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January 19th 2022

McGill University
Intro to Computer Systems

Assignment 1

Section 1 : Number Representation, 2s Complement, and Floating Point

(1) 0.101011_2

$$\begin{aligned}
0.101011_2 &= 0 \cdot 2^0 + 1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} + 0 \cdot 2^{-4} + 1 \cdot 2^{-5} + 1 \cdot 2^{-6} \\
&= \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{64} \\
&= 0.5 + 0.125 + 0.03125 + 0.015625 = 0.671875_{10}
\end{aligned} \tag{1}$$

(2) 8.725_{10}

$$\begin{aligned}
8.725_{10} &= 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 + 0.725_{10} \\
&= 1 \cdot 2^3 + 1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0.225_{10} \\
&= 1 \cdot 2^3 + 1 \cdot 2^{-1} + 1 \cdot 2^{-3} + 0.100_{10} \\
&= 1 \cdot 2^3 + 1 \cdot 2^{-1} + 1 \cdot 2^{-3} + 1 \cdot 2^{-4} + 0.0375_{10} \\
&= 1000_2 + 1 \cdot 2^{-1} + 1 \cdot 2^{-3} + 1 \cdot 2^{-4} + 1 \cdot 2^{-5} + 0.000625_{10} \\
&= 1000_2 + 1 \cdot 2^{-1} + 1 \cdot 2^{-3} + 1 \cdot 2^{-4} + 1 \cdot 2^{-5} + 1 \cdot 2^{-8} + 1 \cdot 2^{-9} + 1 \cdot 2^{-12} \\
&+ 1 \cdot 2^{-13} + 1 \cdot 2^{-16} + \cdots \\
&= 1000.1011100110011001_2 \\
1000_2 &= 1 \cdot 2^3 = 8 \cdot 16^0 = 8_{16} \\
0.1011100110011001_2 &= 1 \cdot 2^{-1} + 1 \cdot 2^{-3} + 1 \cdot 2^{-4} + 1 \cdot 2^{-5} + 1 \cdot 2^{-8} + 1 \cdot 2^{-9} + 1 \cdot 2^{-12} \\
&+ 1 \cdot 2^{-13} + 1 \cdot 2^{-16} + \cdots \\
&= \frac{8}{16} + \frac{2}{16} + \frac{1}{16} + \frac{8}{16^2} + \frac{1}{16^2} + \frac{8}{16^3} + \frac{1}{16^3} + \frac{8}{16^4} + \frac{1}{16^4} \\
&= 11 \cdot 16^{-1} + 9 \cdot 16^{-2} + 9 \cdot 16^{-3} + 9 \cdot 16^{-4} \\
&= 0.\mathrm{B}999_{16} \\
8.725_{10} &= 1000.1011100110011001_2 = 8.\mathrm{B}999_{16}
\end{aligned} \tag{2}$$

(3) $1FA.U06G_{32}$.

$$
\begin{aligned}
1FA.U06G_{32} &= 1\cdot 32^2 + 15\cdot 32^1 + 10\cdot 32^0 + 30\cdot 32^{-1} + 0\cdot 32^{-2} + 6\cdot 32^{-3} + 16\cdot 32^{-4} \\
&= 1\cdot (2^5)^2 + 15\cdot (2^5)^1 + 10\cdot (2^5)^0 + 30\cdot (2^5)^{-1} + 0\cdot (2^5)^{-2} \\
&+ 6\cdot (2^5)^{-3} + 16\cdot (2^5)^{-4} \\
&= 1\cdot 2^{10} + 15\cdot 2^5 + 10\cdot 2^0 + 30\cdot 2^{-5} + 0\cdot 2^{-10} + 6\cdot 2^{-15} + 16\cdot 2^{-20} \\
&= 1\cdot 2^{10} + (14+1)\cdot 2^5 + (4+1)\cdot 2^1 + (14+1)\cdot 2^{-4} \\
&+ (2+1)\cdot 2^{-14} + 1\cdot 2^{-16} \\
&= 1\cdot 2^{10} + 1\cdot 2^8 + 1\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 1\cdot 2^3 + 1\cdot 2^1 + 1\cdot 2^{-1} + 1\cdot 2^{-2} \\
&+ 1\cdot 2^{-3} + 1\cdot 2^{-4} + 1\cdot 2^{-13} + 1\cdot 2^{-14} + 1\cdot 2^{-16} \\
&= 10111101010.1111000000001101_2 \\
10111101010_2 &= 1\cdot 2^{10} + 1\cdot 2^8 + 1\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 1\cdot 2^3 + 1\cdot 2^1 \\
&= 2^2\cdot 16^2 + 1\cdot 16^2 + 2^3\cdot 16^1 + 2^2\cdot 16^1 + 2^1\cdot 16^1 + 2^3\cdot 16^0 + 2^1\cdot 16^0 \\
&= 5\cdot 16^2 + 14\cdot 16^1 + 10\cdot 16^0 = 5EA_{16} \\
0.1111000000001101_2 &= 1\cdot\frac{1}{2^1} + 1\cdot\frac{1}{2^2} + 1\cdot\frac{1}{2^3} + 1\cdot\frac{1}{2^4} + 1\cdot\frac{1}{2^{13}} + 1\cdot\frac{1}{2^{14}} + 1\cdot\frac{1}{2^{16}} \\
&= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{2}\cdot\frac{1}{16^3} + \frac{1}{4}\cdot\frac{1}{16^3} + 1\cdot\frac{1}{16^4} \\
&= \frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16} + \frac{8}{16^4} + \frac{4}{16^4} + \frac{1}{16^4} \\
&= 15\cdot 16^{-1} + 13\cdot 16^{-4} = \mathrm{F00D}_16 \\
1FA.U06G_{32} &= 10111101010.1111000000001101_2 = 5EA.F00D_{16}
\end{aligned} \tag{3}
$$

(4) 3231004_5.

$$
\begin{aligned}
3231004_5 &= 3\cdot 5^6 + 2\cdot 5^5 + 3\cdot 5^4 + 1\cdot 5^3 + 4\cdot 5^0 \\
&= 46375 + 6250 + 1875 + 125 + 4 = 55129_{10} \\
&= 13\cdot 16^3 + 1881 \\
&= 13\cdot 16^3 + 7\cdot 16^2 + 89 \\
&= 13\cdot 16^3 + 7\cdot 16^2 + 5\cdot 16^1 + 9\cdot 16^0 \\
&= D759_{16}
\end{aligned} \tag{4}
$$

(5) -4128786_{10}.

$$
\begin{aligned}
4128786_{10} &= 1\cdot 2^{21} + 1\cdot 2^{20} + 1\cdot 2^{19} + 1\cdot 2^{18} + 1\cdot 2^{17} + 1\cdot 2^{16} + 1\cdot 2^{4} + 1\cdot 2^{1} \\
&= 1111110000000000010010 \\
&\Rightarrow 001111110000000000010010 \text{ (Add two zeros since} \\
&\text{we want a 24 bit number)} \\
&\Rightarrow 110000001111111111101101 \text{ (Flip all the bits)} \\
&\Rightarrow 110000001111111111101110 \text{ (Add one)} \\
110000001111111111101110_2 &= 1\cdot 2^{23} + 1\cdot 2^{22} + 1\cdot 2^{15} + 1\cdot 2^{14} + 1\cdot 2^{13} + 1\cdot 2^{12} + 1\cdot 2^{11} \\
&+ 1\cdot 2^{10} + 1\cdot 2^{9} + 1\cdot 2^{8} + 1\cdot 2^{7} + 1\cdot 2^{6} + 1\cdot 2^{5} + 1\cdot 2^{3} + 1\cdot 2^{2} \\
&+ 1\cdot 2^{1} \\
&= (8+4)\cdot 16^5 + (8+4+2+1)\cdot 16^3 + (8+4+2+1)\cdot 16^2 \\
&+ (8+4+2)\cdot 16^1 + (8+4+2)\cdot 16^0 \\
&= 12\cdot 16^5 + 15\cdot 16^3 + 15\cdot 16^2 + 14\cdot 16^1 + 14\cdot 16^0 \\
&= \text{C0FFEE}_{16}
\end{aligned} \tag{5}
$$

(6) -25.625_{10}

$$
\begin{aligned}
25.625_{10} &= 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^0 + 1\cdot 2^{-1} + 1\cdot 2^{-2} \\
&= 11001.101_2 = 1.1001101\cdot 2^{-4} \\
&\text{Sign is 1 since it is negative ; Exponent is } 4+127=131 \text{ ; Fraction is } 1000011\cdots \\
-25.625_{10} &= 1\ 10000011\ 10011010000000000000000 \\
&\Rightarrow 1\cdot 2^{31} + 1\cdot 2^{30} + 1\cdot 2^{24} + 1\cdot 2^{23} + 1\cdot 2^{22} + 1\cdot 2^{19} + 1\cdot 2^{18} + 1\cdot 2^{16} \\
&= 8\cdot 16^7 + 4\cdot 16^7 + 1\cdot 16^6 + 8\cdot 16^5 + 4\cdot 16^5 + 8\cdot 16^4 + 4\cdot 16^4 + 1\cdot 16^1 \\
&= 12\cdot 16^7 + 1\cdot 16^6 + 12\cdot 16^5 + 13\cdot 16^4 = \text{C1CD000}_16 \\
&\text{This representation is exact.}
\end{aligned} \tag{6}
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Section 2 : Seven Segment Decoder