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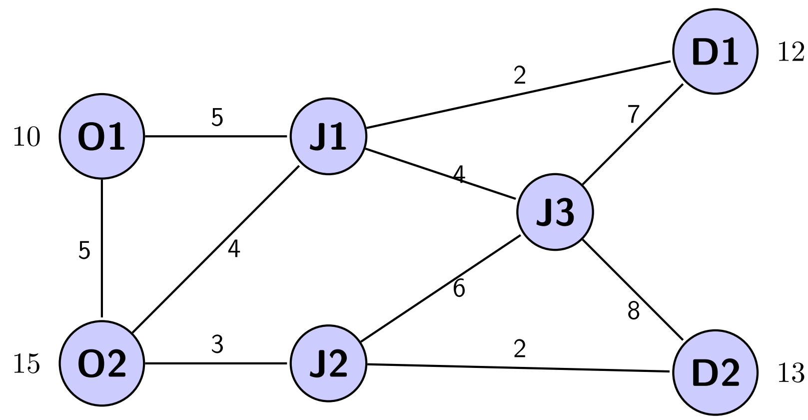
McGill University
MATH470 : Honours Research Project

# Using optimization methods in the context of mine planning

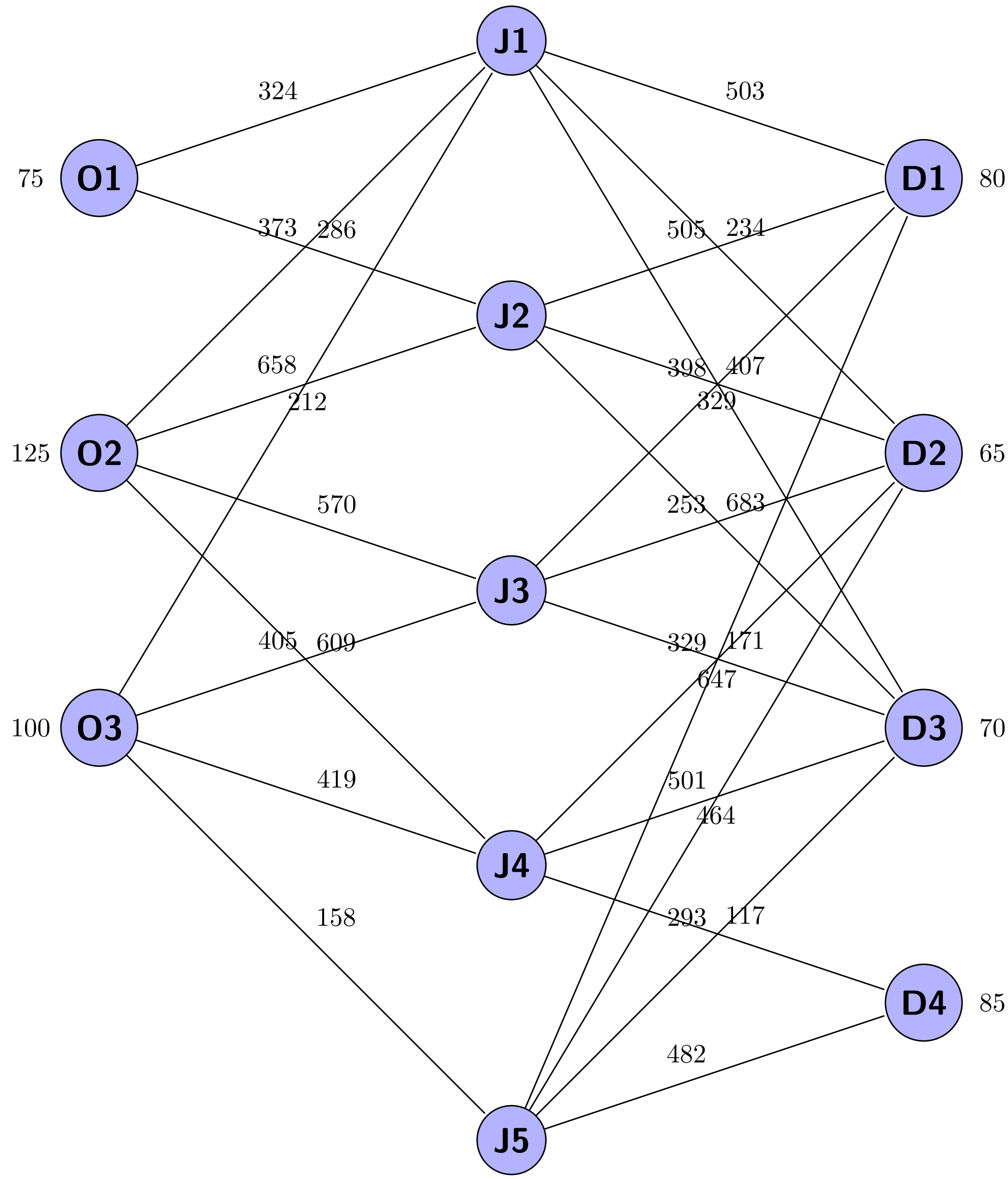
## 1. Transshipment Problem




We can formulate this problem as a linear program in the following way,

$$
\begin{aligned}
\min \quad & \sum_{u,v \in V} t_{u,v} x_{u,v} \\
\text{s.t.} \quad & \sum_{v \in V} x_{u,v} - \sum_{v \in V} x_{v,u} = a_u \quad \forall u \in V \\
& \sum_{v \in V} x_{v,u} - \sum_{v \in V} x_{u,v} = b_u \quad \forall u \in V \\
& \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
& x_{u,v} \geq 0 \quad \forall u, v \in V
\end{aligned}
$$

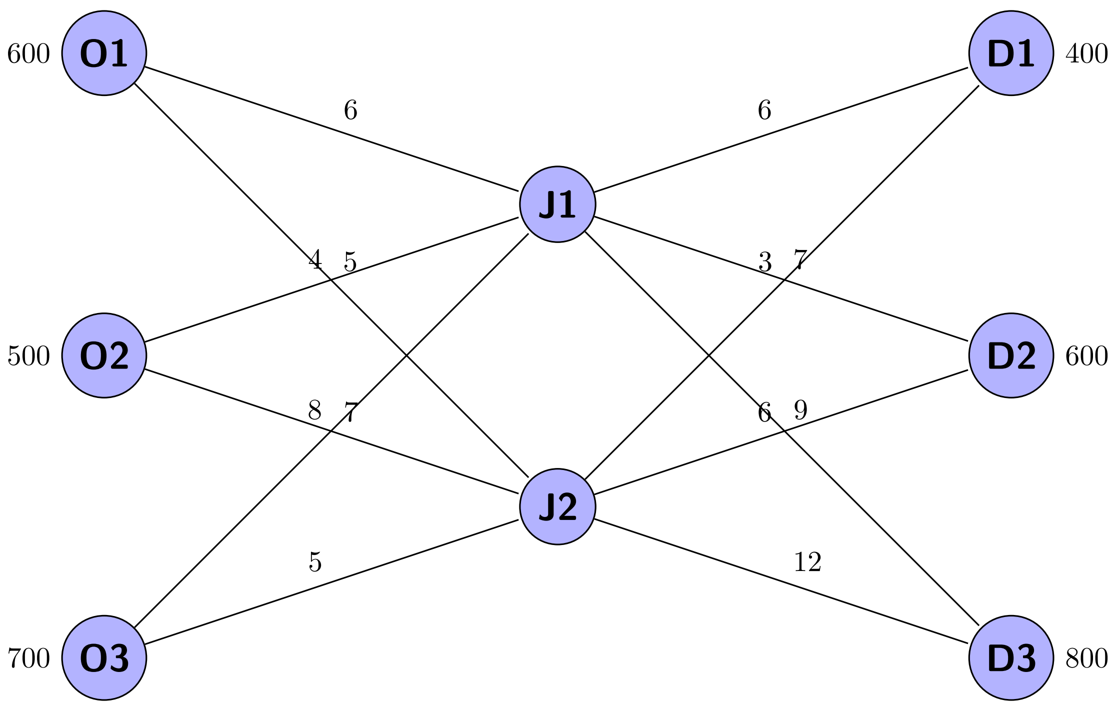
consider the following example,




Additional example.

| | WH 1 | WH 2 | Capacity |
|---|---|---|---|
| Plant 1 | 6 | 5 | 600 |
| Plant 2 | 4 | 7 | 500 |
| Plant 3 | 8 | 5 | 700 |

| | DC 1 | DC 2 | DC 3 |
|---|---|---|---|
| WH 1 | 6 | 7 | 9 |
| WH 2 | 3 | 6 | 12 |
| Demand | 400 | 600 | 800 |




s.t. $x_{o_1,j_1} + x_{o_1,j_2} \leq 600$

$x_{o_2,j_1} + x_{o_2,j_2} \leq 500$

$x_{o_3,j_1} + x_{o_3,j_2} \leq 700$

$x_{o_1,j_1} + x_{o_2,j_1} + x_{o_3,j_1} - x_{j_1,d_1} - x_{j_1,d_2} - x_{j_1,d_3} = 0$

$x_{o_1,j_2} + x_{o_2,j_2} + x_{o_3,j_2} - x_{j_2,d_1} - x_{j_2,d_2} - x_{j_2,d_3} = 0$

$x_{j_1,d_1} + x_{j_2,d_1} = 400$
$x_{j_1,d_2} + x_{j_2,d_2} = 600$
$x_{j_1,d_3} + x_{j_2,d_3} = 800$
$x_{u,v} \geq 0 \quad \forall u, v \in V$

## 2. Multiple Vehicle Routing

$$
\begin{aligned}
\min \quad & \sum_{u \in V} \sum_{v \in V} w_{u,v} x_{u,v} \\
\text{s.t} \quad & \sum_{v \in V} x_{v,u} = 1 \quad \forall u \in V \backslash \{0\} \\
& \sum_{u \in V} x_{v,u} = 1 \quad \forall v \in V \backslash \{0\} \\
& \sum_{v \in V \backslash \{0\}} x_{u,0} = \sum_{u \in V \{0\}} x_{0,v} = k \\
& \sum_{v \notin S} \sum_{u \in S} x_{v,u} \geq r(S), \quad \forall S \subseteq V \backslash \{0\}, S \neq \emptyset \\
& x_{v,u} \in \{0,1\} \quad \forall v, u \in V
\end{aligned}
$$

The 4th constraint can be rewritten as follows,

$$\sum_{v \in S} \sum_{u \in S} x_{v,u} \leq |S| - r(S)$$

## 3. Multidivisional problem

$$
\begin{aligned}
\max \quad & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} \quad & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\
& 2x_1 + 4x_2 + 3x_3 \leq 5 \\
& 2x_4 + 8x_5 + 6x_6 \leq 6 \\
& 3x_7 + 5x_8 + 9x_9 \leq 32 \\
& x_i \geq 0 \quad \forall i \in \{1, \cdots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \cdots, 9\}
\end{aligned}
$$

We then do the benders decomposition, with initial master problem,

$$\begin{array}{rl} \max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\ \text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \end{array}$$

and the following subproblems,

$$\begin{array}{rl} \max & 8x_1 + 5x_2 + 6x_3 \\ \text{s.t.} & 2x_1 + 4x_2 + 3x_3 \leq 5 \end{array}$$

$$\begin{array}{rl} \max & 9x_4 + 7x_5 + 9x_6 \\ \text{s.t.} & 2x_4 + 8x_5 + 6x_6 \leq 6 \end{array}$$

$$\begin{array}{rl} \max & 6x_7 + 5x_8 + 6x_9 \\ \text{s.t.} & 3x_7 + 5x_8 + 9x_9 \leq 32 \end{array}$$

After solving the subproblems, we get the following solutions,

$$\begin{array}{rl} \max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\ \text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\ & 2x_1 \leq 16 \\ & 3x_4 \leq 27 \\ & 10x_7 \leq 60 \\ & x_i \geq 0 \quad \forall i \in \{1, \cdots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \cdots, 9\} \end{array}$$