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Abstract

This report emphasizes the design and analysis of a cantilever wing model, utilizing static aeroelasticity principles. The project involves the evaluation of critical parameters such as wing divergence, aileron effectiveness and reversal, and wing loading. The wing model is assumed to have an aerodynamic center at the quarter chord position, and an elastic axis that varies linearly perpendicular to the flow direction.

The report's primary objective is to develop a wing that has the ability to generate a lift force of 700 N at an airspeed of 70 m/s while adhering to various design constraints. The wing-tip deflection must be kept below 1°, and a wing root bending moment below 300 N-m. The wing divergence speed must also exceed 150 m/s. This report will outline the iterative processes that allowed for the development of a wing that meets the design criteria.

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Introduction

This project aims to design a cantilever wing model that meets specific requirements. The primary constraints are: The wing must generate at least 700 N of lift at an airspeed of 70 m/s. The ratio s/c_{mean} should be greater than 3, the wing-tip deflection may not exceed 1°, and the wing-root bending moment may not exceed 300 N-m. Finally, the wing divergence speed (U_d) must be greater than 150 m/s.

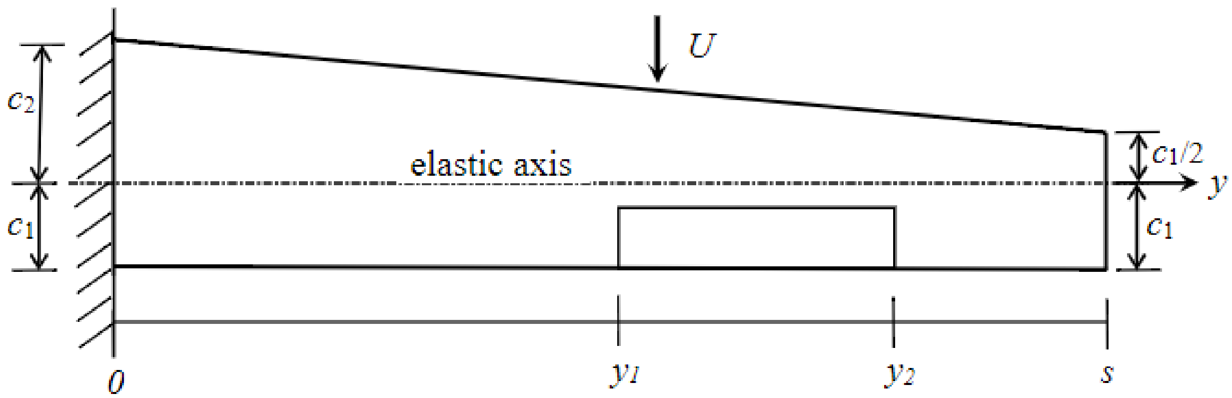

Figure 1: *Wing Planform [3]*

The information provided to develop the wing model is outlined below. The wind-off angle of incidence is given by $\alpha(y) = 5 - 3\eta$ degree where $\eta = \frac{y}{s}$. The lift distribution of the no-camber wing model is given by $C_{L_\alpha} = 2\pi\sqrt{1 - \eta^2}$. The coefficients C_{L_β} and C_{M_β} are assumed to be constant along the span and equal to their airfoil theory values. The torsional stiffness of the model is given by $GJ(\eta) = 8500(1 - K\eta)$. Several design parameters are to be determined to design a wing that meets the above specifications.

Part A

In part A, the divergence dynamic pressure was calculated using the assumed modes method with as many modes as required to find the divergence dynamic pressure to an estimated error of less than 0.1%. To accomplish this, the following sample set of values was given: k = 0.25, c1 = 0.35m, c2 = 0.4m and s = 2m.

The assumed modes method is done using the following series:

$$f_1 = y,\ f_2 = 2y^2,\ ...\ f_n = ny^n \qquad (1)$$

To estimate the error the following equation was used:

$$\%error = \left|\frac{U_n - U_{n-1}}{U_n}\right| \times 100\% \qquad (2)$$

Where U_n is the divergence airspeed, determined through the use of modes. It was determined that employing five modes were then used, achieving an estimated error of 0.028%.

In the equation (6), [E] is the structural stiffness, q[K] denotes the aerodynamic stiffness with the following properties:

- It is proportional to q, i.e. it becomes very significant at high dynamic pressures
- It is proportional to the cube of the aircraft size
- K_{ij} can be negative (a negative aerodynamic spring)
- K_{ij} does not necessarily equal K_{ji}

q{F} is a vector of static aerodynamic loads, also proportional to q and the cube of the aircraft size, and {θ} is a vector of generalized coordinates.

$$E_{ij} = \int_0^s GJ f_i' f_j' dy \qquad (3)$$

Where GJ is the torsional constant (a piecewise function dependent on the spanwise location), and f_n' is the derivative of the mode shape function with respect to the spanwise location y.

$$K_{ij} = -\int_0^s c^2 e C_{L_\alpha} (f_i f_j) dy \qquad (4)$$

Here, c is the chord length at position y, while e denotes the nondimensionalized distance between the aerodynamic center and elastic axis (the aerodynamic center was assumed to be at 0.25c for this project). C_{L_α} represents the lift curve slope of the wing.

Now that the matrices [E] and [K] are known, the divergence dynamic pressure, q, can be readily determined, as the smallest positive eigenvalue of these matrices

$$q_d = eig(E, \ -K) \qquad (5)$$

To calculate wing tip twist, the following equation was solved for the generalized coordinates, denoted as θ_r.

$$[[E] + q[K]]\{\theta_r\} = q\{F\} \qquad (6)$$

$$F_i = q\int_0^s c^2 e C_{L_\alpha} \alpha f_i dy \qquad (7)$$

Wing twist θ is found with:

$$\theta = \theta_1 f_1 + \theta_2 f_2 + ... + \theta_n f_n \qquad (8)$$

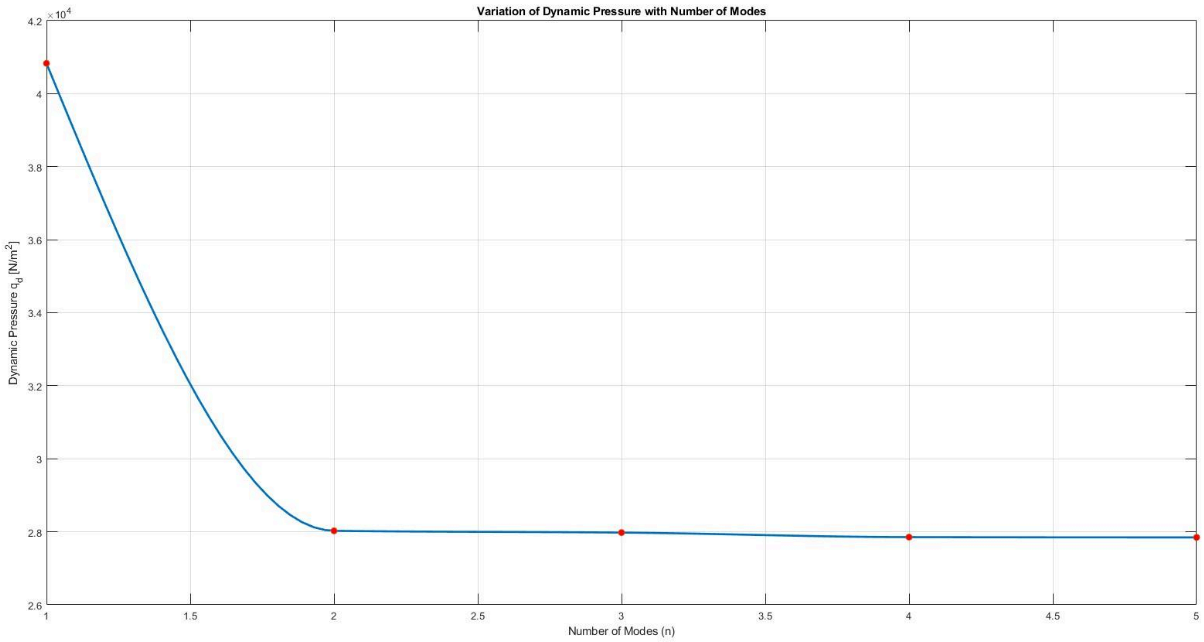

Figure 2: *Variation of Dynamic Divergence Pressure with Number of Modes*

Figure 1 shows the plot of the variation of the divergence dynamic pressure versus the number of modes. It can be seen that the dynamic pressure converges. With 5 modes, the value was calculated to be $27846.25\ N/m^2$ with an estimated error of 0.028%. The wing tip deflection at 50% divergence dynamic pressure was found to be 5.02^o.

To determine the final number of modes, an iterative process was utilized. The number of modes was incrementally changed until equation (2) indicated that a sufficient number of modes had been reached, in this case, the final number of modes amounted to 5.

Part B

In this section of the project, the wing parameters (span s, chord constants c1 and c2, and the torsional stiffness parameter k) must be created so that, while accounting for wing deflection, the model can produce at least 700N of lift at an airspeed of 70 m/s. There must be a ratio of s/c_{mean} greater than 3. The wing-root bending moment should not be greater than 300 N-m, and the wing tip deflection should be less than 1.0^{o} at the same airspeed of 70 m/s. The wing divergence speed must be greater than 150 m/s.

As a last part of the design requirements, the model design should attempt to achieve the lowest possible value of equation (3), shown below:

$$\int_{0}^{1} GJ(\eta)c(\eta)d(\eta) \qquad (9)$$

GJ (torsional rigidity of the wing) describes the ability of the wing to twist under aerodynamic loads. It can be observed from the GJ equation $GJ(\eta) = 8500(1 - K\,\eta)$. It is a function of the span and y, which is the distance from the root to the wing outboard. With further analysis of the equation, it can be seen that the maximum torsion rigidity occurs at the root, while at the tip of the wing η becomes 1, because y=s.

To enable the calculation of lift, wing tip twist, and bending moment, a similar process from part A was used, taking equation (3) to solve for the vector of coefficients representing the variation in wing twist angle, denoted as {θ}.

Once obtained, the divergence speed can easily be solved for, assuming standard sea-level conditions:

$$U_d = \sqrt{\frac{2q_d}{\rho}} \qquad (10)$$

In order to determine the maximum wing twist, the series of modes and the wing twist constant vector {θ} can be put together to find the variation of the wing twist with spanwise location, θ(y)

$$\theta(y) = \theta_1 f_1 + \theta_2 f_2 + \ ...\ \theta_n f_n \quad (11)$$

This equation was solved to find the maximum wing twist, ensuring it remained below the maximum required value of 1 degree. In cases where the wing twist exceeded the allowable threshold, adjustments were made. Reducing the wing chord, increasing the torsional constants, and decreasing the wing span would allow the model to meet the requirements through iteration.

Furthermore, the variation of the wing twist also provides the total angle of attack distribution and allows for the subsequent calculation of lift:

$$L = \frac{1}{2}\rho U^2 \int_0^s cC_{L\alpha}(\alpha + \theta)dy \quad (12)$$

The lift can be assessed with each input change through the MATLAB script, ensuring that it meets the requirement of 700N. In instances where the lift is below the allowable threshold, variations in the wing span, chord, and stiffness (within allowable limits) were made in order to meet the design requirements.

Finally, with the lift distribution of the wing known, the total wing root bending moment can be evaluated. Since the lift distribution followed an elliptical pattern, the moment was found by applying the total lift at a third of the span (13):

$$M = L_{total}\frac{1}{3}s \quad (13)$$

To decrease an excessive bending moment, a reduction in wingspan, or the inward repositioning of the lift distribution by increasing the taper ratio could be used.

In this section, the wing parameters were calculated experimentally, to a given set of constraints, and the final design parameters are tabulated, and graphical representations of the analysis are presented, along with further discussion regarding the results of the analysis.

Property	Final Iteration
S (m)	1.28
S/C_{Mean}	3.05
k	0.25
C_1 (m)	0.24
C_2 (m)	0.24
V_{DIV} (m/s)	272
θ_{MAX} (deg)	0.32
Lift (N)	701.89
Wing Root Bending Moment (N*m)	299.47

Table 1: Final Design Iteration

These values were obtained by guessing in Matlab, outputting a result that would either pass or fail depending on the given constraints:

- Generate at least 700N lift at airspeed 70m/s
- Ratio s/c_{mean} greater than 3
- Wing-tip deflection lower than 1.0^o
- Wing-root bending moment not greater than 300 N-m
- Divergence speed higher than 150 m/s

Part C

Upon the completion of the wing design portion, the wing twist variation, as defined by equation (11), can be graphically represented as a function of the spanwise location. Refer to figure 3 in the discussion section of this report, for a visual representation.

The following graph is the resulting wing twist distribution along the span of the wing. It can be observed that the highest increase in wing twist with respect to the spanwise location ($\frac{dy}{d\theta}$) is near the tip, and decreases closer to the wingtip.

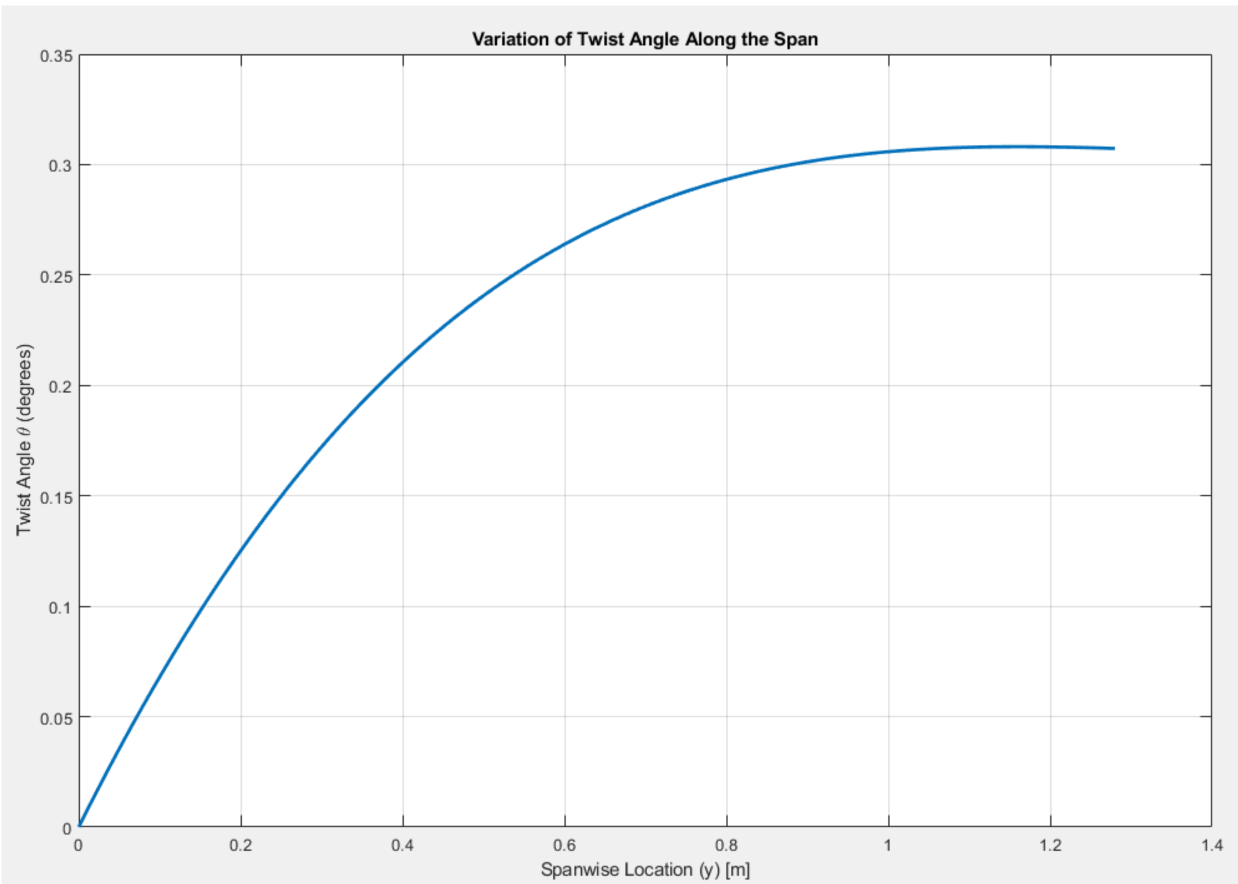

Figure 3: *Variation of Twist Angle Along the Span*

Part D

The final design values determine the lift distribution, as indicated by the equation shown below:

$$L = \frac{1}{2}\rho U^2 c C_{L\alpha}(\alpha + \theta) \qquad (14)$$

It is worth noting that the variables c, α, θ vary with the spanwise location on the wing, while the remaining variables do not. Equation (14) can be plotted to find the lift distribution. See figure 4.

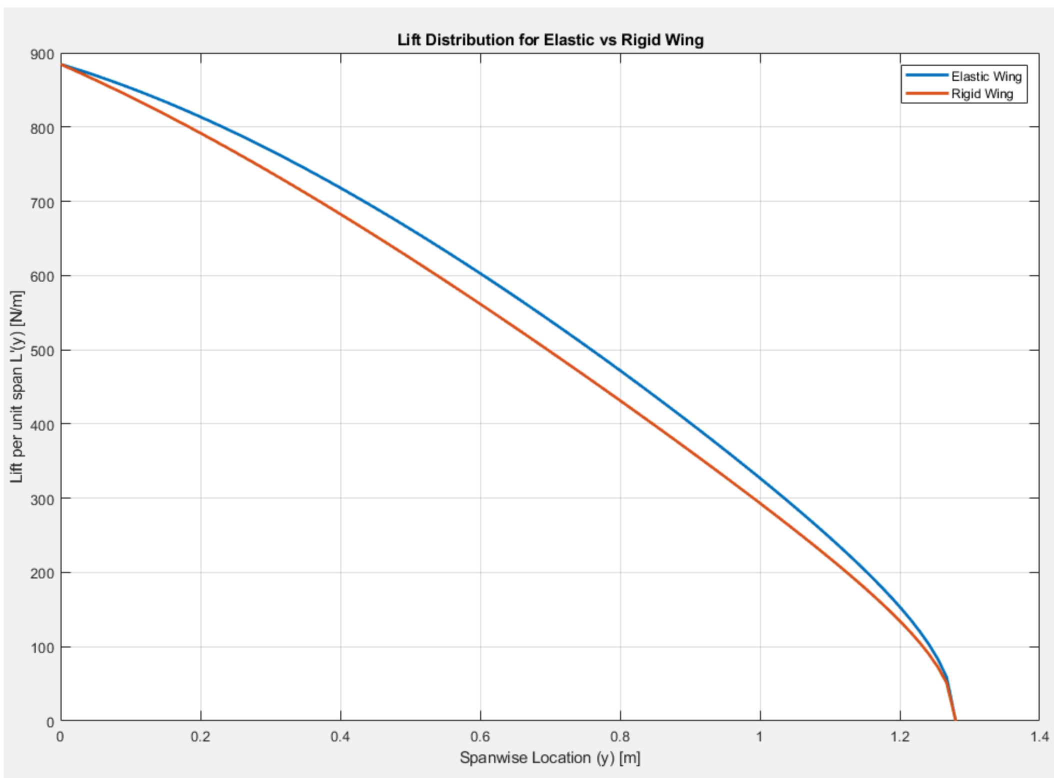

Figure 4: *Lift Distribution For Elastic vs Rigid Wing*

The lift distribution curves for the elastic and rigid wing are shown above. Since the rigid wing has no extra angle of attack due to aeroelastic effects, it produces slightly less maximum lift, as evidenced by the graph. The majority of the lift is produced at the wing root and decreases towards the tip.

Part E

Based on equation (15) it can be noted that two angles of attack induce lift, namely the initial angle of attack (α) and the angle of attack due to wing twist (θ). θ may be omitted from equation (15), allowing for the integration of equation (16) to determine the lift distribution of the rigid wing:

$$L_{Rigid} = \frac{1}{2}\rho U^2 \int_0^s cC_{L\alpha}(\alpha)dy = 1285.22\ N \qquad (15)$$

The value obtained from equation (16) can then be compared to that obtained from the above equation. This will allow us to determine the percentage change in lift due to aeroelasticity:

$$\%\ Increase\ Lift = \left|\frac{L_{Rigid} - L_{Elastic}}{L_{Rigid}}\right| \times 100 = 5.57\% \qquad (16)$$

$$\%\ Increase\ Wing\ Root\ Bending\ Moment = \left|\frac{M_{Rigid} - M_{Elastic}}{M_{Rigid}}\right| \times 100 = 5.57\% \quad (17)$$

Where:

$$M_{Rigid} = L_{Rigid} * ⅓ * s \qquad (18)$$

The percentage increase in total lift and wing-root bending moment was found to be 5.46%. This value was calculated using equations (16) and (17).

Part F

In order to find the aileron reversal dynamic pressure, as well as the aileron rolling power, several integrals must be evaluated, as outlined below:

$$\varphi = cos^{-1}(2F - 1) \qquad (19)$$

$$C_{L_\alpha} = 2\pi \qquad (20)$$

$$C_{L_\beta} = 2(\pi - \varphi + sin\varphi) \qquad (21)$$

$$C_{M_\beta} =- sin\varphi(1 - cos\varphi) \qquad (22)$$

$$C_{M_\alpha} = 0 \qquad (23)$$

$$A = \int_{y1}^{y2} cC_{L_\beta} ydy \qquad (24)$$

$$B = \int_{y1}^{y2} c^2(eC_{L_\beta} + C_{M_\beta})fdy \qquad (25)$$

$$C = \int_{0}^{s} cC_{L_\alpha} fydy \qquad (26)$$

$$D = \int_{0}^{s} c^2 eC_{L_\alpha} ffdy \qquad (27)$$

$$E = \int_{0}^{s} GJf'f'dy \qquad (28)$$

A mode shape function f = y(4s − 2y) was used. The aileron chord was set to 0.3c, and values of 0.5s and 0.8s were used for y1 and y2, respectively. Now, the wing aileron reversal dynamic can be evaluated using the following equation:

$$q_r = \frac{AE}{AD-BC} \qquad (29)$$

Utilizing the equation below, the aileron rolling power, at a velocity of 70 m/s was found to be 3.13 $/sec$ while the dynamic pressure is 6884.74 N/m^2.

$$\frac{p}{\beta_0} = U\frac{A(E/q - D)+BC}{CG+F(E/q - D)} \quad (30)$$

Part G

Using the design parameters and flight conditions from *Part A*. The Rolling Power was calculated using different sizes of the aileron chord, as a percentage of the chord of the wing.

The results were then plotted and the following trend was obtained. As the size of the aileron increases, the Rolling Power increases.

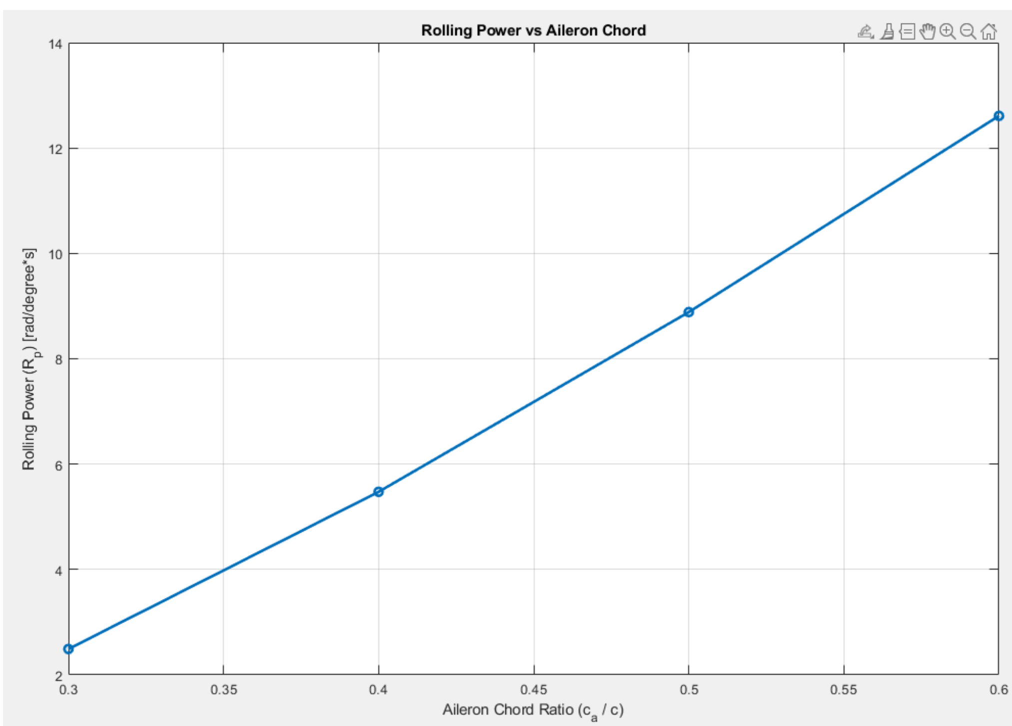

Figure 5: *Rolling Power vs Aileron Chord*

Conclusion

In the beginning of the project, the focus was to calculate the wing's divergence dynamic pressure using the assumed modes method. By employing five modes, the analysis achieved an error margin of less than 0.1%, resulting in a divergent dynamic pressure of 27,846.25 N/m². This value corresponds to a divergence speed of 272 m/s. The analysis involved solving for the torsional and aerodynamic stiffness matrices, ensuring the wing's stability against torsional divergence.

This project achieved the successful design and analysis of a cantilever wing model by applying principles of static aeroelasticity. The primary goal was to develop a wing capable of generating a lift force of 700 N at an airspeed of 70 m/s, while adhering to strict constraints on wing-tip deflection (below 1°), wing-root bending moment (below 300 N-m), and a divergence speed exceeding 150 m/s. Through a series of iterative steps and computational analyses, the final design met or exceeded all performance criteria, with the wing achieving a divergence speed of 272 m/s, a wing-tip deflection of 0.32°, and a wing-root bending moment of 299.47 N-m.

The iterative design process employed a combination of MATLAB simulations and theoretical methods, including the assumed modes method to calculate the wing's divergence dynamic pressure, lift distribution, and structural stiffness. A key aspect of the design was balancing aerodynamic performance with structural integrity, particularly in managing the trade-offs between wing flexibility, lift generation, and the impact of aeroelastic effects.

The project also explored the influence of aeroelasticity on the wing’s performance by comparing the lift distribution and bending moments of both elastic and rigid wing models. It was found that aeroelastic effects led to a 5.57% increase in both lift and wing-root bending moment, which underscored the significance of accounting for wing flexibility in the design process. This increase was primarily attributed to the additional angle of attack induced by wing twist, which improved lift performance but also heightened the structural demands on the wing.

Furthermore, the analysis extended to aileron effectiveness and reversal, where the dynamic pressure and rolling power were evaluated for different aileron sizes. The values obtained seemed unreasonable. This could be due to an error in the code. However, there still was valuable insights into the role of control surfaces in aeroelastic designs, reinforcing the importance of accounting for both aerodynamic forces and structural responses in wing and control surface integration.

Overall, this project not only achieved the specific design goals but also demonstrated the importance of aeroelastic considerations i n modern aircraft design.

References

[1] Dr. Alighanbari, “Wing Aileron Effectiveness and Reversal,” https://courses.torontomu.ca/d2l/le/content/777905/viewContent/5151412/View

[2] Dr. Alighanbari, “Wing Divergence,” https://courses.torontomu.ca/d2l/le/content/777905/viewContent/5151429/View

[3] Dr. Alighanbari, “Project 1,” https://courses.torontomu.ca/d2l/le/content/777905/viewContent/5382229/View