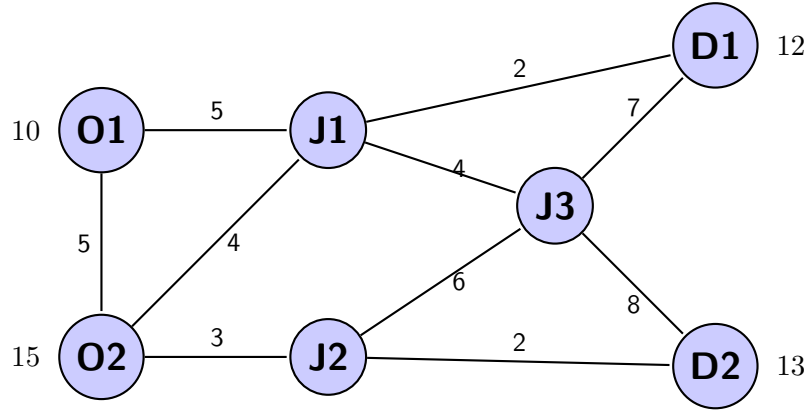


Using optimization methods in the context of mine planning

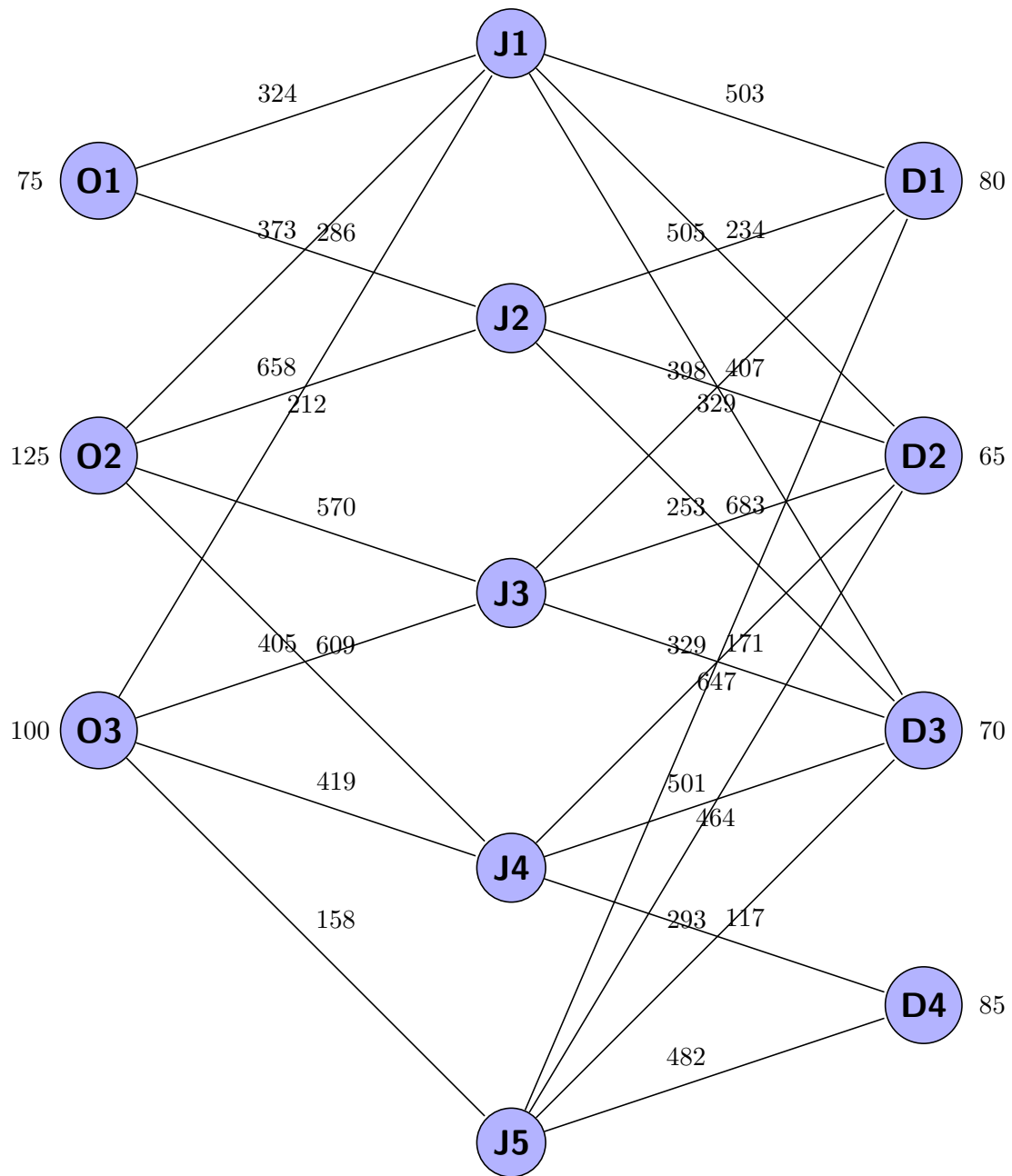
1. Transshipment Problem



We can formulate this problem as a linear program in the following way,

$$\begin{aligned}
 \min \quad & \sum_{u,v \in V} t_{u,v} x_{u,v} \\
 \text{s.t.} \quad & \sum_{v \in V} x_{u,v} - \sum_{v \in V} x_{v,u} = a_u \quad \forall u \in V \\
 & \sum_{v \in V} x_{v,u} - \sum_{v \in V} x_{u,v} = b_u \quad \forall u \in V \\
 & \sum_{i=1}^m a_i = \sum_{j=1}^n b_j \\
 & x_{u,v} \geq 0 \quad \forall u, v \in V
 \end{aligned}$$

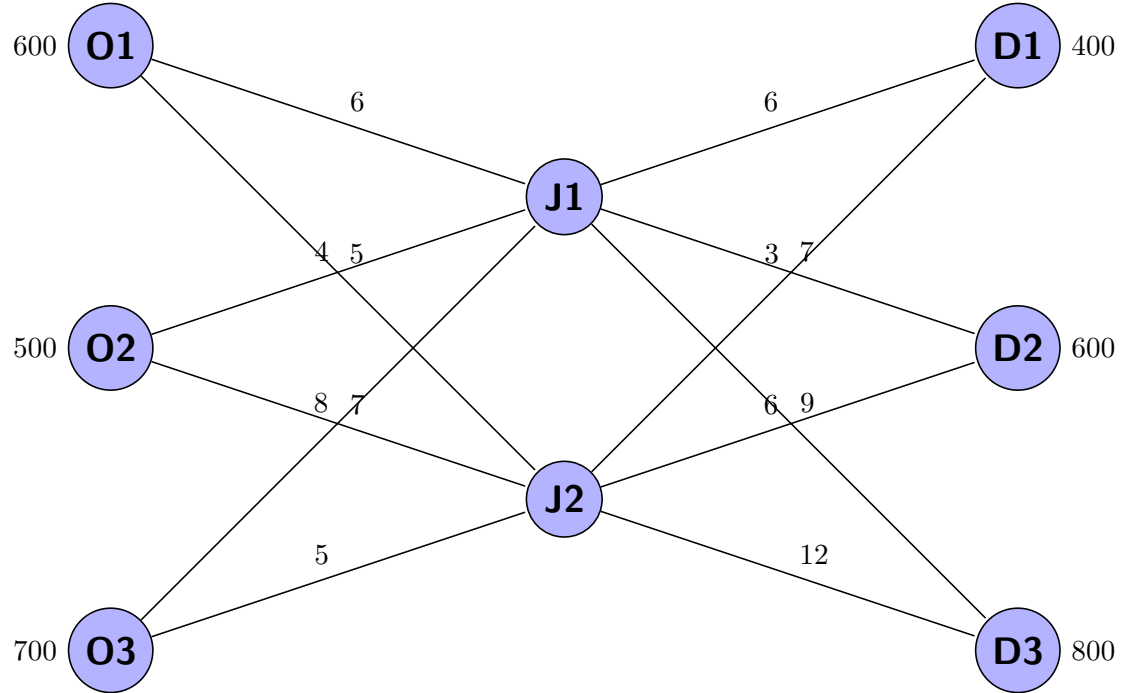
consider the following example,



Additional example.

	WH 1	WH 2	Capacity
Plant 1	6	5	600
Plant 2	4	7	500
Plant 3	8	5	700

	DC 1	DC 2	DC 3
WH 1	6	7	9
WH 2	3	6	12
Demand	400	600	800



$$\begin{aligned}
& \text{s.t.} \quad x_{o_1,j_1} + x_{o_1,j_2} \leq 600 \\
& x_{o_2,j_1} + x_{o_2,j_2} \leq 500 \\
& x_{o_3,j_1} + x_{o_3,j_2} \leq 700 \\
& x_{o_1,j_1} + x_{o_2,j_1} + x_{o_3,j_1} - x_{j_1,d_1} - x_{j_1,d_2} - x_{j_1,d_3} = 0 \\
& x_{o_1,j_2} + x_{o_2,j_2} + x_{o_3,j_2} - x_{j_2,d_1} - x_{j_2,d_2} - x_{j_2,d_3} = 0
\end{aligned}$$

$$\begin{aligned}
x_{j_1,d_1} + x_{j_2,d_1} &= 400 \\
x_{j_1,d_2} + x_{j_2,d_2} &= 600 \\
x_{j_1,d_3} + x_{j_2,d_3} &= 800 \\
x_{u,v} &\geq 0 \quad \forall u, v \in V
\end{aligned}$$

2. Multiple Vehicle Routing

$$\begin{aligned}
\min \quad & \sum_{u \in V} \sum_{v \in V} w_{u,v} x_{u,v} \\
\text{s.t.} \quad & \sum_{v \in V} x_{v,u} = 1 \quad \forall u \in V \setminus \{0\} \\
& \sum_{u \in V} x_{v,u} = 1 \quad \forall v \in V \setminus \{0\} \\
& \sum_{v \in V \setminus \{0\}} x_{u,0} = \sum_{u \in V \setminus \{0\}} x_{0,v} = k \\
& \sum_{v \notin S} \sum_{u \in S} x_{v,u} \geq r(S), \quad \forall S \subseteq V \setminus \{0\}, S \neq \emptyset \\
& x_{v,u} \in \{0, 1\} \quad \forall v, u \in V
\end{aligned}$$

The 4th constraint can be rewritten as follows,

$$\sum_{v \in S} \sum_{u \in S} x_{v,u} \leq |S| - r(S)$$

3. Multidivisional problem

$$\begin{aligned}
\max \quad & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} \quad & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\
& 2x_1 + 4x_2 + 3x_3 \leq 5 \\
& x_4 + 8x_5 + 6x_6 \leq 17 \\
& 3x_7 + 5x_8 + 9x_9 \leq 32 \\
& x_i \geq 0 \quad \forall i \in \{1, \dots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \dots, 9\}
\end{aligned}$$

We then do the benders decomposition, with initial master problem,

$$\begin{array}{ll}
\max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30
\end{array}$$

and the following subproblems,

$$\begin{array}{ll}
\max & 8x_1 + 5x_2 + 6x_3 \\
\text{s.t.} & 2x_1 + 4x_2 + 3x_3 \leq 5
\end{array}$$

$$\begin{array}{ll}
\max & 9x_4 + 7x_5 + 9x_6 \\
\text{s.t.} & x_4 + 8x_5 + 6x_6 \leq 17
\end{array}$$

$$\begin{array}{ll}
\max & 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} & 3x_7 + 5x_8 + 9x_9 \leq 32
\end{array}$$