

Neural Network

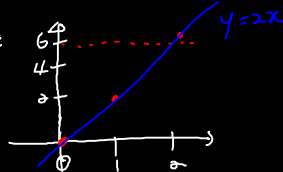
Week1: Basics

Supervised Learning: Need a $x - y$ data set \rightarrow most of Machine Learning (ML)

Example: Decision tree, neural net, SVM, Bayes, etc.

$$X = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad Y = \begin{bmatrix} 0.6 \\ 1.49 \\ 6.2 \end{bmatrix}$$

$$Y = 2x + \epsilon$$

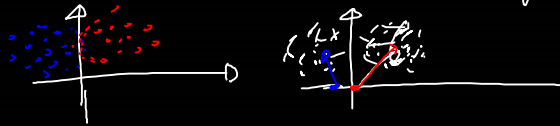


$$X = \begin{bmatrix} \text{img0} \\ \text{img1} \end{bmatrix} \quad Y = \begin{bmatrix} \text{"cup"} \\ \text{"cat"} \end{bmatrix} \rightarrow \begin{bmatrix} \text{"0"} \\ \text{"1"} \end{bmatrix}$$

8 bit unsigned integers.
0 ~ 255
= $w \cdot h + \text{img0}, \text{img1}$

Unsupervised Learning: Need a x data set, no need for label. However, you need a measure to tell whether the output is proper.

Example: genetic algorithm, KNN \rightarrow k-nearest neighbor



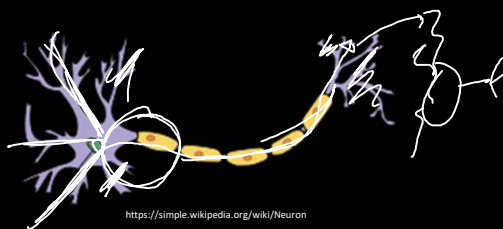
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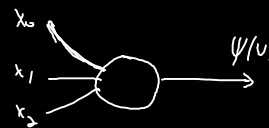
Week1: Basics

Perceptron

Neuron


<https://simple.wikipedia.org/wiki/Neuron>

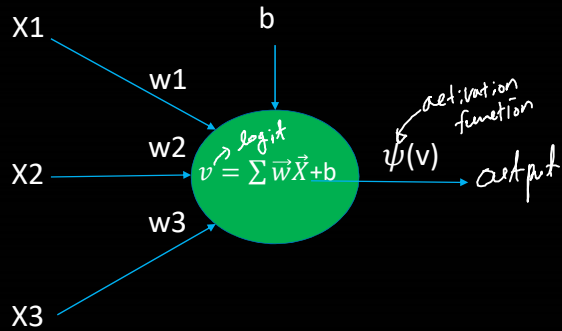
McCulloch-Pits Perceptron



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Week1: Basics

Perceptron: a basic unit for neural net

Math notation:

$$\begin{aligned}
 v &= x_1 w_1 + x_2 w_2 + x_3 w_3 + b \\
 &= \sum_{i=0}^n x_i w_i + b \\
 &= x_1 w_1 + x_2 w_2 + x_3 w_3 + 1 \cdot w_0 \\
 &= [1 \ x_1 \ x_2 \ x_3] \begin{bmatrix} w_0 & w_1 & w_2 & w_3 \end{bmatrix}^T \\
 &= \vec{x} \cdot \vec{w}^T \\
 \psi(v) &= \begin{cases} 1 & v \geq 0 \\ 0 & \text{otherwise} \end{cases}
 \end{aligned}$$

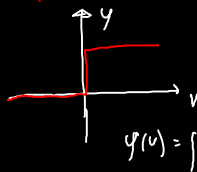
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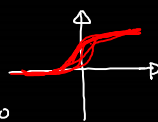
Week1: Basics

Activation Functions

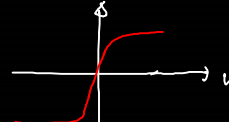
Step Function



Sigmoid Function



Hyperbolic Tangent Function



Relu Function



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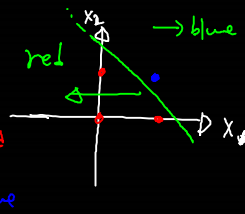
Week1: Basics

Perceptron

Linear Separability

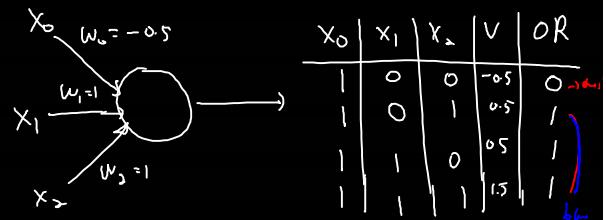
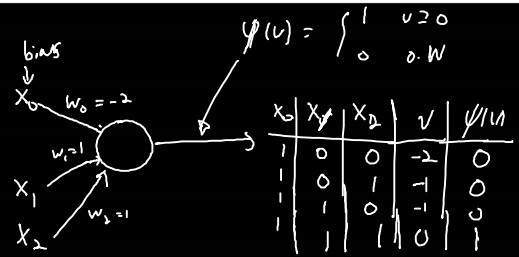
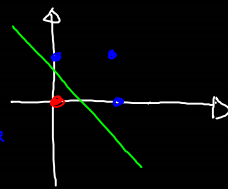
AND Gate

x_1	x_2	AND
0	0	0
0	1	0
1	0	0
1	1	1



OR Gate

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1



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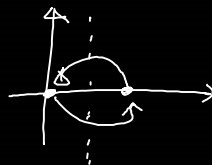
Week1: Basics

Perceptron

Linear Separability

NOT Gate

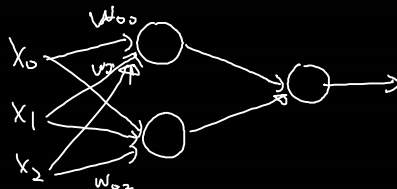
x_1	NOT
0	1
1	0



x	v
0	1
1	0

$$\psi(v) = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 \end{cases}$$

XOR Gate



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Week1: Basics

Perceptron

Perceptron Learning Rule:

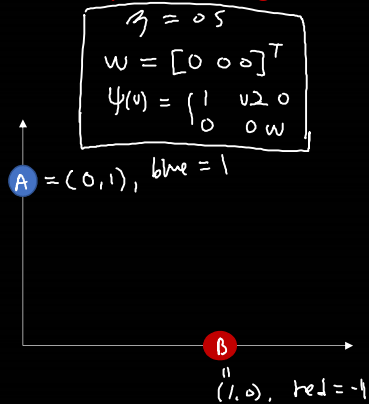
$$w(n+1) = \overline{w(n)} + \Delta w \rightarrow \text{change of } w \text{ val.}$$

$$= \overline{w(n)} + \eta(y - \hat{y})x(n)$$

next weight values
current w. val.

Code: HW

$\eta = \text{learning rate, eta}$
 $y - \hat{y} = \text{desired} - \text{predicted, Error}$
 $x(n) = \text{current input}$ *caused*



	X	W	V = wx	y	\hat{y}	Error $y - \hat{y}$	dw	Next w
A=(0,1) b=1	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	0	1	1	0	$\Delta w = \eta \cdot E \cdot \text{input}$ $= 0.5 \cdot (0) \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$
B=(1,0) b=1	$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	0	-1	1	-2	$\Delta w = 0.5 \cdot (-2) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$	

$$\Sigma \Delta w = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \rightarrow w(n)$$

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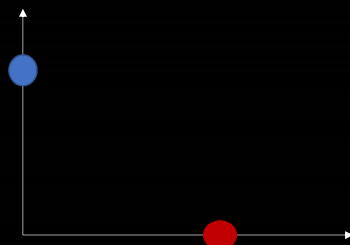
Neural Network

Week1: Basics

Perceptron

Code: HW

Perceptron Learning Rule: $w(n+1) = w(n) + \Delta w$
 $= w(n) + \eta(y - \hat{y})x(n)$



	X	W	V = wx	y	\hat{y}	Error $y - \hat{y}$	dw	Next w
A=(0,1) b=1	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$	$V = -1 + 0 + 0 = -1$	1	-1	2	$\Delta w = 0.5 \cdot (2) \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	$w + \Delta w$ $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
B=(1,0) b=1	$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$	$V = -1 - 1 = -2$	-1	-1	0	$\Delta w = 0$	$= \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

$$\Sigma \Delta w = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

wish i knew

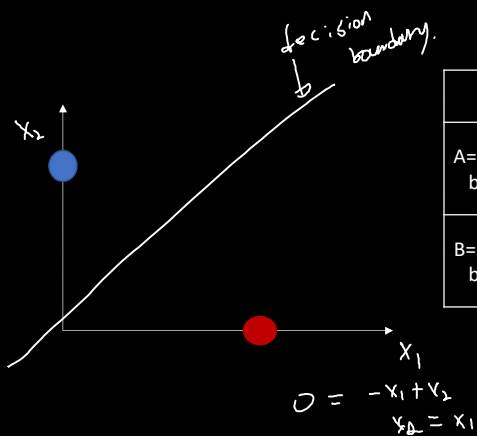
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Week1: Basics

Perceptron

Code: HW

Perceptron Learning Rule: $w(n+1) = w(n) + \Delta w$
 $= w(n) + \eta(y - \hat{y})x(n)$



	X	W	V = wx	y	\hat{y}	Error $y - \hat{y}$	dw	Next w
A=(0,1) b=1	$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$	$v = 1$	1	1	0	0	X
B=(1,0) b=1	$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$	$v = -1$	-1	-1	0	0	

$$V = X \cdot W = \begin{pmatrix} 1 & x_1 & x_2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad W = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad \text{bias} = 0$$

$$= -x_1 + x_2$$

$$\psi = \begin{pmatrix} v \geq 0 \\ v < 0 \end{pmatrix}$$

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Week1: Basics

Numerical Solution

First, let's find a root of second order polynomial. $y = ax^2 + bx + c = 0$

Analytical solution: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Numerical solution: Newton – Rapson method $y = x^2 - 5x + 6 = 0$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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Week1: Basics

Numerical Solution

Code: HW

Numerical solution: Newton – Rapson method $y = x^2 - 5x + 6 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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