Analysis of ATLAS Z^0 Boson Invariant Mass

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Introduction

This report analyzes data gathered from ATLAS about the Z^0 Boson. The report will begin by discussing a fitting of the data gathered to the Breit-Wigner decay function to determine the mass of the Z Boson, including a discussion of the χ^2 (chi squared) value and p-value. Then, since 2 parameters were fitted, the report will discuss a 2-dimensional scan of the parameter-space to determine the most probable values for the mass and the width parameter Γ. The report will then discuss potential sources of error, compare the conclusions of this analysis to literature accepted values for the Z Boson mass, and discuss ways to improve accuracy in the future.

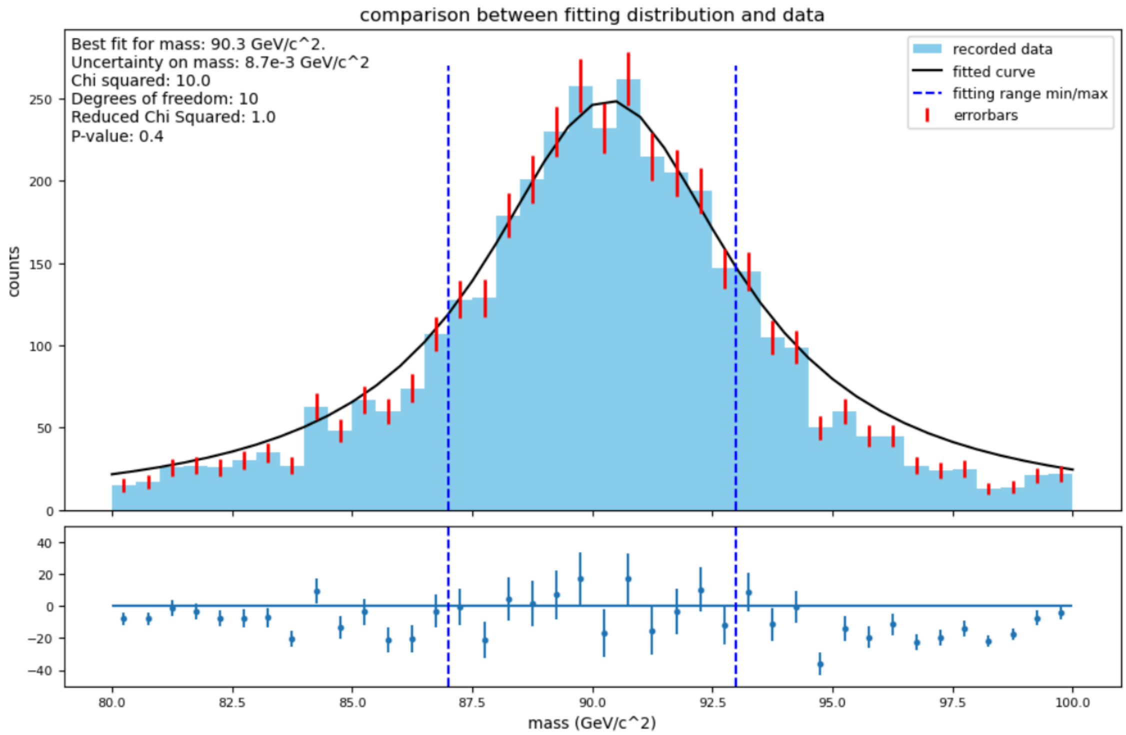
The Invariant Mass Distribution and its Fit

First, for a mathematical background. Particles in the ATLAS detector have four easily measurable properties, which are the total energy E, the transverse momentum p_T, the pseudorapidity η, and the azimuthal angle φ. The pseudorapidity and azimuthal angle measure two different angles within the detector. Using these measurements, the "four momentum" of the particle can be defined as the particle's energy, and its momentum in the x, y, and z directions. The momentum in each direction can be derived as follows: $p_x = p_T \cos(\varphi)$, $p_y = p_T \sin(\varphi)$, $p_z = p_T \sinh(\eta)$, if natural units are used and the speed of light is set to be 1. The invariant mass is derived as follows: $M^2 = E^2 - (p_x + p_y + p_z)$. Taking a square root gives M. This value for M is what is trying to be determined in the experiments. Also, summing the four-momenta of two particles (E, p_x, p_y, p_z) gives the total momenta of those two particles.

Now, methods used within this report will be discussed. The programming language python was used to analyze the data gathered by the detector. In particular, the numpy library (essentially an add-on to the language) was used to help deal with numbers, the matplotlib library was used to generate the figures you will see later in this report, and the curve fit and stats modules from the scipy library were used to create a best fit curve and perform statistical tests on our data.

The report will now discuss the fitting process. We know that the distribution of decays should follow a Breit-Wigner peak, described by the following equation:
$D(m,m_0,\Gamma) = (1/\pi) * (\Gamma/2) / ((m-m_0)^2 + (\Gamma/2)^2)$, where the distribution depends on the true rest mass, m0, and the width parameter Γ. The true width parameter is Γ_0, and is based on the Heisenberg uncertainty principle, however the detector has additional uncertainties which makes the experimental value for Γ larger than Γ_0. Also, since we have 2500 decays in our dataset, we need to multiply this function by 2500 when fitting. The data was loaded into python, and fitted using scipy's curve fit. However, we only fitted the data in the center of the distribution, for mass values between 87 and 93 GeV/c^2. The best fit value for the mass of the Z Boson was 90.3 GeV/c^2, with an uncertainty of 8.7e-3 GeV/c^2, and the best fit value for Γ was 6.4, with an uncertainty of 3.3e-2. Below is a plot of our data and best fit curve.

Figure 1



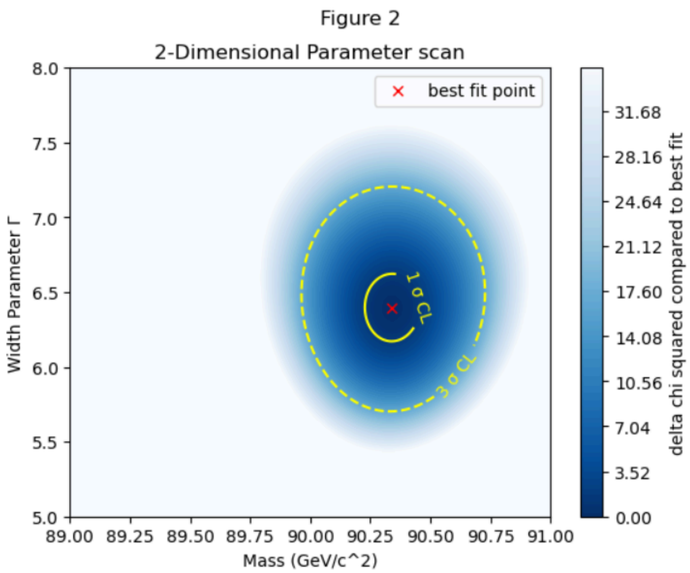
Visually, the fitted distribution appears to be a good fit over the fitting range, as the fitted curve appears to be about 1 standard deviation from most data points in the range. However, once outside the fitting range, the fit appears to become worse, especially on the right tail of the distribution where the fit overestimates the data by over 1 standard deviation for nearly every datapoint.

Analyzing the fit numerically over the fitting range, the χ^2 (chi squared) value was 10.0, and the number of degrees of freedom was 10 (12 fitting points - 2 fitting parameters). using the scipy stats module, this χ^2 and degrees of freedom gives a p-value of 0.4. This p-value indicates that there is a 40% chance that our data and our curve of best fit would disagree by this much purely by random chance if the curve in reality does fit the data. This indicates good agreement between the best fit curve and the data, indicating that this distribution is a good model for our data, at least over the fitting range. Also, we can see from the residuals plot that the residuals appear randomly distributed over the fitting range, which also indicates good agreement between the data and the curve of best fit. Also, it should be noted that χ^2 was computed only using the data within the fitting range.

The 2D Parameter Scan

Now, the report will discuss the two-dimensional nature of the fit, and a two-dimensional parameter scan to visualize the covariance of the two fitting parameters. The plot below shows the value for χ^2 given the values of the fitting parameters at that point minus the minimum value for χ^2.

The x-axis represents the mass, and the y-axis represents the width parameter. Qualitatively, dark blue indicates that the χ^2 at that point is very close to the minimum χ^2 value, while lighter blue



indicates that the χ^2 at that point is very far from the minimum value for χ^2. Also the plot was clipped at a difference of 35, meaning that anything in the region that is completely white has a χ^2 value greater than 35. Also quantitatively, the best fit point was plotted with a red x, and 1 sigma and 3 sigma confidence levels were plotted with solid and dashed yellow lines, respectively.

This plot shows that there is not a large covariance between the two fitting parameters, as the shape made is roughly circular. If this plot were extremely elliptical and angled such that the long end of the ellipse is not parallel with the x or y axis, that would indicate the two parameters have high codependence, and that constraining one more would help significantly to constrain the other.

Discussion and Future Work

This analysis found that the best fit value for the mass of the Z Boson was 90.341 GeV/c^2, with an uncertainty of 8.7e-3. Comparing the Breit-Wigner function evaluated at these values along with the best fit for the width parameter (6.39±0.033) to the data using a χ^2 test resulted in a χ^2 of 10.0. With 10 degrees of freedom, this gives a p-value of 0.4, which indicates good agreement between our best fit curve and the data.

The value generally found in literature for the mass of the Z Boson is 91.188 ± 0.002 GeV/c^2. Given the small uncertainties, our best fit value is 94 standard deviations from the literature accepted value. While this could indicate that the literature could be incorrect, it is more likely that some of the assumptions and simplifications made in this report are not necessarily accurate.

One simplification is that we assumed a fit over a small region of the distribution would provide an accurate fit for all the data and thus for the mass. However, we can see from Figure 1 that this is not true. Looking at the residual plot, outside the fitting range, most of the residuals are under the fit by significantly more than 1 standard deviation. This indicates that the earlier assumption was incorrect, and a fit over the entire range may produce a value closer to the value in literature.

Also, we assumed that the only errors in our measurements were random uncertainties, however this is likely not true. The measurements themselves also probably have some uncertainty. These uncertainties would make the overall uncertainty in our data larger, which would make the difference between our measurements and the literature accepted value less extreme.

Given the above discussion, some ways to help improve the accuracy of the experiment in the future would be to fit over the entire data range, determine the measurement errors of our apparatus, and also to run more trials so we have a larger sample.