Elliot F. Poirier
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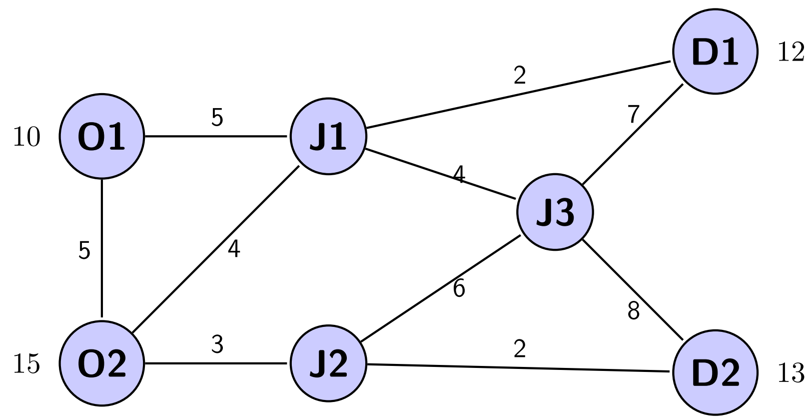
McGill University
MATH470 : Honours Research Project

Using optimization methods in the context of mine planning

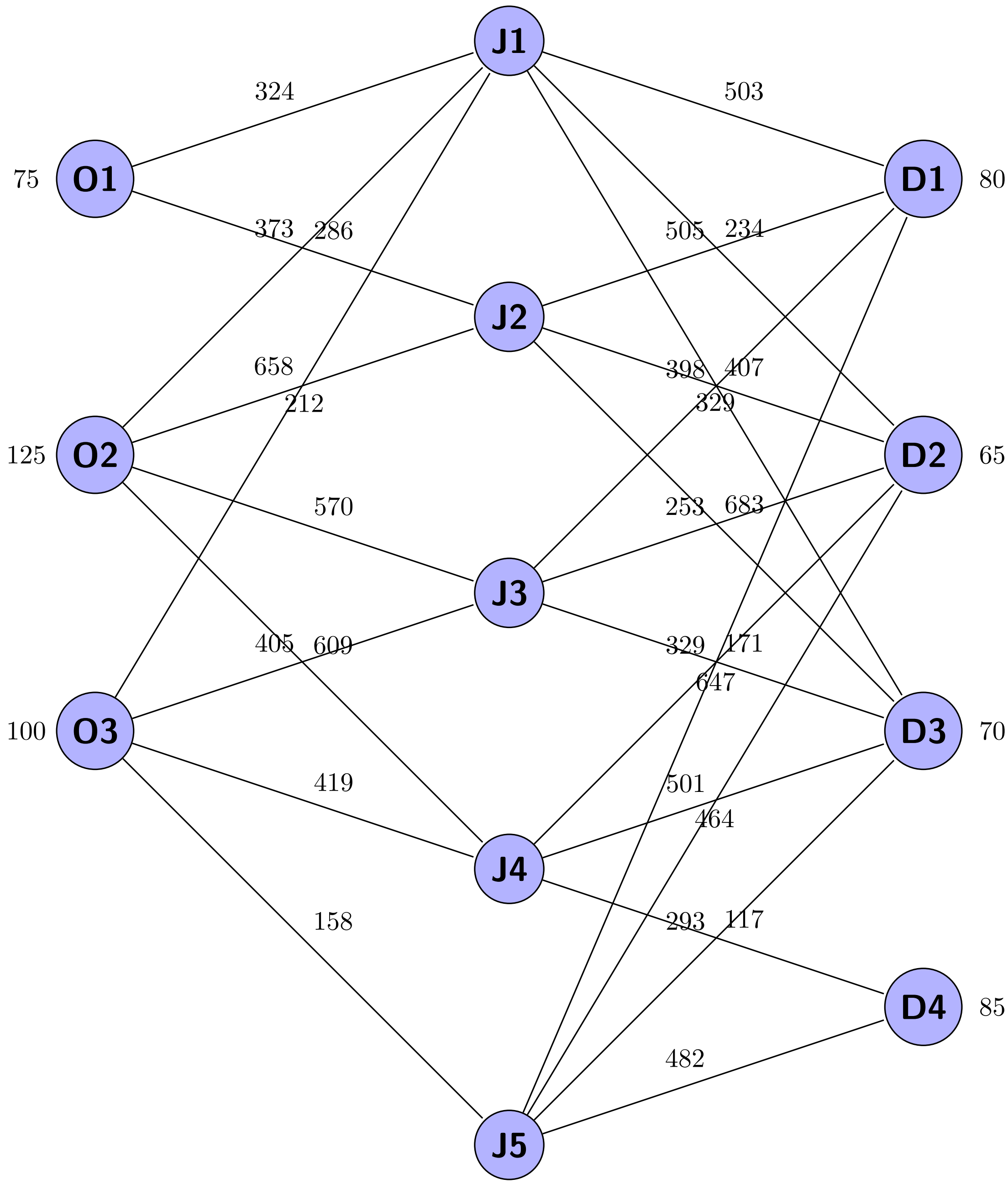
1. Transshipment Problem



We can formulate this problem as a linear program in the following way,

$$
\begin{aligned}
\min \quad & \sum_{u,v \in V} t_{u,v} x_{u,v} \\
\text{s.t.} \quad & \sum_{v \in V} x_{u,v} - \sum_{v \in V} x_{v,u} = a_u \quad \forall u \in V \\
& \sum_{v \in V} x_{v,u} - \sum_{v \in V} x_{u,v} = b_u \quad \forall u \in V \\
& \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
& x_{u,v} \geq 0 \quad \forall u, v \in V
\end{aligned}
$$

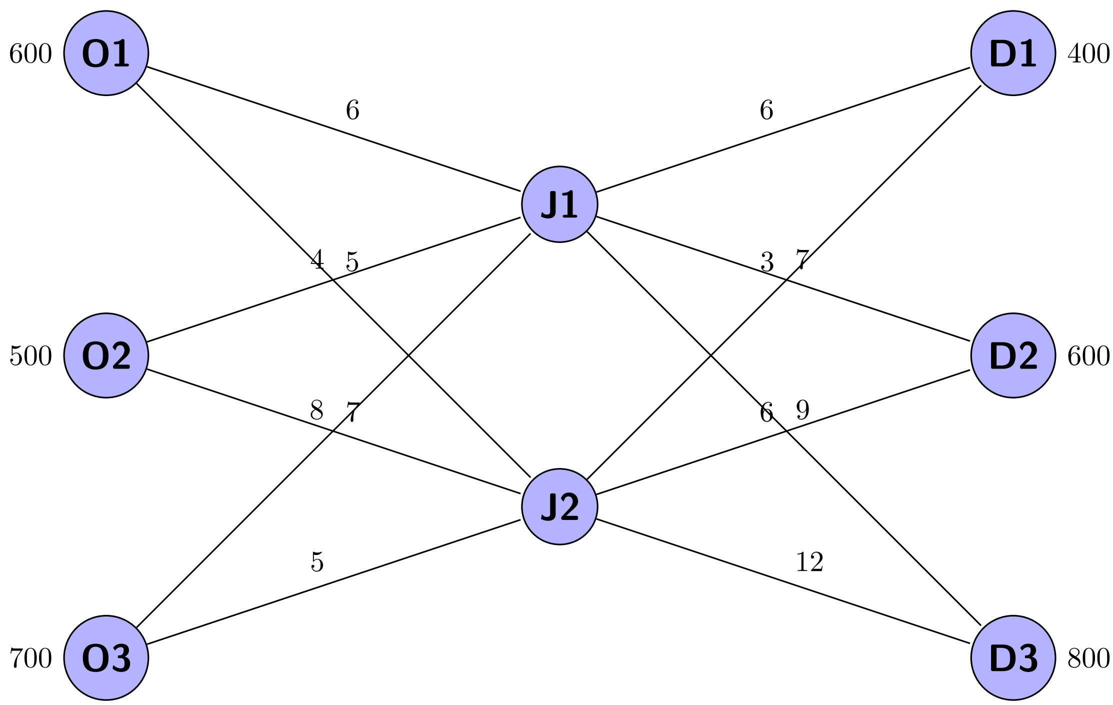
consider the following example,



Additional example.

	WH 1	WH 2	Capacity
Plant 1	6	5	600
Plant 2	4	7	500
Plant 3	8	5	700

	DC 1	DC 2	DC 3
WH 1	6	7	9
WH 2	3	6	12
Demand	400	600	800



s.t. $\quad x_{o_1,j_1} + x_{o_1,j_2} \leq 600$

$x_{o_2,j_1} + x_{o_2,j_2} \leq 500$

$x_{o_3,j_1} + x_{o_3,j_2} \leq 700$

$x_{o_1,j_1} + x_{o_2,j_1} + x_{o_3,j_1} - x_{j_1,d_1} - x_{j_1,d_2} - x_{j_1,d_3} = 0$

$x_{o_1,j_2} + x_{o_2,j_2} + x_{o_3,j_2} - x_{j_2,d_1} - x_{j_2,d_2} - x_{j_2,d_3} = 0$

$x_{j_1,d_1} + x_{j_2,d_1} = 400$
$x_{j_1,d_2} + x_{j_2,d_2} = 600$
$x_{j_1,d_3} + x_{j_2,d_3} = 800$
$x_{u,v} \geq 0 \quad \forall u, v \in V$

2. Multiple Vehicle Routing

$$\begin{aligned}
\min \quad & \sum_{u \in V} \sum_{v \in V} w_{u,v} x_{u,v} \\
\text{s.t} \quad & \sum_{v \in V} x_{v,u} = 1 \quad \forall u \in V \backslash \{0\} \\
& \sum_{u \in V} x_{v,u} = 1 \quad \forall v \in V \backslash \{0\} \\
& \sum_{v \in V \backslash \{0\}} x_{u,0} = \sum_{u \in V \{0\}} x_{0,v} = k \\
& \sum_{v \notin S} \sum_{u \in S} x_{v,u} \geq r(S), \quad \forall S \subseteq V \backslash \{0\}, S \neq \emptyset \\
& x_{v,u} \in \{0, 1\} \quad \forall v, u \in V
\end{aligned}$$

The 4th constraint can be rewritten as follows,

$$\sum_{v \in S} \sum_{u \in S} x_{v,u} \leq |S| - r(S)$$

3. Multidivisional problem

$$\begin{aligned}
\max \quad & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} \quad & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\
& 2x_1 + 4x_2 + 3x_3 \leq 5 \\
& 2x_4 + 8x_5 + 6x_6 \leq 6 \\
& 3x_7 + 5x_8 + 9x_9 \leq 32 \\
& x_i \geq 0 \quad \forall i \in \{1, \cdots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \cdots, 9\}
\end{aligned}$$

We then do the benders decomposition, with initial master problem,

$$
\begin{array}{rl}
\max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30
\end{array}
$$

and the following subproblems,

$$
\begin{array}{rl}
\max & \pi_1\,(5x_1 + 3x_2 + 4x_3) + \pi_2\,(2x_1 + 4x_2 + 3x_3) \\
\text{s.t.} & 2x_1 + 4x_2 + 3x_3 \leq 5 \\
& x_1, x_2, x_3 \geq 0, \quad x_1, x_2, x_3 \in \mathbb{Z}
\end{array}
$$

$$
\begin{array}{rl}
\max & \pi_3\,(2x_4 + 8x_5 + 6x_6) + \pi_4(2x_4 + 7x_5 + 3x_6) \\
\text{s.t.} & 2x_4 + 8x_5 + 6x_6 \leq 6 \\
& x_4, x_5, x_6 \geq 0, \quad x_4, x_5, x_6 \in \mathbb{Z}
\end{array}
$$

$$
\begin{array}{rl}
\max & \pi_5\,(3x_7 + 5x_8 + 9x_9) + \pi_6(4x_7 + 6x_8 + x_9) \\
\text{s.t.} & 3x_7 + 5x_8 + 9x_9 \leq 32 \\
& x_7, x_8, x_9 \geq 0, \quad x_7, x_8, x_9 \in \mathbb{Z}
\end{array}
$$

After solving the subproblems, we get the following solutions,

$$
\begin{array}{rl}
\max & 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 + 6x_9 \\
\text{s.t.} & 5x_1 + 3x_2 + 4x_3 + 2x_4 + 7x_5 + 3x_6 + 4x_7 + 6x_8 + x_9 \leq 30 \\
& 2x_1 \leq 16 \\
& 3x_4 \leq 27 \\
& 10x_7 \leq 60 \\
& x_i \geq 0 \quad \forall i \in \{1, \cdots, 9\} \quad \text{and} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1, \cdots, 9\}
\end{array}
$$

4. Block Selection Linear Program

This is a subproblem of larger mine planning optimization problems. The objective is to maximize the total value of the blocks selected, subject to the constraints that the total volume of the selected blocks does not exceed the available volume, and that the total value of the selected blocks does not exceed the available value. Here is a formal version of the problem for two modes, $\{A, B\}$ and two ore types $\{I, II\}$.

$$
\begin{aligned}
\max \sum_{b \in \mathfrak{B}} & (V_{bAI} m_{bAI} + V_{bAII} m_{bAII} + V_{bBI} m_{bBI} + V_{bBII} m_{bBII}) \\
\text{st.} \quad \sum_{b \in \mathfrak{B}} & \left(\frac{m_{bAI}}{r_{AI}} + \frac{m_{bAII}}{r_{AII}} + \frac{m_{bBI}}{r_{BI}} + \frac{m_{bBII}}{r_{BII}} \right) \leqslant t_{\text{period}} \\
& m_{bAI} + m_{bBI} \leqslant m_{bI} \quad \forall b \in \mathfrak{B} \\
& m_{bAII} + m_{bBII} \leqslant m_{bII} \quad \forall b \in \mathfrak{B} \\
& m_{ijk}, r_{jk} \geqslant 0 \quad \forall i \in \mathfrak{B}, j \in \text{modes}, \ k \in \text{ores}
\end{aligned}
$$

where $\mathfrak{B}$ is the set of blocks, V_{bjk} is the value of block b in mode j and ore k, m_{bjk} is the volume of block b in mode j and ore k, r_{jk} is the recovery rate of ore k in mode j, and t_{period} is the total time available for block selection.

The first constraint represents the time constraint, that is to say that the total volume of blocks divided by the rate of recovery of the ore must be less than or equal to the total time available. The second and third constraints represent the volume constraints, that is to say that the volume of blocks selected in mode j and ore k must be less than or equal to the total volume of blocks in mode j and ore k.