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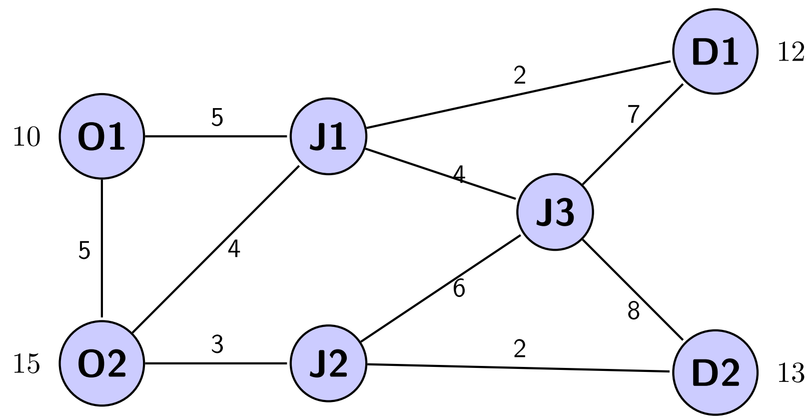
McGill University
MATH470 : Honours Research Project

Using optimization methods in the context of mine planning

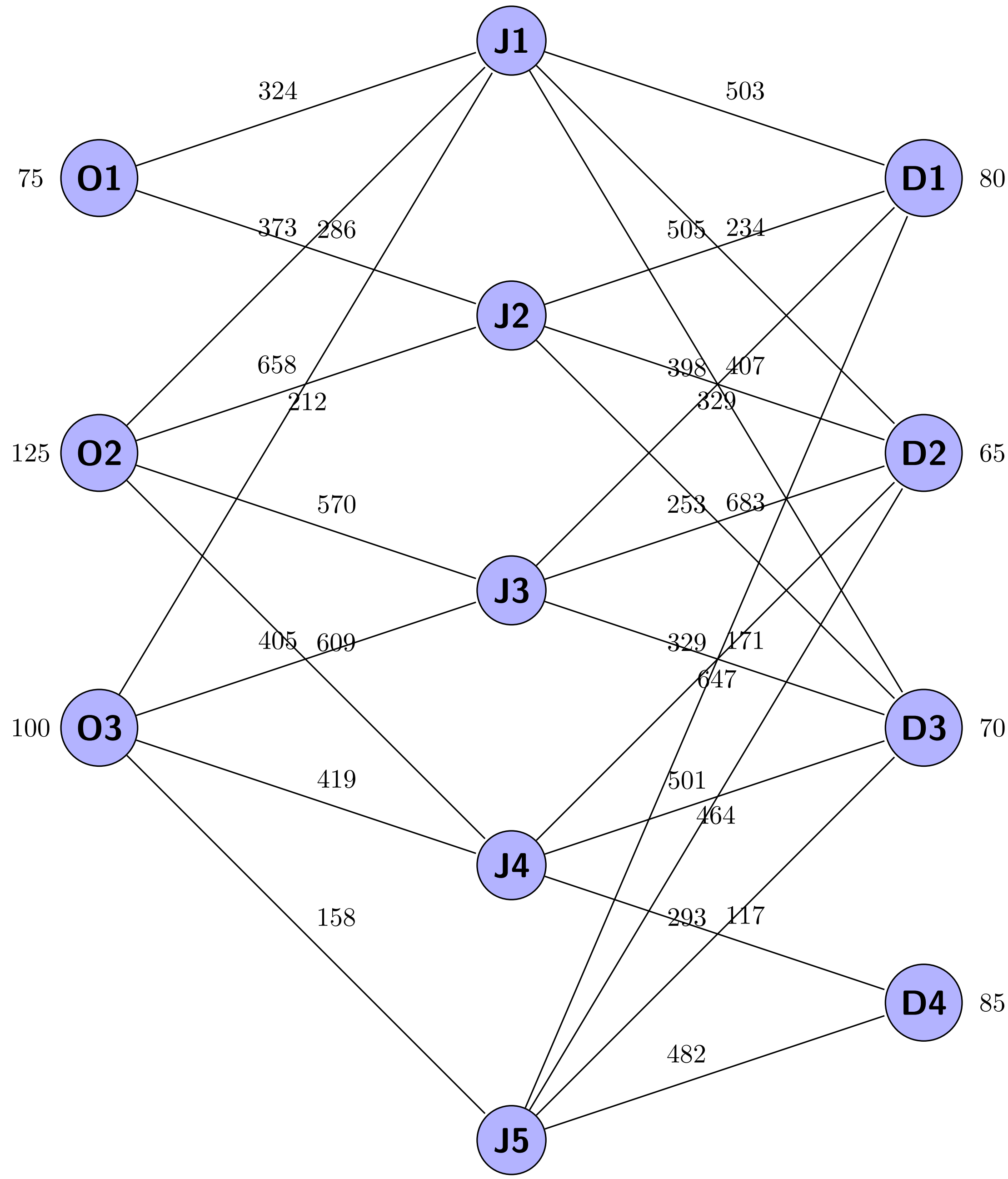
1. Transshipment Problem



We can formulate this problem as a linear program in the following way,

$$
\begin{aligned}
\min \quad & \sum_{u,v \in V} t_{u,v} x_{u,v} \\
\text{s.t.} \quad & \sum_{v \in V} x_{u,v} - \sum_{v \in V} x_{v,u} = a_u \quad \forall u \in V \\
& \sum_{v \in V} x_{v,u} - \sum_{v \in V} x_{u,v} = b_u \quad \forall u \in V \\
& \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
& x_{u,v} \geq 0 \quad \forall u, v \in V
\end{aligned}
$$

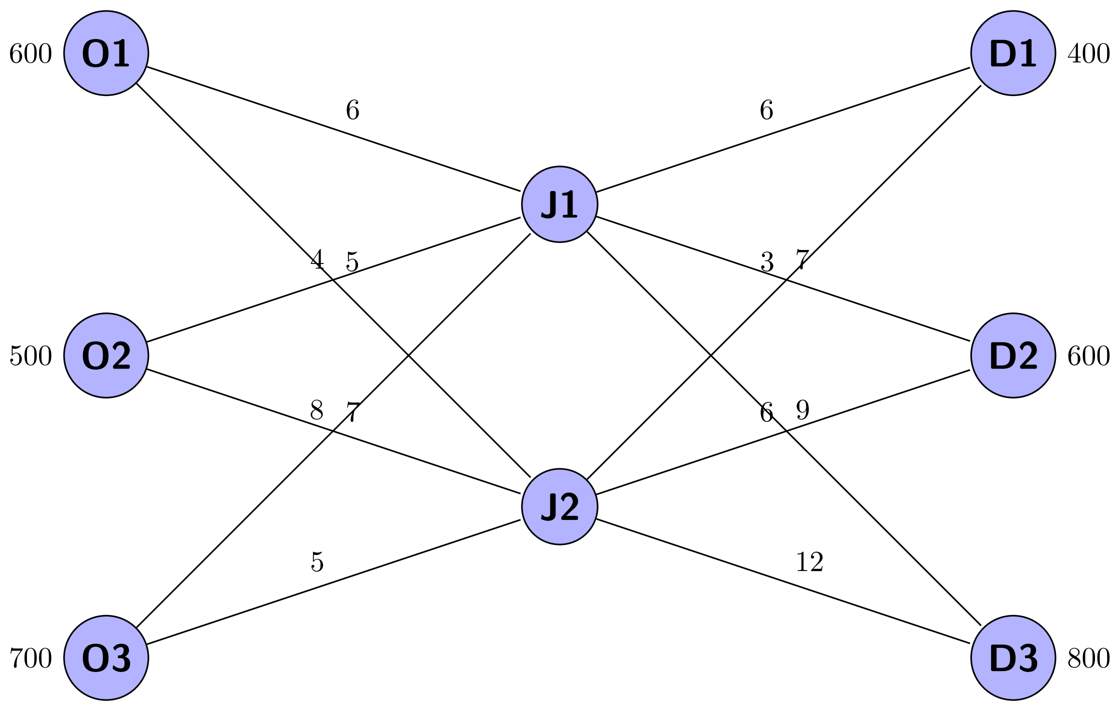
consider the following example,



Additional example.

	WH 1	WH 2	Capacity
Plant 1	6	5	600
Plant 2	4	7	500
Plant 3	8	5	700

	DC 1	DC 2	DC 3
WH 1	6	7	9
WH 2	3	6	12
Demand	400	600	800



s.t. $\quad \mathrm{x}_{o_1,j_1} + x_{o_1,j_2} \leq 600$

$x_{o_2,j_1} + x_{o_2,j_2} \leq 500$

$x_{o_3,j_1} + x_{o_3,j_2} \leq 700$

$x_{o_1,j_1} + x_{o_2,j_1} + x_{o_3,j_1} - x_{j_1,d_1} - x_{j_1,d_2} - x_{j_1,d_3} = 0$

$x_{o_1,j_2} + x_{o_2,j_2} + x_{o_3,j_2} - x_{j_2,d_1} - x_{j_2,d_2} - x_{j_2,d_3} = 0$

$x_{j_1,d_1} + x_{j_2,d_1} = 400$
$x_{j_1,d_2} + x_{j_2,d_2} = 600$
$x_{j_1,d_3} + x_{j_2,d_3} = 800$
$x_{u,v} \geq 0 \quad \forall u, v \in V$

2. Multiple Vehicle Routing

$$
\begin{aligned}
\min \quad & \sum_{u \in V} \sum_{v \in V} w_{u,v} x_{u,v} \\
\text{s.t} \quad & \sum_{v \in V} x_{v,u} = 1 \quad \forall u \in V \backslash \{0\} \\
& \sum_{u \in V} x_{v,u} = 1 \quad \forall v \in V \backslash \{0\} \\
& \sum_{v \in V \backslash \{0\}} x_{u,0} = \sum_{u \in V \{0\}} x_{0,v} = k \\
& \sum_{v \notin S} \sum_{u \in S} x_{v,u} \geq r(S), \quad \forall S \subseteq V \backslash \{0\}, S \neq \emptyset \\
& x_{v,u} \in \{0, 1\} \quad \forall v, u \in V
\end{aligned}
$$

The 4th constraint can be rewritten as follows,

$$
\sum_{v \in S} \sum_{u \in S} x_{v,u} \leq |S| - r(S)
$$

3. Multidivisional problem

$$
\begin{aligned}
\max \quad & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} \quad & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\
& 2x_1 + 4x_2 + 3x_3 \leq 5 \\
& 2x_4 + 8x_5 + 6x_6 \leq 6 \\
& 3x_7 + 5x_8 + 9x_9 \leq 32 \\
& x_i \geq 0 \quad \forall i \in \{1, \cdots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \cdots, 9\}
\end{aligned}
$$

We then do the benders decomposition, with initial master problem,

$$
\begin{array}{rl}
\max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30
\end{array}
$$

and the following subproblems,

$$
\begin{array}{rl}
\max & \pi_1 \left(5x_1 + 3x_2 + 4x_3\right) + \pi_2 \left(2x_1 + 4x_2 + 3x_3\right) \\
\text{s.t.} & 2x_1 + 4x_2 + 3x_3 \leq 5 \\
& x_1, x_2, x_3 \geq 0, \quad x_1, x_2, x_3 \in \mathbb{Z}
\end{array}
$$

$$
\begin{array}{rl}
\max & \pi_3 \left(2x_4 + 8x_5 + 6x_6\right) + \pi_4 (2x_4 + 7x_5 + 3x_6) \\
\text{s.t.} & 2x_4 + 8x_5 + 6x_6 \leq 6 \\
& x_4, x_5, x_6 \geq 0, \quad x_4, x_5, x_6 \in \mathbb{Z}
\end{array}
$$

$$
\begin{array}{rl}
\max & \pi_5 \left(3x_7 + 5x_8 + 9x_9\right) + \pi_6 (4x_7 + 6x_8 + x_9) \\
\text{s.t.} & 3x_7 + 5x_8 + 9x_9 \leq 32 \\
& x_7, x_8, x_9 \geq 0, \quad x_7, x_8, x_9 \in \mathbb{Z}
\end{array}
$$

After solving the subproblems, we get the following solutions,

$$
\begin{array}{rl}
\max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\
& 2x_1 \leq 16 \\
& 3x_4 \leq 27 \\
& 10x_7 \leq 60 \\
& x_i \geq 0 \quad \forall i \in \{1, \cdots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \cdots, 9\}
\end{array}
$$