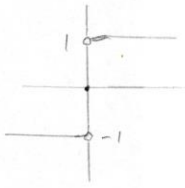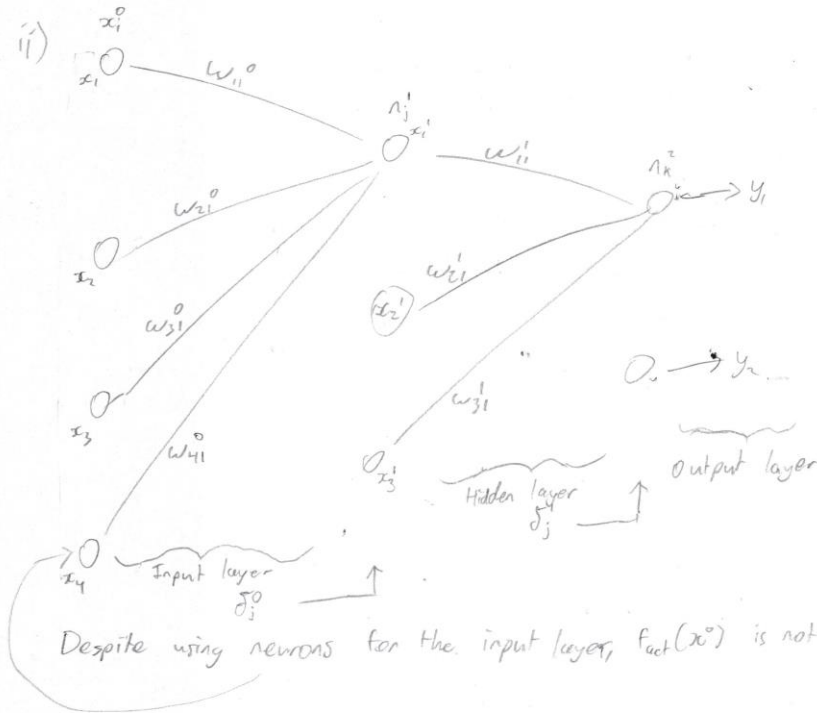Assignment 3

i)



$$\hat{y}_k = f_{SHL}(v_k + b_k) = \begin{cases} -1 & \text{if } v_k + b_k < 0 \\ 0 & \text{if } v_k + b_k = 0 \\ 1 & \text{if } v_k + b_k > 0 \end{cases}$$

ii)



Despite using neurons for the input layer, $f_{act}(x^0)$ is not required

$\bullet$ = dot multiply, $X^T$ = transpose(X), * = cross product/ matrix multiplication (* in matlab)

iii) $x_I^T \cdot w_{IJ} = z_J$

$$= [-1 \;-1 \; 0 \; 0] * \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = [-2 \;-1 \;-2] \qquad f_{act}(z_J) = x_J^T = [-1 \;-1 \;-1]$$

$x_J^T \cdot w_{Jk} = z_k$

$$= [-1 \;-1 \;-1] * \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = [-2 \;-3] \qquad f_{act}(z_k) = y_L^T = [-1 \;-1]$$

iv) $\delta_j' = y_L(1-y_L)(d_L - y_L)$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1--1 \\ 1--1 \end{bmatrix} \cdot \begin{bmatrix} 1--1 \\ 0--1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

v) $w_{Jk}^{new} = w_{Jk}^{old} + h\,\delta_j'\, x_j'$  (for matrix multiplication, use $x_j' * \delta_j'^T$)

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} .5 & .25 \\ .5 & .25 \\ .5 & .25 \end{bmatrix} = \begin{bmatrix} 0.5 & 1.25 \\ 1.5 & 1.25 \\ 1.5 & 1.25 \end{bmatrix}$$

vi) $\delta_i^0 = x_j' \cdot (1-x_j') \cdot (w_{Jk} * \delta_j')$

$$= \left( \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right) \cdot \left( \begin{bmatrix} 0.5 & 1.25 \\ 1.5 & 1.25 \\ 1.5 & 1.25 \end{bmatrix} * \begin{bmatrix} -4 \\ -2 \end{bmatrix} \right) = \begin{bmatrix} 9 \\ 17 \\ 17 \end{bmatrix}$$

vii) $w_{IJ}^{new} = w_{IJ}^{old} + h\,\delta_i^0\, x_i^0$  (again, vectorise with $x_i^0 * \delta_i^{0T}$)

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} -1.125 & -2.125 & -2.125 \\ -1.125 & -2.125 & -2.125 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -0.125 & -2.125 & -1.125 \\ -0.125 & -1.125 & -1.125 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$