A1 for <math>\frac{49}{18}</math> or an exact equivalent.</p>	