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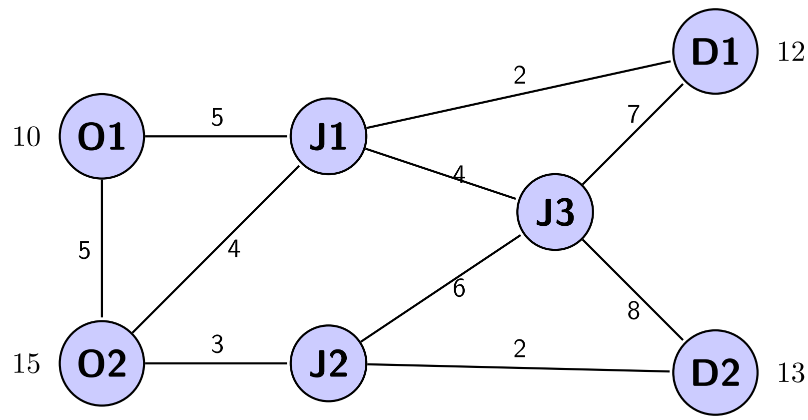
Summer 2024
McGill University
MATH470 : Honours Research Project

Using optimization methods in the context of mine planning

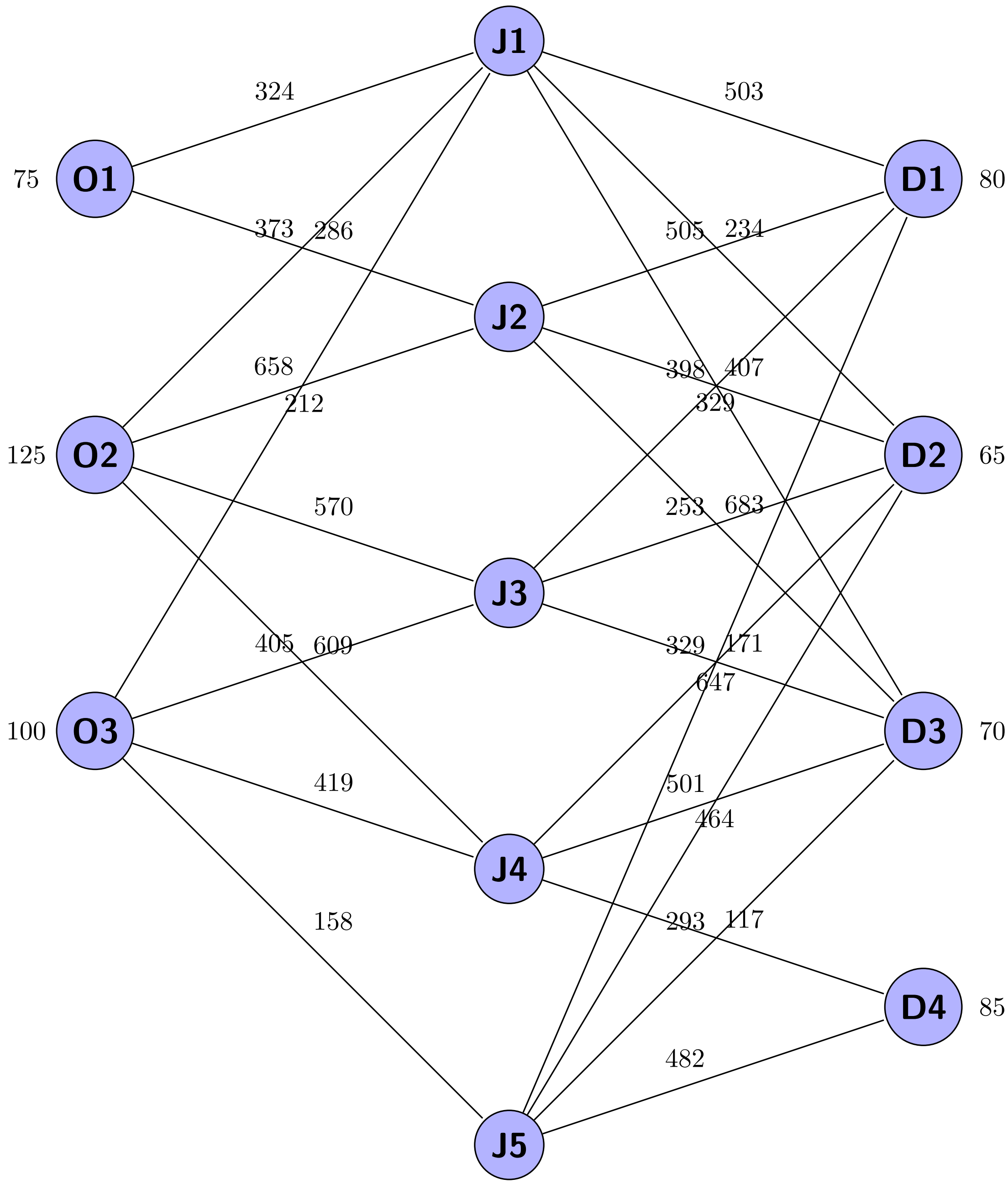
1. Transshipment Problem



We can formulate this problem as a linear program in the following way,

$$\begin{aligned}
\min \quad & \sum_{u,v \in V} t_{u,v} x_{u,v} \\
\text{s.t.} \quad & \sum_{v \in V} x_{u,v} - \sum_{v \in V} x_{v,u} = a_u \quad \forall u \in V \\
& \sum_{v \in V} x_{v,u} - \sum_{v \in V} x_{u,v} = b_u \quad \forall u \in V \\
& \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
& x_{u,v} \geq 0 \quad \forall u, v \in V
\end{aligned}$$

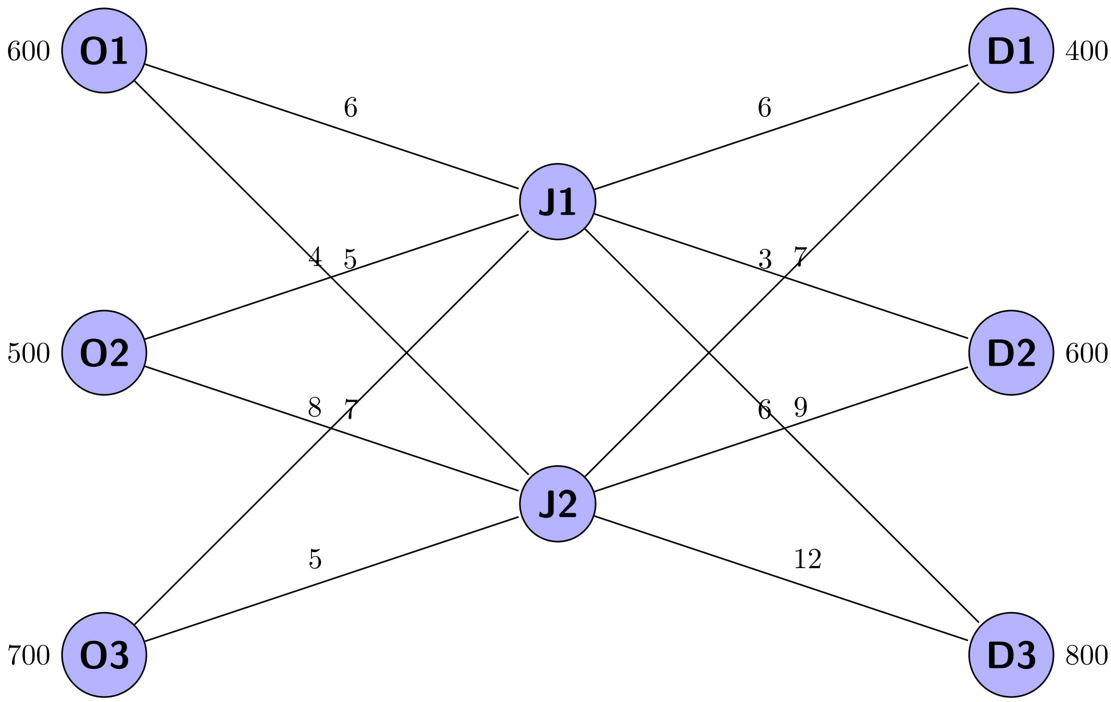
consider the following example,



Additional example.

	WH 1	WH 2	Capacity
Plant 1	6	5	600
Plant 2	4	7	500
Plant 3	8	5	700

	DC 1	DC 2	DC 3
WH 1	6	7	9
WH 2	3	6	12
Demand	400	600	800



$$
\begin{aligned}
\min \quad & 6x_{o_1,j_1} + 5x_{o_1,j_2} + 4x_{o_2,j_1} + 7x_{o_2,j_2} + 8x_{o_3,j_1} + 5x_{o_3,j_2} \\
& + 6x_{j_1,d_1} + 7x_{j_1,d_2} + 9x_{j_1,d_3} + 3x_{j_2,d_1} + 6x_{j_2,d_2} + 12x_{j_2,d_3} \\
\text{s.t.} \quad & x_{o_1,j_1} + x_{o_1,j_2} \leq 600 && (1) \\
& x_{o_2,j_1} + x_{o_2,j_2} \leq 500 && (2) \\
& x_{o_3,j_1} + x_{o_3,j_2} \leq 700 && (3) \\
& x_{o_1,j_1} + x_{o_2,j_1} + x_{o_3,j_1} - x_{j_1,d_1} - x_{j_1,d_2} - x_{j_1,d_3} = 0 && (4) \\
& x_{o_1,j_2} + x_{o_2,j_2} + x_{o_3,j_2} - x_{j_2,d_1} - x_{j_2,d_2} - x_{j_2,d_3} = 0 && (5) \\
& x_{j_1,d_1} + x_{j_2,d_1} = 400 && (6) \\
& x_{j_1,d_2} + x_{j_2,d_2} = 600 && (7) \\
& x_{j_1,d_3} + x_{j_2,d_3} = 800 && (8) \\
& x_{u,v} \geq 0 \quad \forall u, v \in V && (9)
\end{aligned}
$$

2. Multiple Vehicle Routing

$$
\begin{aligned}
\min \quad & \sum_{u \in V} \sum_{v \in V} w_{u,v} x_{u,v} \\
\text{s.t} \quad & \sum_{v \in V} x_{v,u} = 1 \quad \forall u \in V \backslash \{0\} \\
& \sum_{u \in V} x_{v,u} = 1 \quad \forall v \in V \backslash \{0\} \\
& \sum_{v \in V \backslash \{0\}} x_{u,0} = \sum_{u \in V \{0\}} x_{0,v} = k \\
& \sum_{v \notin S} \sum_{u \in S} x_{v,u} \geq r(S), \quad \forall S \subseteq V \backslash \{0\}, S \neq \emptyset \\
& x_{v,u} \in \{0,1\} \quad \forall v, u \in V
\end{aligned}
$$

The 4th constraint can be rewritten as follows,

$$
\sum_{v \in S} \sum_{u \in S} x_{v,u} \leq |S| - r(S)
$$