# Mark Scheme (Pre-Standardisation)

# Summer 2007

**GCE**

# GCE Mathematics (6676/01)

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# June 2007
# 6676 Further Pure Mathematics FP3
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | (a) $\dfrac{y_1 - 0.2}{0.1} \approx \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 0.2 \times \mathrm{e}^0 \quad (= 0.2)$ | M1 |
| | $y_1 \approx 0.22$ | A1 **(2)** |
| | (b) $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_1 \approx 0.22 \times \mathrm{e}^{0.01} \approx 0.2222 \ldots$ awrt 0.2222 | B1 |
| | $\dfrac{y_2 - 0.2}{0.2} \approx 0.2222\ldots$ | M1 |
| | $y_2 \approx 0.2444$ cao | A1 **(3)** |
| | | **[5]** |
| **2.** | (a) $\left(1 - x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} - 2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - \dfrac{\mathrm{d}y}{\mathrm{d}x} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ | M1 |
| | At $x = 0$, $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} = -\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ | M1 A1 **(3)** |
| | (b) $\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 = -4$ Allow anywhere | B1 |
| | $y = \mathrm{f}(0) + \mathrm{f}'(0)x + \dfrac{\mathrm{f}''(0)}{2}x^2 + \dfrac{\mathrm{f}'''(0)}{6}x^3 + \ldots$ | |
| | $= 2 - x - 2x^2, + \dfrac{1}{6}x^3 + \ldots$ | M1 A1, A1**(4)** |
| | | **[7]** |

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| **3.** | (a) $\begin{pmatrix} 3 & 4 & p \\ -1 & q & -4 \\ 1 & 1 & 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ | | |
| | Third row $1-3=-\lambda \;\Rightarrow\; \lambda = 2$ | | M1 A1 **(2)** |
| | (b) $\begin{pmatrix} 3 & 4 & p \\ -1 & q & -4 \\ 1 & 1 & 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4-p \\ q+4 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ | | M1 A1 |
| | First row $4-p=0 \;\Rightarrow\; p=4$ | Method for either | M1 |
| | Second row $q+4=2 \;\Rightarrow\; q=-2$ | Both correct | A1 **(4)** |
| | (c) $\begin{pmatrix} 3 & 4 & 4 \\ -1 & -2 & -4 \\ 1 & 1 & 3 \end{pmatrix} . \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ 3 \end{pmatrix}$ | | |
| | $3l+4m+4n=10$<br>$-l-2m-4n=-4$<br>$l+m+3n=3$ | Obtaining 3 linear equations | M1 |
| | $2l+2m=6$<br>$3l+2m=8$ | Reducing to a pair of equations and solving for one variable | M1 |
| | $l=2,\ m=1,\ n=0$ | Solving for all three variables. | M1 A1 **(4)**<br>**[10]** |
| | Alternative to (c) | | |
| | $\mathbf{A}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 8 & 8 \\ -1 & -5 & -8 \\ -1 & -1 & 2 \end{pmatrix}$ | | M1 M1 |
| | $\frac{1}{6}\begin{pmatrix} 2 & 8 & 8 \\ -1 & -5 & -8 \\ -1 & -1 & 2 \end{pmatrix} . \begin{pmatrix} 10 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ | | M1 A1 **(4)** |

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| --- | --- | --- |
| **4.** | (a) $z^n = (\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$ | |
| | $z^{-n} = (\cos\theta + \mathrm{i}\sin\theta)^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$ both | M1 |
| | Adding $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ ∗ cso | A1 **(2)** |
| | (b) $\left(z + \dfrac{1}{z}\right)^6 = z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$ | M1 |
| | $= z^6 + z^{-6} + 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) + 20$ | M1 |
| | $64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$ | M1 |
| | $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$ | A1, A1 |
| | $(p = 1, q = 6, r = 15, s = 10)$ A1 any two correct | **(5)** |
| | (c) $\int \cos^6\theta \, d\theta = \left(\dfrac{1}{32}\right)\int (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)\, d\theta$ | |
| | $= \left(\dfrac{1}{32}\right)\left[\dfrac{\sin 6\theta}{6} + \dfrac{6\sin 4\theta}{4} + \dfrac{15\sin 2\theta}{2} + 10\theta\right]$ | M1 A1 |
| | $[\ ....\ ]_0^{\frac{\pi}{3}} = \dfrac{1}{32}\left[-\dfrac{3}{2}\times\dfrac{\sqrt{3}}{2} + \dfrac{15}{2}\times\dfrac{\sqrt{3}}{2} + \dfrac{10\pi}{3}\right] = \dfrac{5\pi}{48} + \dfrac{3\sqrt{3}}{32}$ or exact equivalent | M1 A1 **(4)** |
| | | **[11]** |

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|---|---|---|
| **5.** | $n=1$: $\quad 1^2=\frac{1}{3}\times1\times1\times3$ | B1 |
| | (Hence result is true for $n=1$.) | |
| | $\sum_{r=1}^{k+1}(2r-1)^2=\sum_{r=1}^{k}(2r-1)^2+(2k+1)^2$ | |
| | $=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$, by induction hypothesis | M1 |
| | $=\frac{1}{3}(2k+1)(2k^2-k+6k+3)$ | |
| | $=\frac{1}{3}(2k+1)(2k^2+5k+3)$ | |
| | $=\frac{1}{3}(2k+1)(2k+3)(k+1)$ | M1 A1 |
| | $=\frac{1}{3}(k+1)\left[2(k+1)k-1\right]\left[2(k+1)+1\right]$ | |
| | (Hence, if result is true for $n=k$, then it is true for $n=k+1$.) | |
| | By Mathematical Induction, above implies the result is true for all $n\in\mathbb{Z}^+$. ∗ cso | A1 **(5)** **[5]** |
| **6.** | (a) $\mathrm{f}(k+1)-\mathrm{f}(k)=3^{4k+4}+2^{4k+6}-3^{4k}-2^{4k+2}$ | |
| | $=3^{4k}(3^4-1)+2^{4k+2}(2^4-1)$ | M1 |
| | $=3^{4k}\times80+2^{4k+2}\times15$ can be implied | A1 |
| | $=3^{4k-1}\times240+2^{4k+2}\times15=15(16\times3^{4k-1}+2^{4k+2})$ | M1 |
| | Hence $15\mid\mathrm{f}(k+1)-\mathrm{f}(k)$ ∗ cso | A1 **(4)** |
| | Note: $\mathrm{f}(k+1)-\mathrm{f}(k)$ is divisible by 240 and other appropriate multiples of 15 lead to the required result. | |
| | (b) $n=1$: $\quad \mathrm{f}(1)=3^4+2^6=145=5\times29 \;\Rightarrow\; 5\mid\mathrm{f}(1)$ | B1 |
| | (Hence result is true for $n=1$.) | |
| | From (a) $\mathrm{f}(k+1)-\mathrm{f}(k)=15\lambda$, say. By induction hypothesis $\mathrm{f}(k)=5\mu$, say. | |
| | $\mathrm{f}(k+1)=\mathrm{f}(k)+15\lambda=5(\mu+3\lambda) \;\Rightarrow\; 5\mid\mathrm{f}(k+1)$ | M1 |
| | (Hence, if result is true for $n=k$, then it is true for $n=k+1$.) | |
| | By Mathematical Induction, above implies the result is true for all $n\in\mathbb{Z}^+$. ∗ Accept equivalent arguments cso | A1 **(3)** |
| | (c) $\mathrm{f}(1)=145=5\times29$ is not divisible by 15, so result is not true for all $\mathbb{Z}^+$. | B1 **(1)** |
| | Note: There is no integer for which $\mathrm{f}(n)$ is divisible by 15 and any specific example should be accepted. | **[8]** |

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| **7.** | (a) $\overrightarrow{AB} = -\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{AC} = 4\mathbf{j} + 2\mathbf{k}$ both | B1 |
| | $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 3 \\ 0 & 4 & 2 \end{vmatrix} = -6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ | M1 A1 A1 |
| | Give A1 for any two components correct or the negative of the correct answer. | **(4)** |
| | (b) Cartesian equation has form $3x - y + 2z = p$ | |
| | $(2, -1, 0) \quad \Rightarrow \quad 6 + 1 = p$ or use of another point | M1 |
| | $3x - y + 2z = 7$ or any multiple | A1 **(2)** |
| | (c) Parametric form of line is $\mathbf{r} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ or equivalent form | M1 A1 |
| | Substituting into equation of plane | |
| | $3(5 + 2\lambda) - (5 - \lambda) + 2(3 - 2\lambda) = 7$ | M1 |
| | Leading to $\lambda = -3$ | A1 |
| | $T : (-1, 8, 9)$ | A1 **(5)** |
| | (d) $\overrightarrow{AT} = -3\mathbf{i} + 9\mathbf{j} + 9\mathbf{k}, \quad \overrightarrow{BT} = -2\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$ both | M1 |
| | These are parallel and hence $A$, $B$ and $T$ are collinear ∗ (by the axiom of parallels) | M1 A1 **(3)** |
| | | **[14]** |
| | Alternative to (d) | |
| | The equation of $AB$: $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mu(-\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$ or equivalent | |
| | $\mathbf{i}$: $-1 = 2 - \mu \quad \Rightarrow \quad \mu = 3$ | M1 |
| | $\mu = 3 \quad \Rightarrow \quad \overrightarrow{OT} = -\mathbf{i} + 8\mathbf{j} + 9\mathbf{k}$ | M1 |
| | Hence $A$, $B$ and $T$ are collinear ∗ cso | A1 **(3)** |
| | Note: Column vectors or bold-faced vectors may be used at any stage. | |

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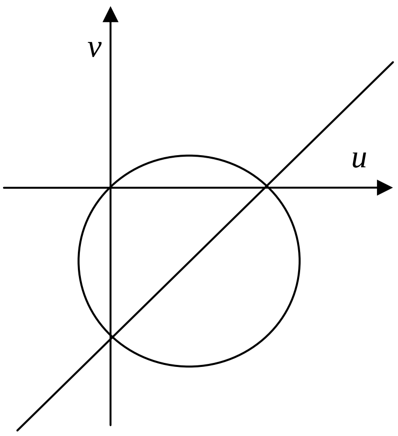
| **8.** | (a) Let $z=\lambda+\lambda\mathrm{i}$; $\quad w=\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda(1+\mathrm{i})}$ | M1 |
| | $=\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda(1+\mathrm{i})}\times\dfrac{1-\mathrm{i}}{1-\mathrm{i}}$ | M1 |
| | $u+\mathrm{i}v=\dfrac{(2\lambda+1)+\mathrm{i}}{2\lambda}$ | A1 |
| | $u=1+\dfrac{1}{2\lambda},\quad v=\dfrac{1}{2\lambda}$ | M1 |
| | Eliminating $\lambda$ gives a line with equation $v=u-1$ or equivalent | A1 **(5)** |
| | (b) Let $z=\lambda-(\lambda+1)\mathrm{i}$; $\quad w=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}$ | M1 |
| | $=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}\times\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda+(\lambda+1)\mathrm{i}}$ | M1 |
| | $u+\mathrm{i}v=\dfrac{\lambda(2\lambda+1)+\lambda\mathrm{i}}{2\lambda^2+2\lambda+1}$ | A1 |
| | $u=\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1},\quad v=\dfrac{\lambda}{2\lambda^2+2\lambda+1}$ | M1 |
| | $\dfrac{u}{v}=2\lambda+1$ | |
| | $v=\dfrac{2\lambda}{4\lambda^2+4\lambda+2}=\dfrac{(2\lambda+1)-1}{(2\lambda+1)^2+1}=\dfrac{\frac{u}{v}-1}{\left(\frac{u}{v}\right)^2+1}$ | M1 |
| | Reducing to the circle with equation $u^2+v^2-u+v=0$ ∗ cso | M1 A1 **(7)** |
| | (c)  | |
| | ft their line | B1ft |
| | Circle through origin, centre in correct quadrant | B1 |
| | Intersections correctly placed | B1 **(3)** |
| | | **[15]** |

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| **8.** | Alternative for (b) | |
| | Let $z=\lambda-(\lambda+1)\mathrm{i}$; $\quad w=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}$ | M1 |
| | $=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}\times\dfrac{\lambda+(\lambda+1)\mathrm{i}}{\lambda+(\lambda+1)\mathrm{i}}$ | M1 |
| | $u+\mathrm{i}v=\dfrac{\lambda(2\lambda+1)+\lambda\mathrm{i}}{2\lambda^2+2\lambda+1}$ | A1 |
| | $u=\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1},\quad v=\dfrac{\lambda}{2\lambda^2+2\lambda+1}$ | M1 |
| | $u^2+v^2-u+v=\left(\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1}\right)^2+\left(\dfrac{\lambda}{2\lambda^2+2\lambda+1}\right)^2-\dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1}+\dfrac{\lambda}{2\lambda^2+2\lambda+1}$ | |
| | $=\dfrac{(4\lambda^4+4\lambda^3+\lambda^2)+\lambda^2-2\lambda^2(2\lambda^2+2\lambda+1)}{(2\lambda^2+2\lambda+1)^2}$ | M1 |
| | $=0$ ∗ | M1 A1 **(7)** |
| **8.** | Alternative for (b) | |
| | Let $z=\lambda-(\lambda+1)\mathrm{i}$; $\quad u+\mathrm{i}v=\dfrac{\lambda-\lambda\mathrm{i}}{\lambda-(\lambda+1)\mathrm{i}}$ | M1 |
| | $(u+\mathrm{i}v)(\lambda-(\lambda+1)\mathrm{i})=\lambda-\lambda\mathrm{i}$ | M1 |
| | $u\lambda+v(\lambda+1)+[v\lambda-u(\lambda+1)]\mathrm{i}=\lambda-\lambda\mathrm{i}$ | A1 |
| | Equating real & imaginary parts | |
| | $u\lambda+v(\lambda+1)=\lambda$ (i) $\qquad v\lambda-\lambda u-u=-\lambda$ (ii) | M1 |
| | From (i) $\lambda=\dfrac{v}{1-u-v}$ $\qquad$ From (ii) $\lambda=\dfrac{u}{1-u+v}$ | |
| | $\dfrac{v}{1-u-v}=\dfrac{u}{1-u+v}$ | M1 |
| | Reducing to the circle with equation $u^2+v^2-u+v=0$ ∗ | M1 A1 **(7)** |