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1 Past and ongoing research projects

My research interests have always been at the junction of group theory, geometry and combinatorics. For my D.Phil thesis and for three years subsequently I worked on Tits buildings and related structures; these provide a geometrical picture of simple groups. More recently, I have worked on geometrical, combinatorial and computational aspects of group theory.
My work is motivated by questions which arise within central strands of group theory. I have worked largely with groups defined by finite presentations (which are generally infinite), but also with matrix groups specified by a set of generating matrices. For finitely presented groups I have looked at problems of decidability and computability; of my most recent projects, one examines connections between group theory and formal language theory, the other studies quantum computation within a group theoretic perspective. For both finitely presented and matrix groups I have also worked on the theoretical development and analysis of high performance algorithms, and further I have developed practical implementations of these, which have been freely and widely distributed within the mathematical community.
Finitely presented groups arise naturally from problems in geometry and topology, and hence techniques which allow computation with them are of particular interest to geometers and topologists. Some of the techniques involved also use ideas from the theory of formal languages. Thus this branch of group theory invites interaction both with mathematicians from other areas of mathematics and with computer scientists, and my work in this area has been very collaborative in its nature.
The family of **automatic groups**, originally defined by W. Thurston in an attempt to abstract certain finiteness properties of the fundamental groups of hyperbolic manifolds, has been of interest for some time to geometers and topologists. The defining properties of the family give a geometrical viewpoint on the groups and facilitate computation with them; to such a group is associated a set of paths in the Cayley graph of the group (a 'language' for the group) which both satisfies a geometrical 'fellow traveller condition' and lies in the formal language class of regular languages. Examples of automatic groups are provided by the fundamental groups of compact Euclidean and hyperbolic manifolds, and of geometrically finite hyperbolic manifolds, also by Coxeter groups, braid groups, many Artin groups, and groups satisfying various small cancellation conditions. While working as an RA in Warwick, I developed (in conjunction with David Epstein and Derek Holt of Warwick Mathematics Institute, [24]) a suite of programs known as *automata* to compute with these groups. The basic algorithms are still widely used (recently in a new implementation, by Holt, known as *kbmag*, which is available within the GAP computer algebra system). My own work in [11] generalises them to deal with a wider family of automatic groups; experiments with these new algorithms already show some very interesting results.
The study of the larger family of **combable groups**, in particular in relation to **formal language classes** is motivated by the failure of some very interesting groups to be automatic. The fundamental groups of compact 3-manifolds based on the Nil and Sol geometries are not automatic, nor is any nilpotent group (probably also any soluble group) which is not virtually abelian. Some of my recent theoretical work was based on an attempt to find a generalisation

of the definition which admits more examples, but preserves the best properties of automatic groups, building on work of Bridson and Gilman. A combable group is defined to be a group equipped with a language satisfying some kind of fellow traveller condition (for some types of combings this condition may be weaker than that satisfied by automatic groups); for practical purposes it is often interesting to restrict attention to groups with languages in some formal language classes. Bridson and Gilman found appropriate languages in the formal language class of indexed languages for the fundamental groups of all compact, geometrisable 3-manifolds. Gilman, Holt and I extended this work in [9] to find combings in the class of real-time languages (recognised by Turing machines which process their input in 'real-time', that is, which complete the processing as they finish reading the input) for many nilpotent groups. We showed further that any finitely generated nilpotent or even polycyclic group embeds in a real-time combable group. In related work in [8], I investigated the connections between the group theoretic and formal language theoretic properties of combings.

Real-time algorithms also feature in my recent work with Holt, on the language theoretic complexity of the word problem, described in more detail in the section below. Our first results on the family of groups with word problem solvable in real-time are described in [5]. Recent work with Holt, Röver and Thomas [2] looked at the family of groups for which the complement of the word problem is a context-free language; the groups with context-free word problem form a proper subset of this family.

The examination of the **quotient structure of finitely presented groups** was the central theme of two programs developed by myself and Derek Holt. It played an important part in the program *testisom*([23, 37]), which is effective in practice for the theoretically insoluble problem of isomorphism testing; basically a comparison of finite quotients was used to search for a proof of non-isomorphism. Subsequently, the ideas were developed further in an interactive graphics program *quotpic* ([22, 35, 10]) for the construction of finite quotients. Both programs have been widely used, and frequently cited. These ideas have now been taken up by the developers of GAP, and similar facilities provided within that system. *quotpic* itself continues to be developed ([10]), following demands from users.

Over the last ten years, the development of **fast random algorithms** has opened up new directions in computational group theory. In particular it has become feasible to develop algorithms which depend on the generation of random elements, and thus **matrix group**s, for which there were previously very few efficient algorithms, have become tractable. In collaboration with others I developed and implemented (in GAP) random (Monte Carlo) algorithms to investigate irreducibility, imprimitivity and decomposition with respect to a normal subgroup of matrix groups ([16, 15, 14]).

Joint work with Leonard Soicher (QMW, London) in [6] on the **effective computation of topological invariants of cell complexes** took my research in a somewhat different, but complementary, direction. A viewpoint originally due to Reidemeister was further investigated. Effective algorithms to compute the fundamental group, first homology group and covers of a combinatorial cell complex have now been developed, and implementations in the computer algebra system GAP are publicly and freely available.

Very recently I have started work with Mike Batty and Andrew Duncan on a project to study **quantum computation** in a group theoretical context. We hope that group theory will provide a fruitful testbed for quantum computational ideas, and that the perspective of quantum computation will bring us insight into computation with groups even within a classical computational framework.

My work in the development of algorithms has been a mix of theory and implementation, and has naturally led me to be involved in the development of larger scale **computer algebra systems**, in addition to my own, widely distributed, standalone software. For many years I have collaborated with the authors of the computer algebra system GAP. Many of the algorithms I have been involved with are implemented within that system, or (like *quotpic*) have influenced the development of GAP in other ways. I have had a lot of interaction with Australian leaders in the field of computational group theory, such as Mike Newman (Australian National University), George Havas (Queensland) and Eamonn O'Brien (formerly ANU, now Auckland). Further I have been a major contributor to the MAGNUS system for computation with finitely presented groups, which is developed from CCNY in New York by a team led by Gilbert Baumslag, and have incorporated many of the automatic group algorithms into that system.

2 Publications

NB: Submitted articles are available as preprints from http://www.mas.ncl.ac.uk/~nser

[1] Quantum algorithms in group theory, (with Michael Batty, Samuel L. Braunstein, and Andrew J. Duncan), in preparation.

[2] Groups with context-free co-word problem, (with Derek F Holt, Claas E Röver and Richard M Thomas), submitted to the London Mathematical Society.

[3] How hard is the word problem?, refereed Proceedings of Conference for European Women in Mathematics, Varna, Bulgaria, September 2002.

[4] Quasigeodesics in word hyperbolic groups (with Derek Holt), accepted in March 2002 for Internat. J. Alg. Comp.

[5] Solving the word problem in real time, (with Derek F. Holt), J. London Mathematical Society 63 (2001), 623–639.

[6] An algorithmic approach to fundamental groups and covers of combinatorial cell complexes (with Leonard Soicher), *J. Symbolic Computation* 29 (2000), 59–77.

[7] Some challenging group presentations, (with George Havas, Derek F. Holt, P. E. Kenne), in *J. Austral. Math. Soc. Ser. A* 67 (1999), 206–213.

[8] A language theoretic analysis of combings, Groups, Languages and Groups, 117-136, *Contemporary Mathematics* 250, AMS, Providence RI, 1999.

[9] Combing nilpotent and polycyclic groups, (with Robert H. Gilman and Derek F. Holt), *Internat. J. Algebra Comput.* 9 (1999) 135–155.

[10] Computing with abelian sections of finitely presented groups, (with Derek F. Holt), *J. Algebra* 214 (1999) 714–728.

[11] Automatic groups associated with word orders other than shortlex, *Internat. J. Algebra Comput.* 8 (1998), 575–598.

[12] Hairdressing in groups; a survey of combings and formal languages, *Geometry and Topology Monographs* 1 (1998), The Epstein Birthday Schrift, 493–509.

[13] Free quotients of finitely presented groups, (with D.F. Holt) *Experimental Mathematics* 5 (1996) 49–56

[14] Computing module decompositions with respect to a normal subgroup, (with D.F. Holt, C.R. Leedham-Green, E.A. O'Brien) *J. Algebra* 184 (1996) 818 – 838.

[15] Testing matrix groups for primitivity, (with D.F. Holt, C.R. Leedham-Green, E.A. O'Brien) *J. Algebra* 184 (1996) 795 – 817.

[16] Testing modules for irreducibility (with D.F. Holt) *J. Austral. Math. Soc. Ser. A* 57 (1994) 1 – 16.

[17] Recognizing badly presented Z-modules (with G. Havas and D.F. Holt) *J. Lin. Alg. and its Applications* 192 (1993) 137 – 163.

[18] Recognition of Matrix Groups, *Proc. Int. Conf. Group Theory, Timişoara, 1992*, An. Univ. Timişoara Ser. Ştiinţ. Mat. (1993), Special Issue, 133 – 140.

[19] Finding subgroups and quotients of finitely presented groups (with D.F. Holt) *Proc. Int. Conf. Group Theory, Timişoara, 1992*, An. Univ. Timişoara Ser. Ştiinţ. Mat. (1993), Special Issue, 99 – 107.

[20] An implementation of the Neumann-Praeger algorithm for the recognition of special linear groups (with D.F. Holt) *Experimental Mathematics* 1 (1993) 237 – 242.

[21] Software for automatic groups, isomorphism testing and finitely presented groups (with D.F. Holt) *Geometric Group Theory, Vol. 1, London Math. Soc. Lecture Note Ser.* 181 (1993) 120 – 125.

[22] A graphics system for displaying finite quotients of finitely presented groups (with D.F. Holt) *Groups and Computation, DIMACS Series in Discrete Mathematics and Theoretical Computer Science* 11 (1993) 113 – 126.

[23] The isomorphism problem for finitely presented groups (with D.F. Holt) *Groups, Combinatorics and Geometry, London Math. Soc. Lecture Note Ser.* 165 (1992) 459 – 475.

[24] The use of Knuth-Bendix methods to solve the word problem in automatic groups (with D.B.A. Epstein and D.F. Holt) *J. Symbolic Computation* 12 (1991) 397 – 414.

[25] Weak buildings of spherical type, *Geom. Dedicata* 27 (1988) 15 – 47.

[26] Embeddings of an M_{12} geometry: some simple combinatorial descriptions, *European J Comb.* 9 (1988) 183 – 187.

[27] The subgroup structure of the sporadic group M_{12} (with F. Buekenhout) *Math. of Computation* 50 (1988) 595 – 605.

[28] A classification of a class of C_3 geometries, *J. Combinatorial Theory Ser. A* 44 (1987) 173 – 181.

[29] A theorem on geometries of type C_3 (with A.L. Pasini) *J. Geometry* 30 (1987) 124 – 143.

[30] Products of Tits geometries, *J. Geometry* 25 (1985) 77 – 87.

[31] Finite C_3 geometries in which all lines are thin, *Math. Zeitschrift* 189 (1985) 263 – 271.

[32] C_3 geometries arising from the Klein quadric, *Geom. Dedicata* 18 (1985) 67 – 85.

Mathematical software

All software is freely and publicly available.

[33] Major contribution to the MAGNUS computational system for finitely presented groups, publicly released in 1997.

[34] *smash*, algorithms to decompose a matrix groups with respect to a normal subgroup, and to test for imprimitivity, written in GAP with D.F. Holt, C.R. Leedham-Green, E.A. O'Brien, and distributed with the GAP package, since 1993.

[35] *quotpic*, an interactive graphics display program to calculate finite quotients of a finitely presented group (written in C, with D.F. Holt, available by anonymous *ftp* since 1991 updated versions released since).

[36] a program to demonstrate encoding and decoding techniques for Reed-Solomon codes (written in C, with P. van Assche, Institut für Experimentelle Mathematik, Essen) available since 1990.

[37] *testisom*, a program to test for isomorphism between two finitely presented groups (with D.F. Holt, available by anonymous *ftp* since 1991).

[38] *automata*, a program to calculate the automatic structure of automatic groups (written in C, with D.B.A. Epstein and D.F. Holt, available by anonymous *ftp* since 1990).