Nonlinear Approach To Searing Steak

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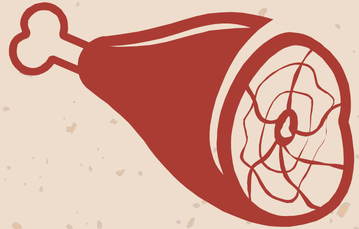

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General Approach

Background

Cooking the perfect steak has always been an elusive goal for many cooks

It is both a science and an art, making it an interesting opportunity to apply nonlinear control

Figure 1 [2]

Doneness

Temperature ranges for different cooks of steak

Rare	120-125℉
Medium Rare	130-135℉
Medium	140-145℉
Medium Well	150-155℉
Well Done	160-165℉
Overcooked	160℉ >

For more info:
[1]

Previous Work

[3]

Development and Validation of a Computational Model for Steak Double-Sided Pan Cooking

[4]

A Mathematical Model for Meat Cooking Based on Polymer-Solvent Analogy

[5]

Extreme Steak: Wild and Crazy Ways to Get a Killer Sear

New Approach

Approach real-world problem with theoretical understanding

1. Derive approx. equations for
   - Internal temperature
   - Searing Index
2. Gather data across different temperatures and times
3. Nonlinear param. estimation
4. Model as nonlinear system
5. Derive control to approach goal temp/sear at time t_{final}

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Internal Temperature

Modeling Temp

Based on broad research and intuition

- Temperature change starts slowly
- Speeds up over time
- Finally, it slows down again

It could be approximately estimated as a sigmoid [3].

$$Tint(T,t) \approx Tinit + \frac{(Tpan - Tinit)}{1 + e^{-k\frac{t+c}{thickness}}}$$

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Searing Index

Modeling Sear

Searing starts much more initially

- Directly exposed to heat, instant browning
- Searing hits an upper limit (asymptote) where it doesn't get any darker (burnt)

Might be modeled effectively using an exponential

- Base EQ. on color (visual inspection)

$$C(T,t) \approx Cmax * (1 - e^{-a(Tpan-b)t})$$

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System Representation

Initial System

X_1

$$X_1 = C = 100(1 - e^{-a(T_{pan} - b)t}) = \int_0^t \widehat{X}_1 dt$$

$\dot{X}_1$

$$\widehat{X}_1 = \widehat{C} = 100a(T_{pan} - b + ut) \times e^{-a(T_{pan} - b)t}$$

Explanation

X_1 equation and derivative for sear control.

X_2

$$X_2 = T_{int} = T_{int\ init} + \frac{T_{pan} - T_{int\ init}}{1 + e^{-c(d + f(T_{pan} - T_{int\ init}))(t-g)}} - \frac{T_{pan} - T_{int\ init}}{1 + e^{c(d + f(T_{pan} - T_{int\ init}))g}} = T_{int\ init} + \int_0^t \widehat{X}_2 dt - \frac{T_{pan} - T_{int\ init}}{1 + e^{c(d + f(T_{pan} - T_{int\ init}))g}}$$

$\dot{X}_2$

$$\widehat{X}_2 = \widehat{T}_{int} = u\sigma(H(t)) + cT_{diff}\sigma(H(t))(1 - \sigma(H(t)))[d + fT_{diff} + fu(t - g)]$$

$$Where:\ H(t) = c(d + f(T_{diff}))(t - g),\ T_{diff} = T_{pan} - T_{int\ init}$$

Explanation

X_2 equation and derivative for internal temp control

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Finding Constants

Experiment Setup

- Grill 1in thick steaks cut 1 in^3
- Thermometer probe for internal temp (check every 10 s)
- Thermometer heat gun for pan temp (verify stable temp every 30 s).
  - Assume gaps are linear.
- Visual inspection for sear (Check every 30 s)

Figure 2

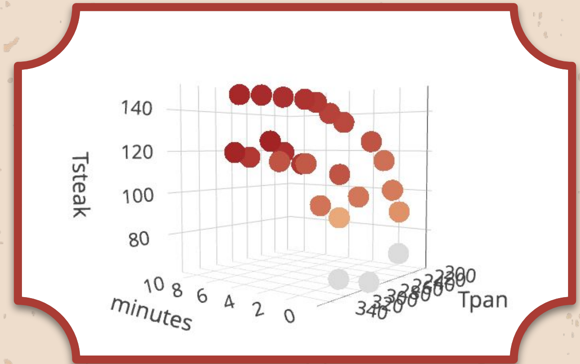

Figure 3

Internal Temp

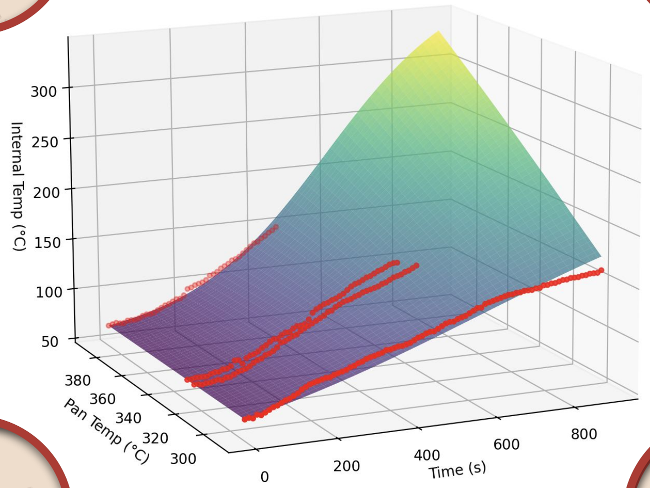

Figure 4

$$\underbrace{Tint = Tinit}_{\text{Init}} + \underbrace{\frac{Tpan - Tinit}{1+e^{-a(b+c(p-Tinit))(t-d)}}}_{\text{time-dep.}} - \underbrace{\frac{Tpan - Tinit}{1+e^{a(b+c(p-Tinit))d}}}_{\text{time-indep.}}$$

$a = 1.1$
$b = -3.908e{-}03$
$c = 2.742e{-}05$
$d = 587.7$
$T_{init} = 66$ ℉

In reality, the function required:

- a time independent offset and
- extra variables

Searing Index

$$C(T,t) \approx Cmax * \left(1 - e^{-a(Tpan-b)t}\right)$$

a = 3.315151e-05
b = 170.604
C_{max} = 100

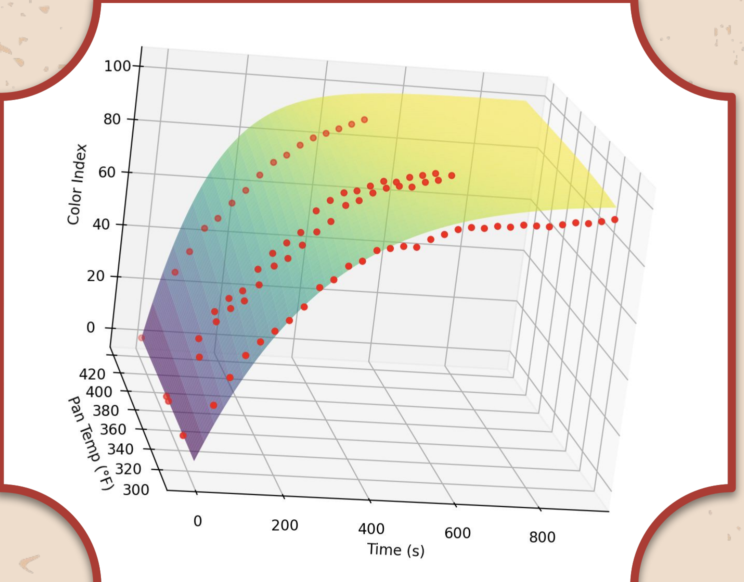

Figure 5

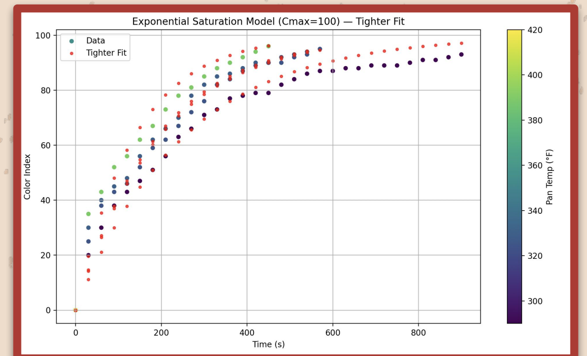

Figure 6

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Control Derivation

Initial System

$X_3 = T_{pan}$

$$X_3 = T_{pan} = \int_0^t \widehat{X}_3 dt + T_{pan\ init}$$

$\dot{X}_3 = \check{T}_{pan}$

$$\widehat{X}_3 = \widehat{T}_{pan} = u$$

Explanation

Because we can control the heating rate of the pan, we can T_{pan} as a state variable and introduce controller u to the system.

Discretized System

$$x_{1,k+1} = x_{1,k} + \dot{C}(x_{3,k}, t_k)\,\Delta t$$

$$x_{2,k+1} = x_{2,k} + \dot{T}_{\text{int}}(x_{3,k}, t_k)\,\Delta t$$

$$x_{3,k+1} = x_{3,k} + u_k\,\Delta t$$

How do chefs do it?

Generally, a combination of PID control and feed-forward scheduling is done. Chefs use three main combinations of these:

- Forward Sear
- Reverse Sear
- **Static Temperature**

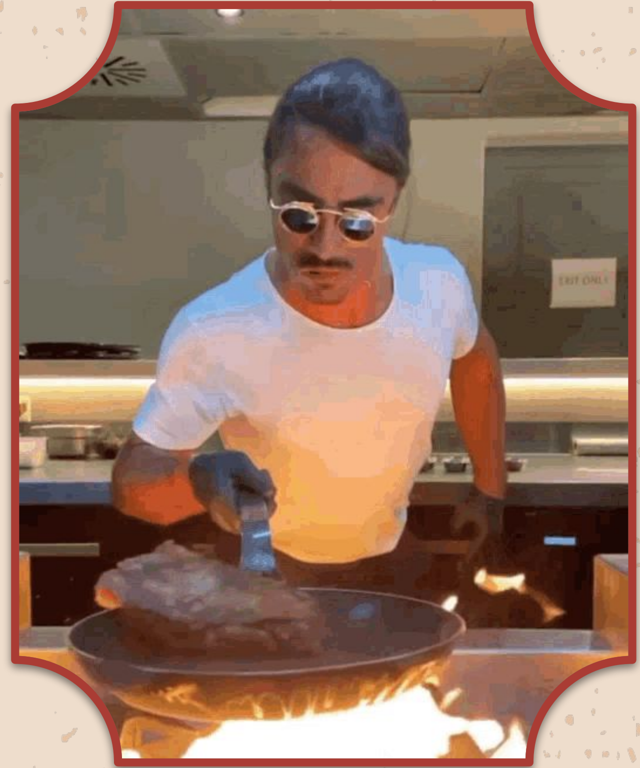

Figure 7 [6]

Nonlinear Optimal Control

Find Optimal Temp

Pre-calculate optimal temperature based on system representation. Find matched finish time for sear and internal temp.

PID as Control

Use a PID to control temperature, as convection, heat transfer, etc remove heat from the system.

Simulation

Capture ‘measurements’ by discretizing system at much smaller time steps (approx 10 ms).

Capture system data and adjust u at larger time steps for approximation (1 second).

```python
def temp_PID_controller():
    k=(0.1, 0.005, 0.001)
    pid = PID(*k)

    return pid

def cost(Tpan, Tint, C, Tinit, x_meas):
    return abs(t_core(Tpan, Tint, Tinit, x_meas[1]) - t_crust(Tpan, C, x_meas[0]))

def optimal_pan_temp(targets, t, x_meas, bracket=(250.0, 450.0), xatol=1e-3):

    # unpack targets
    Tgoal   = targets.Tgoal
    Cgoal   = targets.Cgoal
    Tinit   = targets.Tsteak_init

    sol = minimize_scalar(cost, bounds=bracket, args=(Tgoal, Cgoal, Tinit, x_meas), method='bounded', options={'xatol': xatol})
    print(f"optimal pan temperature: {sol.x:.1f} °F")
    print(f"residual time difference: {sol.fun:.1f} s")
    if not sol.success:
        raise RuntimeError("optimisation failed: " + sol.message)
    return sol.x, sol.fun   # (optimal Tpan, residual |Δt|)
```

Figure 8

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Results

Initial Static Temperature

Matches Initial temperature

- Let the pan get up to temperature

```
optimal pan temperature: 404.1 °F
residual time difference: 0.0 s
Crust index finished: 94.92 (goal: 95.00) at time 385.0
Core temperature finished: 135.00 (goal: 135.00) at time 387.0
Ending time: 387.0 s

Crust index ended at: 95.00 (goal: 95.00)
Core temperature ended at: 135.00 (goal: 135.00)
Finished at t = 387.0 s with 388 samples.
elliotweiner@rfc1918 steak % 
```

Figure 9

Results

Basic Simulation

Initialized system with precalculated 'perfect' temperature.

Both achieve goal in <1 sec of each other.

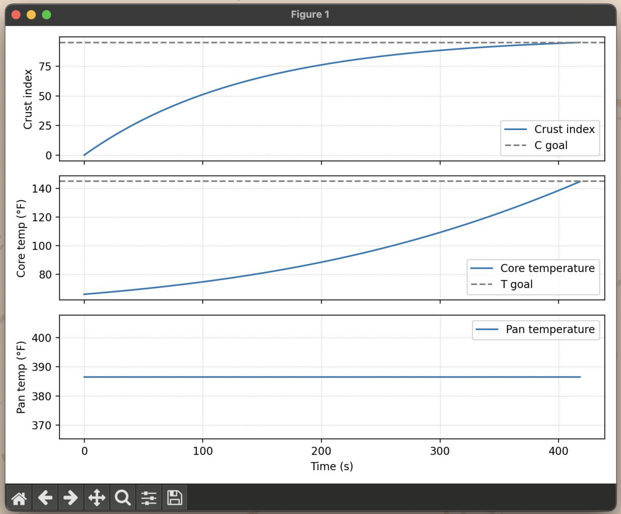

Figure 10

Results

Added Heat Loss

Added PID controller to counteract heat loss from pan. Robust to larger pieces of meat and convective loss.

Both achieve goal in <5 sec of each other.

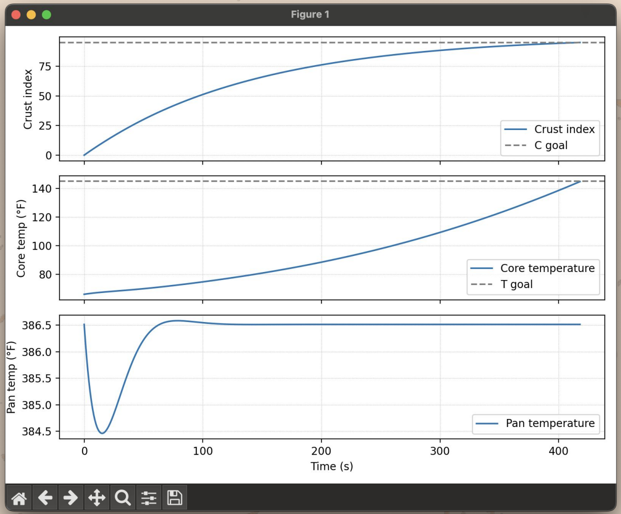

Figure 11

Results

Altered Initial Temp

Added initial temperature offsets to show larger.

Both achieve goal in <1 min of each other at maximum temp offsets.

Potential solution: gain scheduling.

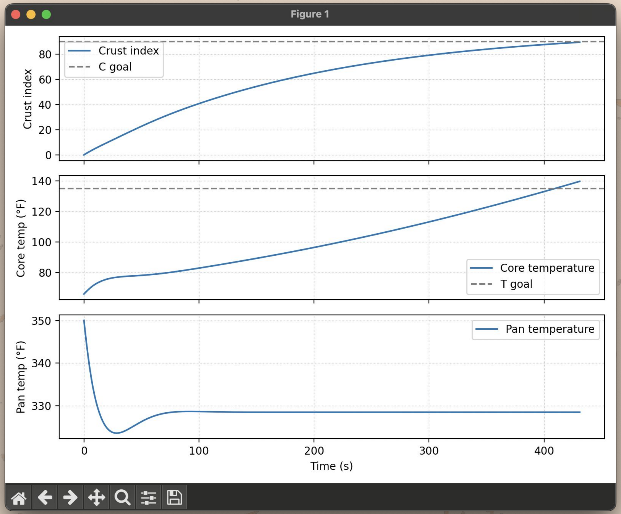

Figure 12

Results

Altered Initial Temp

Added initial temperature offsets to show larger.

Both achieve goal in <1 min of each other at maximum temp offsets.

Potential solution: gain scheduling.

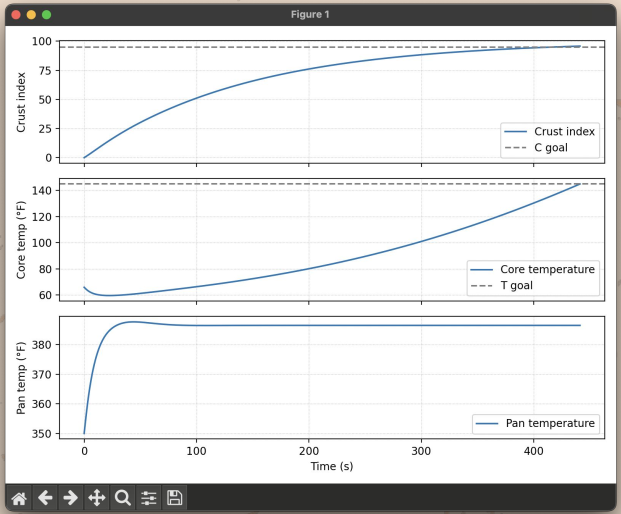

Figure 13

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Conclusions & Future Work

Three ideas

Reinforcement Learning

Reinforcement learning could yield a better approximation of internal temperature, sear index, and temperature control

Better Modeling

Using Steak-unique features such as fat and muscle content, we can derive more precise control [7]

Integrated System

With the right resources, it should be fairly trivial to build an embedded system to control steak searing

Control Graveyard

Dual PID control ❌

- Dynamic gain scheduling
- Achieve forward/reverse sear

MPC ❌

- Leverage internal model
- Lookahead (horizon)

Online Temp Planner with PID control ❌

8

Sources

Resources

1. https://ruthschris.net/blog/steak-temperature-tips/
2. https://searedandsmoked.com/reverse-seared-ribeye/
3. https://www.sciencedirect.com/science/article/pii/S0260877421000236
4. https://www.sciencedirect.com/science/article/pii/S0307904X14006830
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Questions?