Definitions from the Humphreys Book

Elliot Costi

June 2006

1

Let $\mathbb{K}$ be an algebraicly closed field. The set $\mathbb{K}^n$ it called the affine n-space and denoted A^n. An affine variety is the set of common zeroes in A^n of a finite collection of polynomials.

The *Zariski topology* on A^n is defined to be the topology whose closed sets are the sets

$$V(I) := \{x \in A^n \mid f(x) = 0, \forall f \in I\} \subset A^n,$$

where $I \subset K[X_1, \ldots, X_n]$ is any ideal in the polynomial ring $K[X_1, \ldots, X_n]$. For any affine variety $V \subset A^n$, the *Zariski topology* on V is defined to be the subspace topology induced on V as a subset of A^n. So we are saying that the closed sets under the Zariski topology are the affine varieties.

An algebraic group G is a variety endowed with the structure of a group and the maps $\mu : G \times G \to G$, where $\mu(x, y) = xy$ and $\iota : G \to G$, where $\iota(x) = x^{-1}$ are morphisms of varieties.

A topological space X is irreducible if it cannot be written as the union of two proper, nonempty, closed subsets.

A noetherian space X can be written as a union of finitely many of its maximal irreducible subspaces. These are called the irreducible components of X.

The identity component of G, denoted G°, is the unique irreducible component of e. It normal subgroup of finite index in G whose cosets are the connected as well as irreducible components of G.

G is connected if $G = G^\circ$.

A rational representation ϕ is one that maps from some group to $\mathrm{GL}(n, \mathbb{K})$.

Let M be a subset of the algebraic group G. The group closure of M is the smallest closed subgroup containing M.

A derivation $\delta : \mathbb{E} \to \mathbb{L}$ ($\mathbb{E}$ a field, $\mathbb{L}$ an extension field of $\mathbb{E}$) is a map satisfying $\delta(x + y) = \delta(x) + \delta(y)$ and $\delta(xy) = x\delta(y) + \delta(x)y$. If $\mathbb{F}$ is a subfield of $\mathbb{E}$ then δ is called an $\mathbb{F}$-derivation if in addition $\delta(x) = 0$ for all $x \in \mathbb{F}$. So δ is $\mathbb{F}$-linear. The space $Der_{\mathbb{F}}(\mathbb{E}, \mathbb{L})$ of all $\mathbb{F}$-derivations $\mathbb{E} \to \mathbb{L}$ is a vectorspace over $\mathbb{L}$.

Let G be an algebraic group and $A = \mathbb{K}[G]$. G acts on A by left translation: $(\lambda_x f)(y) = f(x^{-1}y)$, $f \in A$. $\lambda : G \to \mathrm{GL}(A)$ and $\lambda(x) = \lambda_x$. λ_x is the comorphism attached to the morphism $y \mapsto x^{-1}y$.

Let Der A be the set of all $\mathbb{K}$-derivations of A. The Lie Algebra of G is $L(G) = \{\delta \in \text{Der } A | \delta\lambda_x = \lambda_x\delta \text{ for all } x \in G\}$.