Nonlinear Analysis and Control of Searing a Perfect Steak

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Figure 1. The Perfect Steak [2]

1. Abstract
Cooking the perfect steak has always been an elusive goal for many home chefs. While professionals make it look easy, getting the perfect combination of internal temperature, sear, seasoning, etc can be, for lack of a better term, a kitchen nightmare. For those who don't have the prowess of a culinary master, even getting one of these aspects of a perfect steak correct can be daunting. For that reason, this project sought to study the nonlinear aspects of cooking steak, model them as a system, and construct a controller capable of balancing multiple key aspects of a good steak.

3. Approach

To approach the problem with little to no knowledge of thermodynamics or chemistry, an abstracted approach was used to model steak's cooking dynamics. A standard 1 inch thick sirloin steak was used for all testing purposes, all from the same cut for consistency. Internal temperature $T_{int}(T_{pan},t)$ and sear $C(T_{pan},t)$ were used as the two controllable metrics, in the hopes of focusing on two of the most important characteristics of a well-cooked steak. Equations were approximated based on prior cooking knowledge and experience, from which a nonlinear system was derived.

Once data was gathered on each characteristic's change over pan temperature and time, equation parameters were estimated. From there multiple control methods were considered and one was implemented, with the goal being to match the time to achieve a specified goal combination of each characteristic (T_{goal}, C_{goal}). This system was then simulated in Python with a custom-made simulator aptly named 'SteakSim'.

4. Initial Equation Approximation

Given little theoretical knowledge, equations were entirely based on prior knowledge from cooking. This meant approximating 'shapes' of what equations could model internal temperature and sear, then adding constants to allow for fitting to collected data on the actual behavior of a steak as it cooks.

$$C = 100 * (1 - e^{-a(T_{pan} - b)t})$$

Equation 1. Sear Approximation

Sear was the simpler of the two to be modeled via equations. There is no way aside from visual inspection to measure the sear, thus a 1 to 100 scale seemed like a logical base for assigning numerical 'searedness'. Additionally, it was clear from prior experience that steak sear starts immediately after contact with a hot surface and gradually slows. This is in part supported by the involvement of the Maillard reaction, which is key to the browning process which occurs during the cooking process [6]. For this reason, a negative decaying exponential seemed like a reasonable choice of starting point for a numerical model. After adding constants for proper fitting, Equation 1 was chosen as the approximation for searing.

$$T_{int} = T_{init} + \frac{T_{pan} - T_{init}}{1+e^{-c(d+f(T_{pan}-T_{init}))(t-g)}} - \frac{T_{pan} - T_{init}}{1+e^{c(d+f(T_{pan}-T_{init}))g}}$$

Equation 2. Internal Temperature Approximation

Internal temperature proved to be slightly more complicated. A sigmoid was immediately picked as the most likely modeling equation: it's ability to start increasing slowly, reach a 'critical zone' where the rate of change peaks, and decrease to a plateau seemed ideal for mimicking the behavior of non-surface temperatures. In addition, two offsets had to be added to ensure correct initial internal temperatures. An initial temperature value was added, followed by a negative time-invariant copy of the sigmoid to remove initial effects of varying pan temperatures. This led to the creation of the constant-included version seen in Equation 2.

5. System Representation

These equations were expanded into a full system including first order derivatives. Doing so allowed for deeper analysis of the system, discretized simulation, and better control. Initial system variables were assigned as seen in Equations 3 and 4, followed by derivatives in Equations 5 and 6.

$$X_1 = C$$

$$X_2 = T_{int}$$

Equations 3 and 4. System Variables

$$\widehat{X}_1 = \widehat{C} = 100a * (T_{pan} - b + ut) \times e^{-a(T_{pan}-b)t}$$

$$\widehat{X}_2 = \widehat{T}_{int} = u\sigma(H(t)) + cT_{diff}\sigma(H(t)) * (1 - \sigma(H(t)))[d + fT_{diff} + fu(t-g)]$$

$$Where:$$

$$H(t) = c(d + f(T_{diff}))(t - g)$$

$$T_{diff} = T_{pan} - T_{init}$$

Equations 5 and 6. Derivative System Variables

6. Data Collection

To properly fit these models, 4 measurements would be needed: pan temperature, time, internal temperature, and time. For this purpose, a thermometer gun was used to spot check the pan, which was assumed to be at a constant heat. Meat probes were used to keep track of internal temperatures and a timer kept measurement timing intervals steady.

Two pieces of steak were seared at a time, each measuring 1 inch cubed for consistency. The pan was checked every 30 seconds to verify constant heat. Internal temperature was checked every 10 seconds using meat probe readings averaged together. Searing index (0 to 100) was visually determined every 30 seconds by alternating pieces of steak picked up. Using two pieces reduced the effects of time off the stove, which presents clear issues in terms of result validity. Figure 2 showcases the experimental setup.

Figure 2. Data Collection Setup

3 steak trials were run initially with thinner cuts (1 cm thick) to test the environment. All three were initialized at a temperature of 66 degrees Fahrenheit. Figure 3 displays the visualized results, where color represents sear while the other axes are labelled. The test collection proved the method's effectiveness in capturing high-quality data, as the results both mirrored my expected equations and showed consistent results across several pan temperatures.

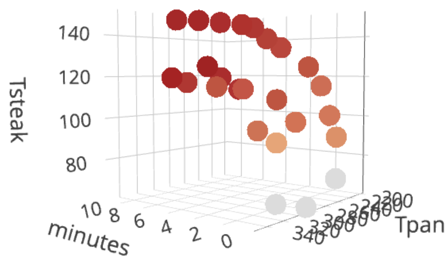

Figure 3. Validation Data

Drawing from the success of the test data collection, 4 trials were attempted with 1 inch thick sirloin steak (Star Market clearance isle steak). 86 data samples were gathered for sear and 250 were gathered for internal temperature.

7. Constant Approximation

Using Python scripts leveraging the Scipy Optimize.curve_fit() function, the data was fitted to curves based on the approximated equations found initially. Implementing equal bias to each steak (distributed among its data points) added additional accuracy in the fits, leading to the plots shown in Figures 4, 5, and 6 and the calculated constants in Table 1.

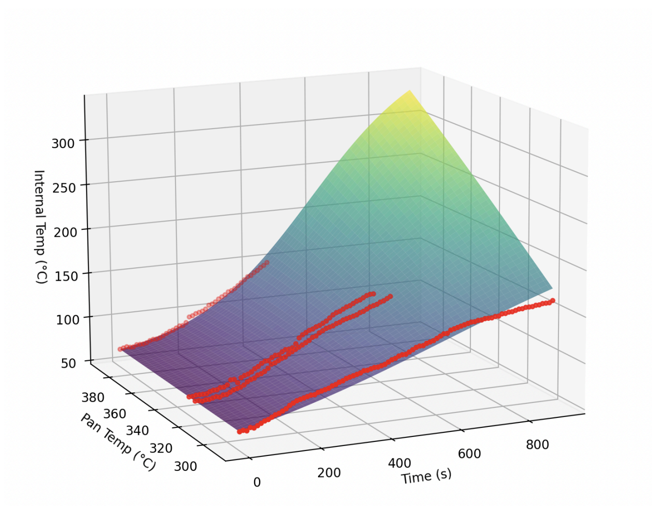

Figure 4. Internal Temperature: Fitted Model

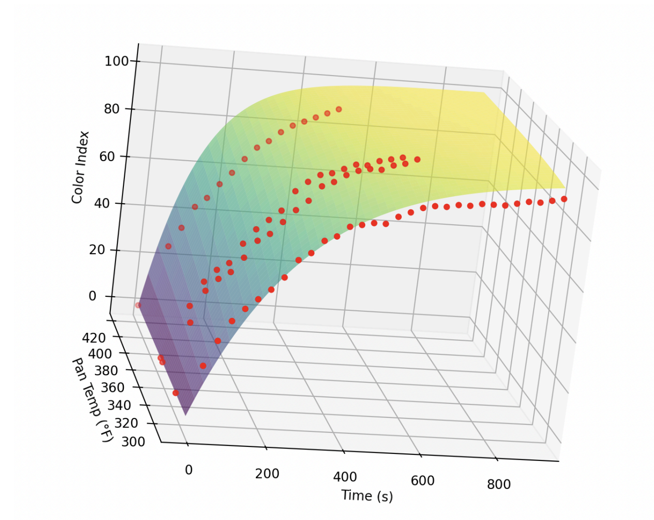

Figure 5. Sear: Fitted Model

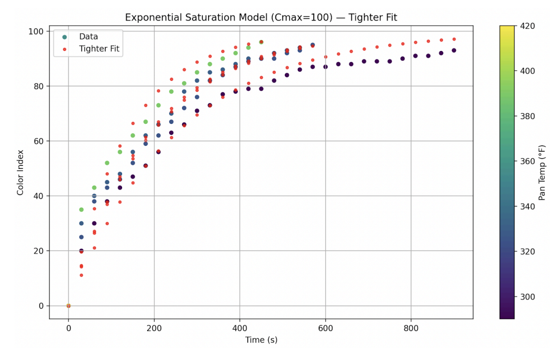

Figure 6. Sear: Fitted Model in 2D

Constant	Value
a	3.315151e-05
b	170.604
c	1.1
d	-3.908e-03
f	2.742e-05
g	587.7

Table 1. Constants

8. Control Derivation

Multiple controllers were attempted to model the common approaches to steak searing. Dual PIDs were considered for mimicking a forward/reverse sear, but were eventually abandoned due to the case-by-case fine tuning required by PIDs. Model Predictive Control was also looked into, but required large overhead and complex implementation. After further research, it also became clear that forward/reverse searing was not optimal in terms of final cooking time (which is likely a prime consideration for anyone hoping to automate their steak cooking).

An alternative approach prioritizing a minimal cooking time would be to find the ideal static cooking temperature. Such would minimize time and the difference between final cooking times (to achieve their respective goals). As such, we derive the equation:

$$e = abs(t_{f\,Tint} - t_{f\,C})$$

Equation 7. Time Error

Where $t_{f\,Tint}$ and $t_{f\,C}$ are calculated by setting the left-hand side of Equations 1 and 2 equal to their goals and solving for t. By then finding an optimal T_{pan} to minimize e, a pan can be initialized to an optimal temperature. An additional PID can be used to stabilize this temperature and ensure minimized error.

9. SteakSim Simulation

$$C_{(k+1)} = C_{(k)} + \widehat{C}_{(k)} * dt$$

$$T_{int(k+1)} = T_{int(k)} + \widehat{T}_{int(k)} * dt$$

$$T_{pan(k+1)} = T_{pan(k)} + \widehat{T}_{pan(k)} * dt$$

Equations 8, 9, and 10. Discretized System

Using the discretized system (Equations 8, 9, and 10), a simulator was created to model the approach of sear and internal temperature to their respective goals. The 'true' measurements were based on a time increment of 10 milliseconds (*dt_true = .01*) and controller measurements were taken every second (*dt_ctrl = 1.0*). A convergence mechanism was also used to stop the simulation when both goals had been achieved, with error in internal temperature and sear being displayed as post-simulation results. Plotted visualizations of the result were also displayed after the simulation ended.

The controller was implemented as an offline optimal pan temperature estimator coupled with a PID controller. The estimator leveraged Scipy Optimize.minimize_scalar() function to minimize *e*. It took in parameters T_{goal}, C_{goal}, and T_{init} to make the only tunable parameter T_{pan}. Once an optimal temperature was found (nearly instantaneously), a PID handled online control. It used parameters $k = (0.1,\ 0.005,\ 0.001)$ on their respective functions: 0.1 for proportional, 0.005 for integral, and 0.001 for derivative control.

10. Results

Several test cases were used to verify proper function of this controller. The first of these was an optimal T_{pan} check via static temperature control, meaning no pan temperature variation. Because the system was discretized and the control updated at a different rate than the 'true' measurement, some error was expected. However, Figure 7 showed excellent results, with error *e* being less than 1 second.

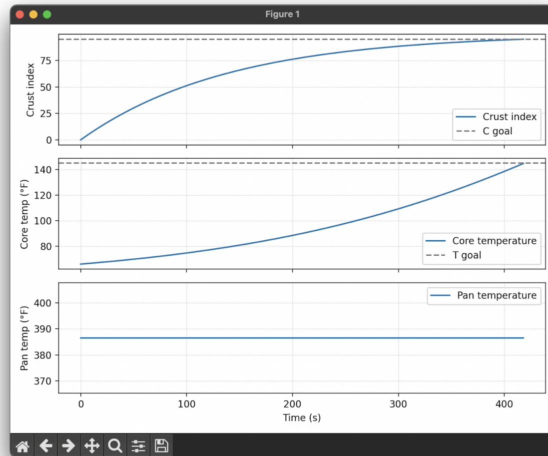

Figure 7. Static Temperature Results

Figure 8 represented the addition of heat loss to the system, in hopes to mimic the temperature variation in a real world system. This was thought to come from convective loss to the surrounding air and heat exchange between the pan and steak. It also served as a test of the PID performance. Results showed a smooth temperature control, quickly applying additional heating to the pan. The *e* in this test was below 5 seconds for most cases, indicating a proper control well within most chef's margins for error.

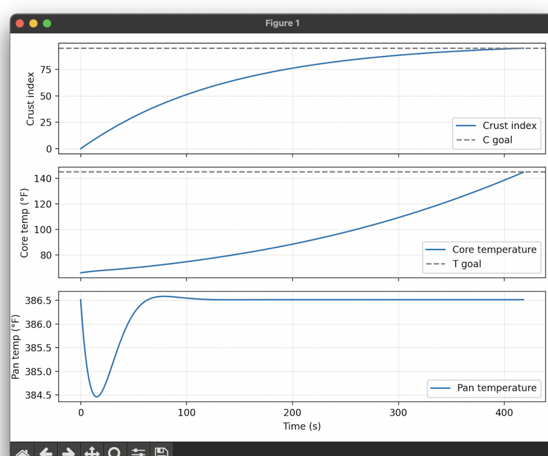

Figure 8. Heat Loss Results

To further push the controller, a large initial temperature offset was added. Figure 9 shows the overshot offset example, where the pan started approximately 22 degrees hotter than the optimal temperature. The system was able to return to the optimal temperature within about 2 minutes of initialization, indicating worse performance than prior cases. The sear and internal temperature were able to achieve their goals within approximately 1 minute of each other. While this demonstrates room for improvement, likely in the tuned PID, it shows consistent results in terms of control.

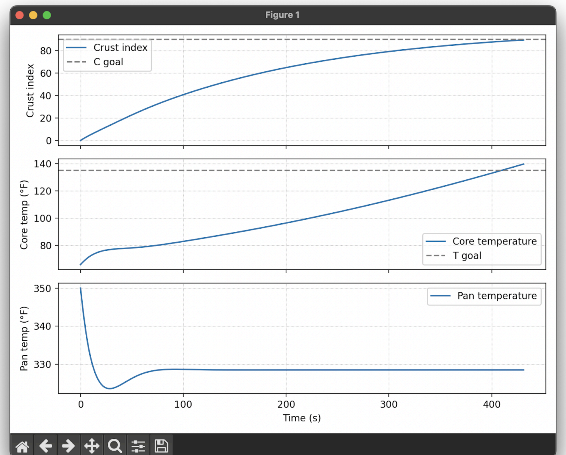

Figure 9. Substantial Offset Results

Figure 10 highlights a major drawback of the mathematical model derived in this project. While it models steak fairly well, it is entirely capable of simulating invalid temperature responses. In the figure, internal temperature drops immediately after being put on the stove. This was due to a constant offset in the T_{int} equation (Equation 2).

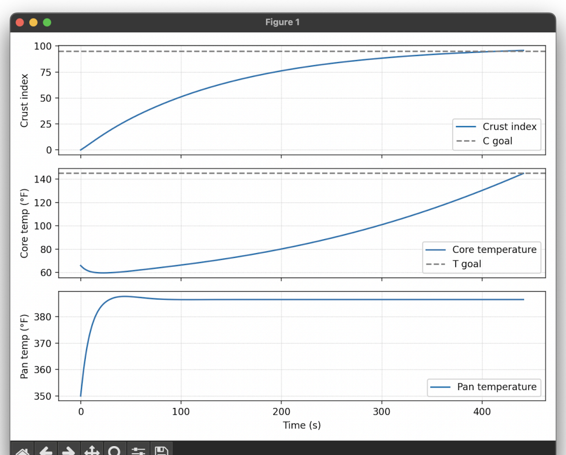

Figure 10. Problematic Case Results

With a grounding in thermodynamics and chemistry, the model in this paper would have likely been much more accurate across a broad range of temperatures. Future experiments would require such a basis, allowing for greater expansion on the work. Larger amounts of data, expanding characteristic modeling (such as fat/muscle ratio, water content, etc), and reinforcement learning could all improve results and control over a complicated and underactuated system such as this. While this research only served as an introductory exploration of control in steak searing, there is potentially a large amount of academic and commercial backing to attempt more in-depth approaches to the problem.

Appendix. Code repository

This report is a comprehensive review of a Boston University graduate project evaluating nonlinear control approaches to searing steak.

For more details, please refer to: https://github.com/ElliotWeiner/steak.

References

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