GCE

Edexcel GCE

Mathematics

Further Pure Mathematics 3 FP3 (6676)

June 2008

advancing learning, changing lives

Mark Scheme

Edexcel GCE

Mathematics

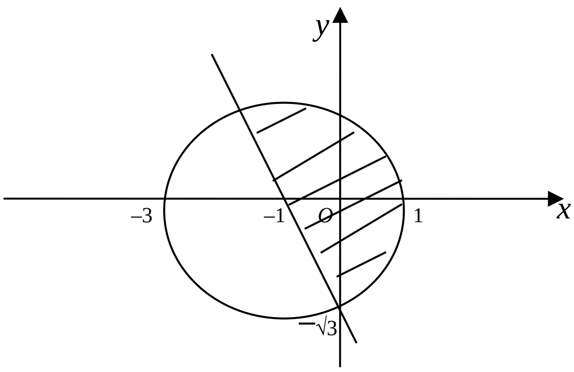
General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

6676 Further Pure FP3
Mark Scheme (post QPEC)

Question Number	Scheme	Marks
1.	$\left(\frac{dy}{dx}\right)_0 = 0 + \cos 0.6$ (= 0.825335…) May be implicit	B1
	$y_1 \approx 0.05\left(\frac{dy}{dx}\right)_0 + y_0 = 0.05 \times 0.825335\ldots + 0.6$	M1
	$y_1 \approx 0.641266\ldots$ $= 0.6413$ (4 d.p.)	A1
	$\left(\frac{dy}{dx}\right)_1 = 0.05 + \cos 0.641266...$	
	$= 0.851338\ldots$	A1ft
	$y_2 \approx 0.05\left(\frac{dy}{dx}\right)_1 + y_1 = 0.05 \times 0.851338\ldots + 0.641266\ldots$	M1
	$y_2 \approx 0.683833\ldots$ $= 0.6838$ (4 d.p.)	A1
		(6)

Question Number	Scheme	Marks
2. (a)	$\begin{pmatrix} 1 & p & 2 \\ 0 & 3 & q \\ 2 & p & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$	
	$1 + 2p + 2 = \lambda, \quad 6 + q = 2\lambda$	M1, A1
	$\therefore \quad 6 + q = 6 + 4p \quad \Rightarrow \quad q = 4p \quad (\ast)$	A1 (3)
(b)	$\begin{vmatrix} -4 & p & 2 \\ 0 & -2 & 4p \\ 2 & p & -4 \end{vmatrix} = 0$	
	$-4(8 - 4p^2) - p(0 - 8p) + 2(0 + 4) = 0$	M1, A1
	$p^2 = 1$	
	$p < 0, p = -1, q = -4$	M1, A1 (4)
	[Alt. for 1st M1 A1: $-4(8 - pq) - p(0 - 2q) + 2(0 + 4) = 0 \Rightarrow pq = 4$]	
(c)	$\begin{pmatrix} -4 & -1 & 2 \\ 0 & -2 & -4 \\ 2 & -1 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	
	$-4x - y + 2z = 0, \ -2y - 4z = 0, \ 2x - y - 4z = 0$ Any 2 eqns	M1
	$2x = -y = 2z$	M1
	E.vector is $k\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$	A1 (3)
		(10)
	Alt. for (b) and (c)	
	$\begin{pmatrix} 1 & p & 2 \\ 0 & 3 & 4p \\ 2 & p & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 5\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1
	$x + py + 2z = 5x, 3y + 4pz = 5y, 2x + py + z = 5z$ Any 2 eqns	A1
	$py + 2z = 4x$ (i), $2pz = y$ (ii), $2x + py = 4z$ (iii)	
	From (i) and (iii) $py = 2z$ Use $q = 4p$ & elim. one of $x/y/z$	M1
	From (ii) $p^2 = 1$	
	$p < 0, p = -1, q = -4$	M1 A1
	$p = -1 \Rightarrow y + 2z = 4x, -2z = y, 2x - y = 4z$ giving $2x = -y = 2z$	M1
	E.vector is $k\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$	A1

Question Number	Scheme	Marks
3. (a)	$\left(x^2+1\right)\dfrac{d^3y}{dx^3}+2x\dfrac{d^2y}{dx^2}=4y\dfrac{dy}{dx}+(1-2x)\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}$	M1, A1
	$\left(x^2+1\right)\dfrac{d^3y}{dx^3}=\left(1-4x\right)\dfrac{d^2y}{dx^2}+\left(4y-2\right)\dfrac{dy}{dx}$ (✱)	A1 (3)
(b)	$\left(\dfrac{d^2y}{dx^2}\right)_0=3$	B1
	$\left(\dfrac{d^3y}{dx^3}\right)_0=5$	B1
	$y\approx 1+x+\dfrac{3}{2}x^2,+\dfrac{5}{6}x^3$	M1, A1 (4)
(c)	$x=-0.5,\quad y\approx 1-0.5+0.375-0.104166\ldots$	
	$=0.77$ (2 d.p.) [awrt 0.77]	B1 (1)
		(8)
4. (a)	$\lvert(x-3)+\mathrm{i}y\rvert=2\lvert x+\mathrm{i}y\rvert\Rightarrow(x-3)^2+y^2=4x^2+4y^2$	M1, A1
	$\therefore\ x^2+y^2+2x-3=0$	
	$(x+1)^2+y^2=4$	M1
	Circle, centre $(-1, 0)$, radius 2	A1, A1 (5)
(b)	 Circle, centre on x-axis B1 C $(-1, 0)$, $r = 2$ B1ft	B1, B1
	Straight line B1 Through $(-1, 0)$ or perp. bis. of $(-3, 0)$ and $(0, \sqrt{3})$ B1 Through pt of int. of circle & –ve y-axis B1	B1, B1, B1 (5)
(c)	Shading inside circle	B1
	Inside correct circle and on the correct side of the correct line	B1 (2)
		(12)
Alt. (a)	The locus is a circle of Apollonius, which is a circle with diameter XY where the points X and Y cut $(3, 0)$ and $(0, 0)$ internally and externally in the ratio 2:1	M1, A1
	X $(1, 0)$ Y $(-3, 0)$	M1, A1
	C $(-1, 0)$ $r = 2$	A1

Question Number	Scheme	Marks
5. (a)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} t \\ 2t \end{pmatrix} = \begin{pmatrix} t(k-4) \\ t(1+k) \end{pmatrix}$	M1
	$t(1+k) = 2t(k-4)$	M1
	$k = 9$	A1 (3)
(b)	det $\mathbf{A} = k^2 + 2(1-k)$	M1
	$= (k-1)^2 + 1$	M1
	$> 0 \ \forall\ k \quad \Rightarrow \mathbf{A}$ is non-singular $\forall\ k$	A1 (3)
(c)	$\mathbf{A}^{-1} = \dfrac{1}{k^2-2k+2}\begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$	M1, A1 (2)
(d)	$k = 3, \quad \mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$	B1
	$\mathbf{Ap} = \mathbf{q} \Rightarrow \mathbf{p} = \mathbf{A}^{-1}\mathbf{q} \qquad \mathbf{p} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 6 \\ -1 \end{pmatrix}$	M1, A1 (3)
	Alt. $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow 3x - 2y = 4, -2x + 3y = -3$ B1	
	M1 A1 for solving two sim. eqns. in x and y to give $x = 1.2$, $y = -0.2$ (o.e.)	**(11)**
6. (a)	$(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta \quad \therefore$ true for $n = 1$	B1
	Assume true for $n = k$, $(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$	
	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	M1
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1
	$\therefore$ true for $n = k + 1$ if true for $n = k$	
	$\therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1 (5)
(b)	$\cos 5\theta = \mathrm{Re}\left[(\cos\theta + \mathrm{i}\sin\theta)^5\right]$	
	$= \cos^5\theta + 10\cos^3\theta\,\mathrm{i}^2\sin^2\theta + 5\cos\theta\,\mathrm{i}^4\sin^4\theta$	M1, A1
	$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
	$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$	M1
	$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta \quad$ (✱)	A1 (5)
(c)	$\dfrac{\cos 5\theta}{\cos\theta} = 0 \quad \Rightarrow \quad \cos 5\theta = 0$	M1
	$5\theta = \dfrac{\pi}{2}, \ldots$	
	$\theta = \dfrac{\pi}{10}, \ldots$	A1
	$x = 2\cos\theta, \qquad x = 2\cos\dfrac{\pi}{10}$ is a root (✱)	A1 (3)
		(13)

Question Number		Scheme	Marks
7.	(a)	$\overrightarrow{PQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{PR} = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1
		$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	M1, A1ft (3)
	(b)	$\mathbf{r} \, . \, (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} - \mathbf{k}) \, . \, (3\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1
		$\mathbf{r} \, . \, (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = 4$ [o.e. may use $\overrightarrow{OQ}$ or $\overrightarrow{OR}$]	A1 (2)
	(c)	$3x + y - z = 4$ (i), $x - 2y - 5z = 6$ (ii)	
		(i) × 2 + (ii) $7x - 7z = 14$, $x = z + 2$	M1
		In (ii) $z + 2 - 2y - 5z = 6$, $y + 2 = -2z$	M1
		$\therefore$ $x = z + 2$ and $y + 2 = -2z$	A1ft
		$\dfrac{x-2}{(1)} = \dfrac{y+2}{-2} = \dfrac{z}{(1)}$ o.e.	M1, A1 (5)
	(d)	Vector parallel to line through P and S = $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	M1
		$\overrightarrow{QR} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ $\therefore PS$ // QR	A1 (2)
	(e)	$\overrightarrow{PT} = 4\mathbf{i} + 2\mathbf{j}$	
		Volume = $\dfrac{1}{3}\left\lvert \overrightarrow{PQ} \times \overrightarrow{PR} . \overrightarrow{PT} \right\rvert = \dfrac{1}{3} \lvert (3\mathbf{i} + \mathbf{j} - \mathbf{k}) \, . \, (4\mathbf{i} + 2\mathbf{j}) \rvert$	M1, A1
		$= \dfrac{1}{3}(12 + 2)$	
		$= 4\frac{2}{3}$ o.e.	A1 (3)
			(15)