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1.0 Problem Description

The problem at hand is to compare two engine systems for a commercial supersonic aircraft. This engine system must be capable of flying from NY to London in under 3 hours. The first engine system is my "fictional" design tailored to the mission requirements, which includes a supersonic inlet that is optimized by the Oswatitsch principle. The second engine system is off-the-shelf. This comparison will be made by a trade-off study, which includes a parametric cycle analysis.

1.1 Supersonic Inlet Design

For the supersonic ramp inlet of the engine system, the goal is to decelerate airflow at Mach 3.2 using a series of 3 oblique shocks, followed by a normal shock. It is required that prior to the normal shock, the flow must be at Mach 1.3. In order to achieve high efficiency, the Oswatitsch principle is used to maximize the total pressure recovery (π_d), which states this occurs when the shocks are of equal strength [1]. This will be done through solving a system of equations involving the shock angle (β), Mach numbers (M), and flow deflection angles (θ). Below is a table of the required inputs and outputs [2].

Table 1.1: Inlet Design Inputs & Outputs.

Inputs	
Number of Shocks	4 (3 oblique + 1 normal)
Flight Mach Number (M_1)	3.2
Normal Shock Upstream Mach Number (M_n)	1.3
Ratio of Specific Heats γ	1.4
Outputs	
Mach numbers (M_2 to M_{n-1})	

Oblique shockwave angles (β_1 to β_{n-1})
Flow Deflection angles (θ_1 to θ_{n-1})
Stagnation pressure ratios (π_1 to π_n)
Total pressure recovery ratio (π_d)

1.2 Parametric Cycle Analysis

A one-dimensional Parametric Cycle Analysis is done to calculate the performance parameters of the “fictional” engine for a supersonic mission. Then, an analysis will be done on the off-the-shelf engine retrieving its performance parameters. These two engines will then be compared through a trade-off study and conclusions will be made taking into consideration the supersonic mission requirements.

2.0 Approach

In order to solve the problems involving the design of the supersonic inlet and the parametric cycle analysis; also to perform the comparison of the “fictional” engine and the off-the-shelf engine, the following approaches were taken.

2.1 Supersonic Inlet Design

For the design of the supersonic inlet, an external compression model was selected for the ramp. The following schematic provides a 2D visual of the model used to solve the problem showing the key parameters.

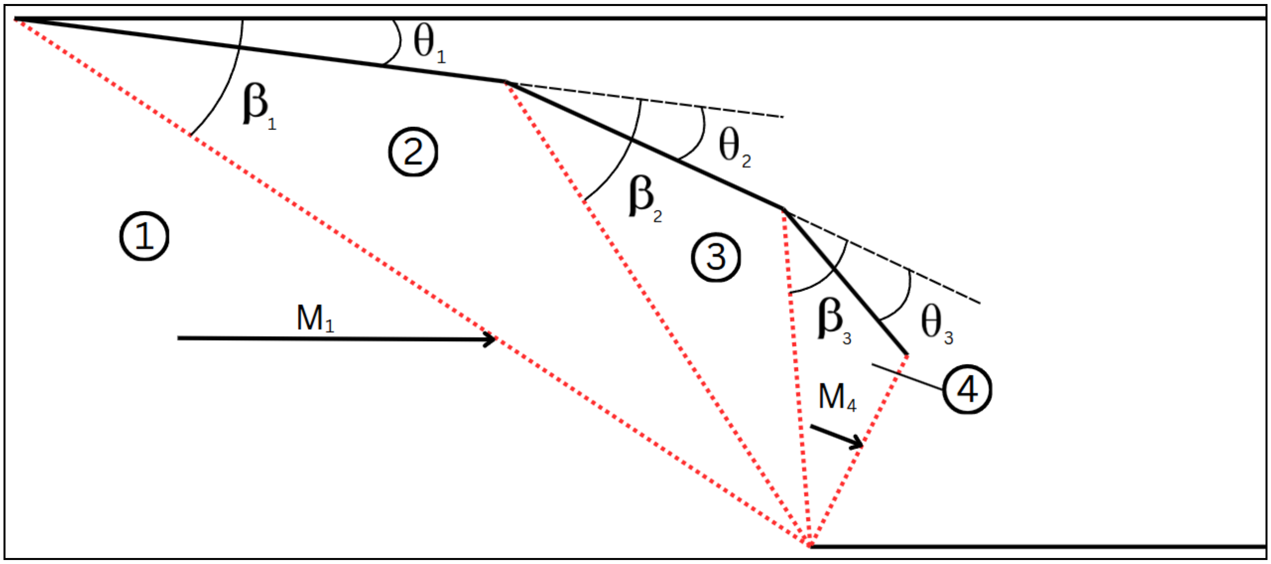

Fig. 2.1. External Compression Supersonic Inlet Model.

To solve the problem, a series of equations was solved for regions 1 to 2, regions 2 to 3, and regions 3 to 4. To do this, a MATLAB program was created which can be seen in Appendix A. The program iterated the flow turning angle (θ_1) from 0 to 25 degrees, performing the following calculations for each iteration:

Region 1 to 2 (Oblique Shock 1):

1. Shock 1 beta calculation (using the fzero function to solve for beta):

$$cot\,\theta_1 = tan\left(\beta_1\right)\frac{(\gamma+1)M_1^{\,2}}{2(M_1^{\,2}sin^2\beta_1 - 1)} \tag{1}$$

2. Normal Mach component before shock 1:

$$M_{n1b} = M_1 sin\left(\beta_1\right) \tag{2}$$

3. Normal Mach component after shock 1:

$$M_{n1a}^{\;\;2} = \frac{M_{n1b}^{\;\;2}\,2/(\gamma-1)}{[2\gamma/(\gamma-1)]M_{n1b}^{\;\;2} - 1} \tag{3}$$

4. Region 2 Mach:

$$M_2 = \frac{M_{n1a}}{sin\left(\beta_1 - \theta_1\right)} \tag{4}$$

5. Stag. pressure ratio of Region 2 rel. to Region 1:

$$\frac{P_{02}}{P_{01}} = \pi_1 = \left[\frac{(\gamma+1)M_1^{\ 2}sin^2\beta_1}{(\gamma-1)M_1^{\ 2}sin^2\beta_1+2}\right]^{\gamma/(\gamma-1)}\left[\frac{(\gamma+1)}{2\gamma M_1^{\ 2}sin^2\beta_1-(\gamma-1)}\right]^{1/(\gamma-1)} \tag{5}$$

Region 2 to 3 and Region 3 to 4 (Oblique Shock 2 and 3):

Letting n = current region:

1. Set the normal mach component before nth shock equal to the normal shock component before previous shock.

$$M_{Nbn} = M_{Nb(n-1)} \tag{6}$$

2. Calculated the nth shock angle (β_n) using eq. (2).
3. Calculated the nth flow deflection angle (θ_n) using eq. (1).
4. Calculated the normal mach component after nth shock (M_{Nan}) using eq. (3).
5. Calculated the Mach after the nth shock (M_{n+1}) using eq. (4).
6. Calculated the stag. Pressure ratio (π_n) using eq. (5).

Region 4 to 5 (Normal Shock):

1. Calculated M_5 using eq. (3).
2. Calculated π_4 using eq. (5).

2.2 Parametric Cycle Analysis

For the “fictional engine”, a turbojet was selected. This was done because the turbojet is able to reach Mach 3.2 and fly with sufficient efficiency (with an afterburner). The following diagram shows each stage of the turbojet, which will be examined in the parametric cycle analysis.

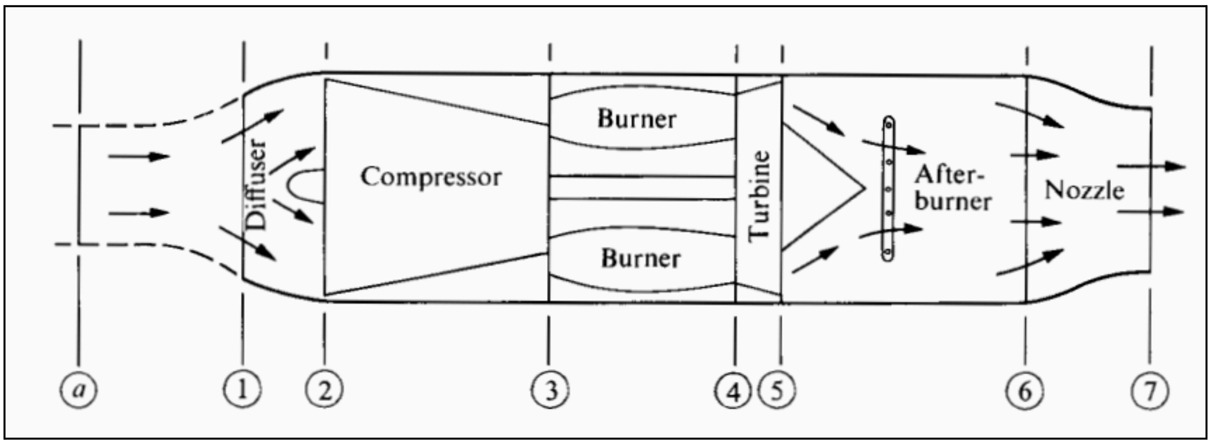

Fig. 2.1. Turbojet Engine Schematic [3].

In order to perform the parametric cycle analysis, it was necessary to define the ambient conditions for each flight Mach, and the specific heat ratios (γ) and component efficiencies (η) for each turbojet stage (for a non-ideal analysis). These parameters can be seen in the table below.

The analysis was done using 3 different maximum temperatures at the compressor exit: 1500 K, 1600K, and 1700 K. It was also performed at three different Mach numbers: 0.85, 2, and 3.2. A MATLAB program was created to complete the calculations, iterating across compressor ratios (0 to 100) and max temperatures. The program is provided in Appendix B.

Table 2.1: Turbojet calculation parameters [3]

Component	Adiabatic efficiency	Average specific heat ratio
Diffuser	$\eta_d = 0.97$	1.40
Compressor	$\eta_c = 0.85$	1.37
Burner	$\eta_b = 1.00$	1.35
Turbine	$\eta_t = 0.90$	1.33
Nozzle	$\eta_n = 0.98$	1.36
Fuel heating value, 45,000 kJ/kg		
Flight altitude (cruise Mach no.)	**Ambient pressure (kPa)**	**Ambient temperature (K)**
Sea level (0)	101.30	288.2
40,000 ft (12,200 m) (0.85)	18.75	216.7
60,000 ft (18,300 m) (2.0)	7.170	216.7
80,000 ft (24,400 m) (3.0)	2.097	216.7

The formulas used for each stage are shown below [3].

Compressor inlet conditions (Stage 2):

$$T_{02} = T_{0a}\left(1 + \frac{\gamma_d - 1}{2} M_a^2\right) \tag{7}$$

$$P_{02} = P_{0a}\pi_d \tag{8}$$

Compressor outlet conditions (Stage 3):

$$P_{03} = P_{02} r_c \tag{9}$$

$$T_{03} = T_{02}\left[1 + \frac{1}{\eta_c}\left(r_c^{(\gamma_c - 1/\gamma_c)} - 1\right)\right] \tag{10}$$

Burner fuel-air ratio:

$$f = \frac{T_{04}/T_{03} - 1}{Q_R/c_p T_{03} - T_{04}/T_{03}} \tag{11}$$

Turbine inlet pressure (Stage 4):

$$P_{04} = P_{03} * \pi_b \tag{12}$$

π_b was assumed to be 0.96 [3].

Turbine outlet conditions (Stage 5):

$$T_{05} = T_{04} - (T_{03} - T_{02}) \tag{13}$$

$$P_{05} = P_{04}\left[1 - \frac{1}{\eta_t}\left(1 - T_{05}/T_{04}\right)\right]^{\gamma_c/(\gamma_c - 1)} \tag{14}$$

Nozzle inlet conditions (Stage 6):

$$T_{06} = T_{05} \tag{15}$$

$$P_{06} = P_{05} \tag{16}$$

Inoperative afterburner.

Nozzle exit velocity (Stage 7):

$$u_e = \sqrt{2\eta_n \frac{\gamma_n}{\gamma_n - 1} R T_{06}[1 - (P_a/P_{06})^{(\gamma_n - 1)/\gamma_n}} \tag{17}$$

Specific Thrust per unit mass flow & Thrust Specific Fuel Consumption:

$$\frac{F}{\dot{m}} = \left[(1 + f)\, u_e - u\right] \tag{18}$$

$$TSFC = \frac{f}{(1+f)\, u_e - u} \tag{19}$$

Efficiency Calculations:

$$\eta_o = \frac{\left(u_e - u\right) u}{f\, Q_r} \tag{20}$$

$$\eta_{th} = \frac{0.5(1+f)u_e^2 - 0.5u^2}{f\, Q_r} \tag{21}$$

$$\eta_p = \frac{2\left(u/u_e\right)}{1 + \left(u/u_e\right)} \tag{22}$$

2.2.1 Off-the-Shelf Engine

The off-the-shelf engine selected for comparison to the “fictional” engine is the Pratt & Whitney J58*,* as used in the SR-71 aircraft. This engine was selected because of its ability to fly at Mach 3.2, despite relying on an afterburner and ramjet-like qualities. Below is an image of this engine.

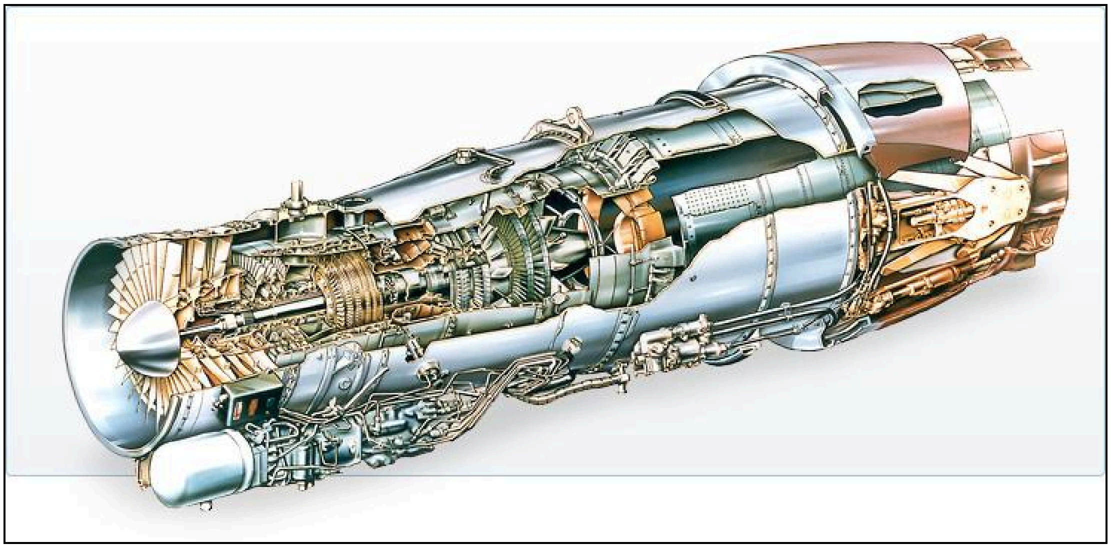

Fig. 2.2. Pratt & Whitney J58 Engine Cutaway [4].

2.2.2 Engine Comparison

The approach for performing this comparison was to analyze the Thrust Specific Fuel Consumption (TSFC), specific thrust-to-mass flow rate, and to perform a tradeoff study based on these parameters. For a more fair comparison with the J58.

3.0 Final Design

The final designs for both the supersonic ramp inlet and engine are both shown in the subsequent sub-chapters; along with a comparison of the “fictional” engine design with the off-the-shelf engine.

3.1 Supersonic Inlet Design

The results from the supersonic inlet MATLAB program produced the following parameters:

Table 3.1: Designed Supersonic Ramp Inlet Parameters

n	M_n	β_n	θ_n	π_n
1	3.2	27.8	11.9	0.9320
2	2.6	35.6	14.4	0.9320
3	2.0	50.0	17.2	0.9320
4	1.3			0.9792

The Overall Pressure Recovery Ratio (π_d) was found to be 0.79. The figure below displays a rendition of the designed supersonic ramp inlet.

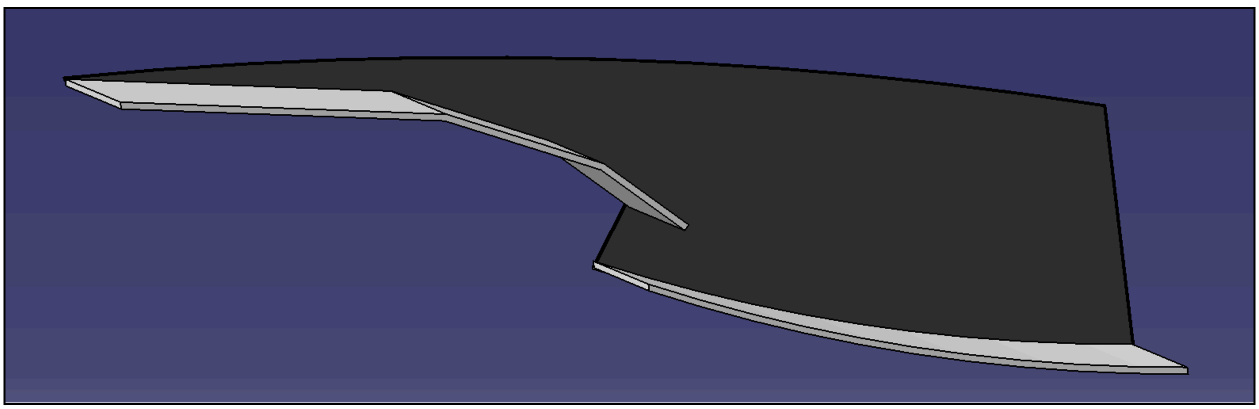

Fig. 3.1. Fictional Supersonic Inlet Rendition.

3.2 Parametric Cycle Analysis

The following figures display the results of the parametric cycle analysis. For the Mach 3.2 condition, the designed pressure recovery across the inlet of **0.79** was used from the previous section.

Mach 0.85 Conditions:

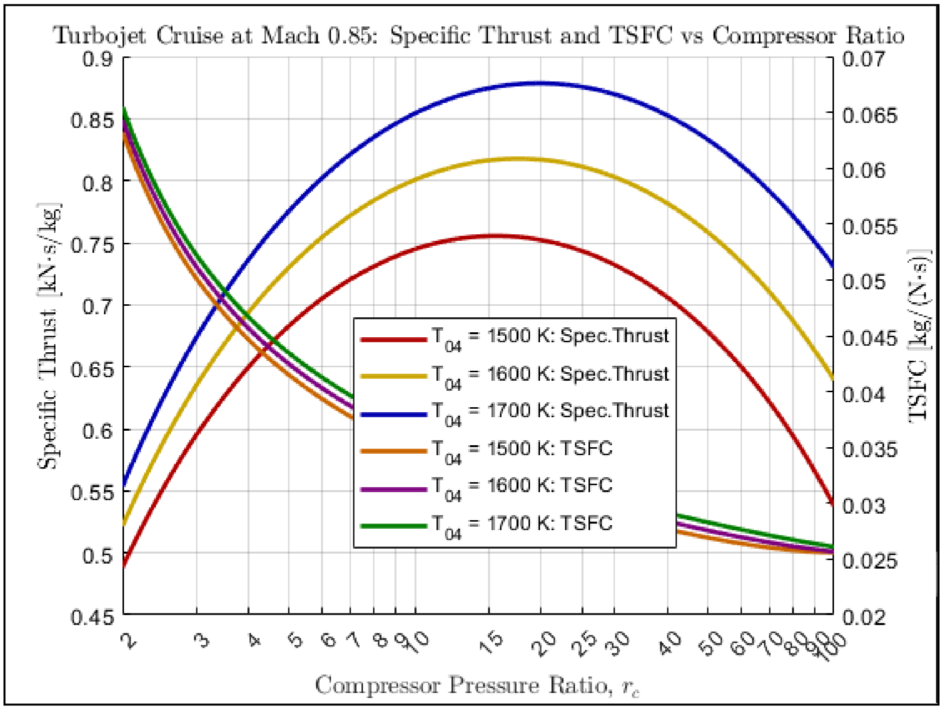

Fig. 3.2. Specific Thrust and TSFC vs Compressor Ratio (M = 0.85)

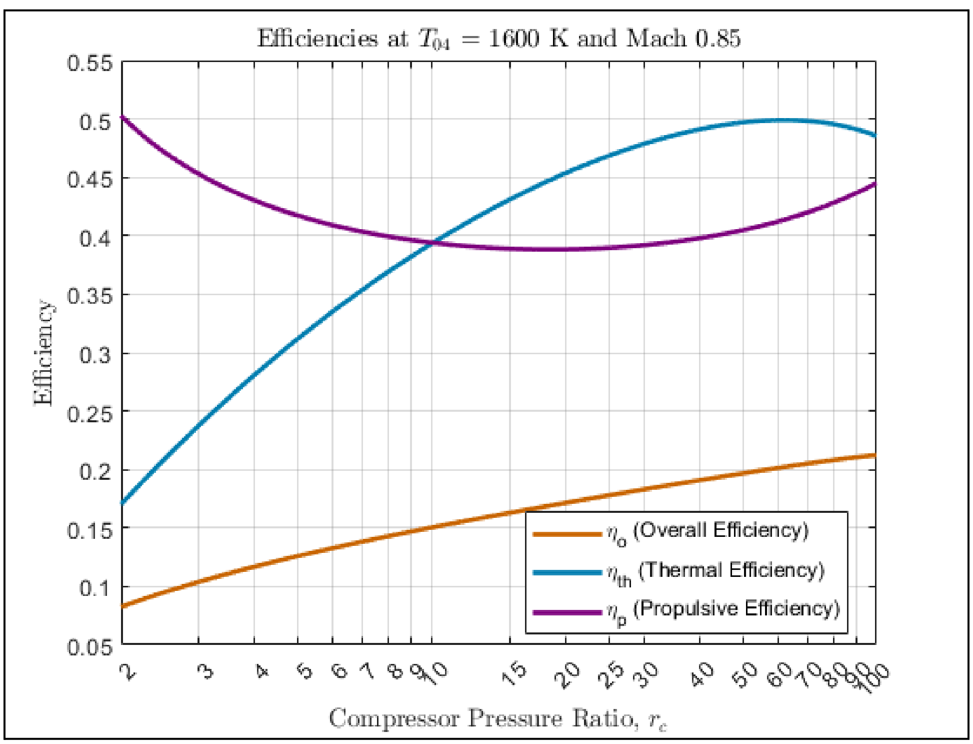

Fig. 3.3. Overall, Thermal, and Propulsive efficiency vs Compressor Pressure Ratio (M = 0.85)

Mach 2 Conditions:

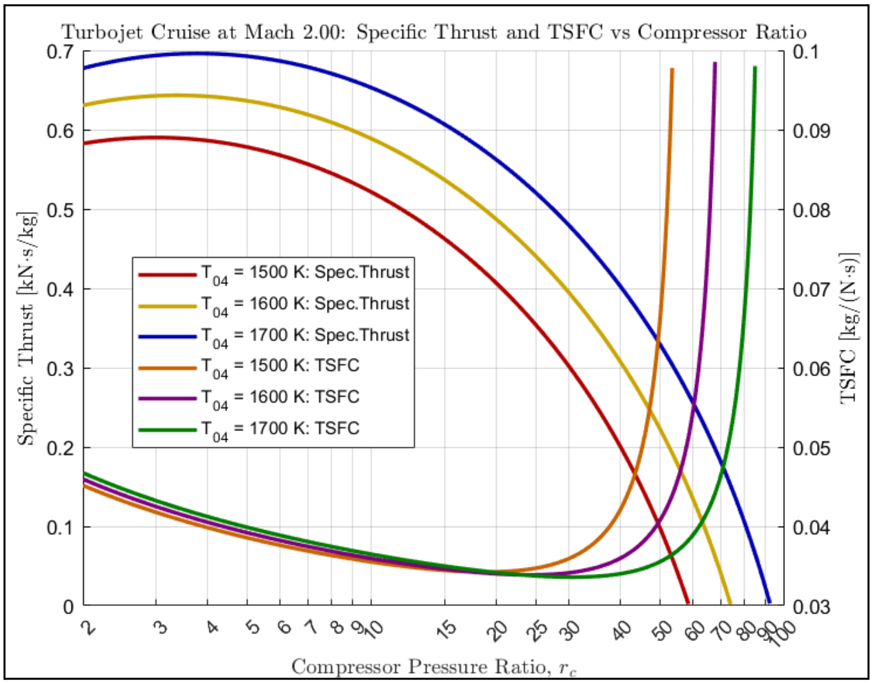

Fig. 3.4. Specific Thrust and TSFC vs Compressor Ratio (M = 2)

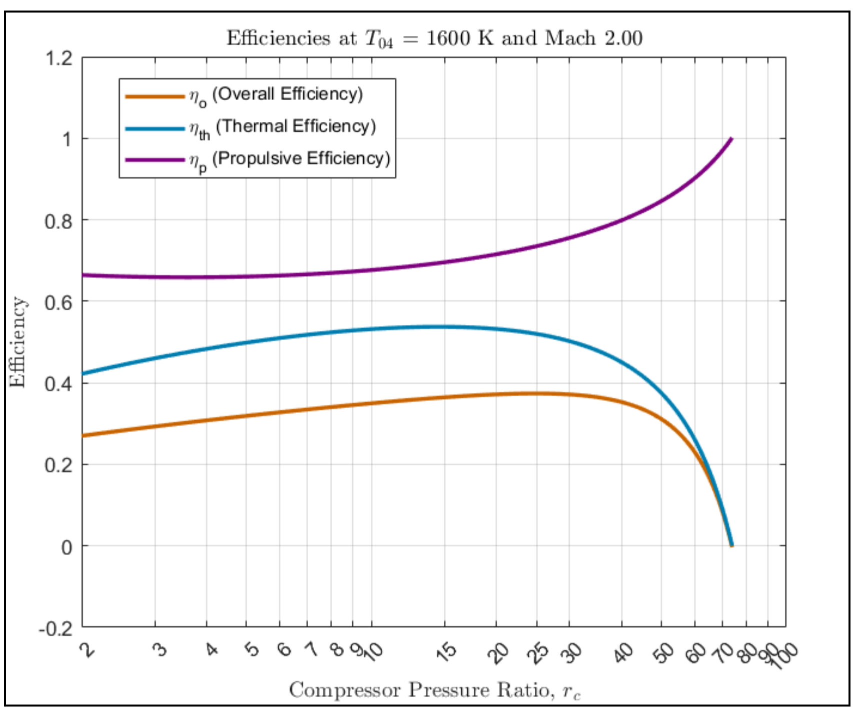

Fig. 3.5. Overall, Thermal, and Propulsive efficiency vs Compressor Pressure Ratio (M = 2)

Mach 3.2 Conditions:

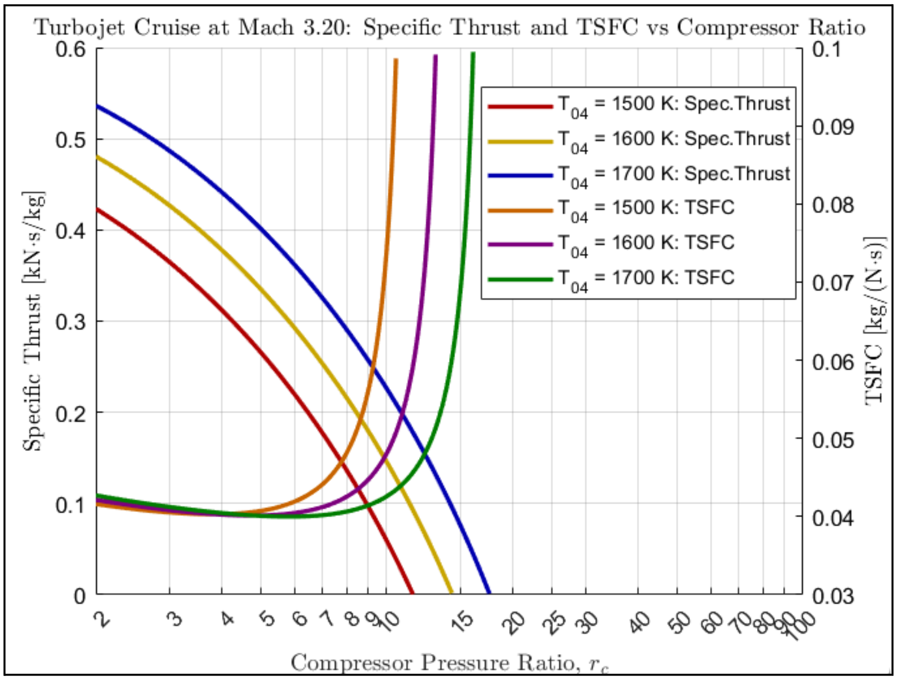

Fig. 3.6. Specific Thrust and TSFC vs Compressor Ratio (M = 3.2)

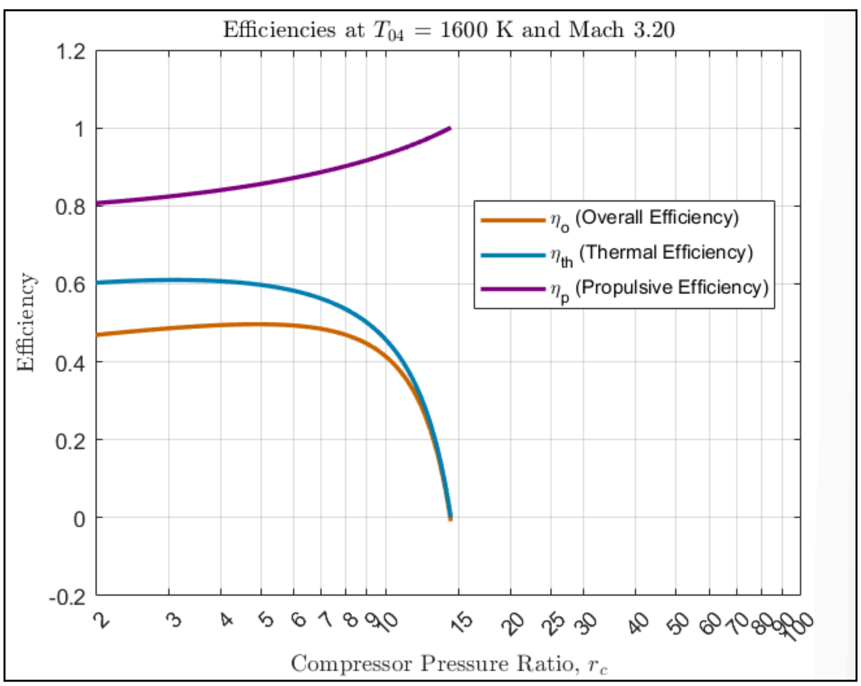

Fig. 3.7. Overall, Thermal, and Propulsive efficiency vs Compressor Pressure Ratio (M = 3.2)

The results from the parametric cycle analysis were highly reasonable. Since the following traits can easily be seen. One, optimizing specific thrust does not minimize fuel consumption - indicating a balance in compressor pressure ratio is needed for best cruising range. Two, raising turbine inlet temperature improves thrust - but obviously to the detriment of the selected materials. Three, higher temperatures only slightly reduce fuel consumption at the optimal pressure ratios. Four, for supersonic flight, lower compressor pressure ratios are much more efficient [3].

3.2.1 Compressor Pressure Ratio Selection

Under the mission requirements, the engine must be able to fly from NY to Paris in under 3 hours. This affects the cruise Mach number and thus the compressor pressure ratio that will be selected for design. Since the parametric cycle analysis was performed with an inoperative afterburner, the cruise Mach for this design will be set at Mach 2 - which has been historically proven by the Concorde to meet the mission requirements [5]. With this information in mind, along with the results from the parametric analysis, a compressor pressure ratio of **9.5:1** is selected.

3.2.2 Maximum Combustion Temperature Selection

For the maximum combustion temperature a value of **1700 K**, as this is the highest typical inlet temperature, and seems feasible given modern cooling techniques [3].

3.3 Engine Comparison

The following table presents the comparison between the performance parameters of the Pratt & Whitney J58 engine and the Fictional engine. The thrust to mass flow rate and TSFC were compared at sea-level conditions.

Table 3.2: J58 and Fictional Engine Performance Parameters

Performance Parameter	Pratt & Whitney J58 Engine	Fictional Engine
Type	Turbojet	Turbojet
Max Combustion temp.	1600 K [6]	1700 K
Compressor Pressure ratio	8.8:1 [6]	9.5:1
Thrust to mass flow rate (sea level) kNs/kg	0.708 [7]	0.91 [Appendix C]
TSFC (takeoff) kg/Nh	*Data not publicly available, 0.075 - 0.11 - typical values [2]*	.03 [Appendix C]

From examining the performance parameters, the designed fictional engine has a higher thrust-to-mass flow rate. This suggests at the lower Mach range, well before Mach 3, it is safe to say the fictional engine would have greater power density. This is especially true considering the higher compressor pressure ratio that was selected for the fictional engine. However, these differences are very marginal. The largest performance gap can be seen in the area of TSFC. Although the numbers for TSFC were not available for the J58, it is safe to assume it's in the region of the typical values. Given this, the fictional engine highly outperforms any off-the-shelf engine, having a TSFC at sea-level of 0.03, more than half of the lowest limit of the typical value. Which is expected given the greater amount of inefficiencies and variables that exist in the real world.

4.0 Conclusion

This project addressed the design and analysis challenge of a fictional supersonic engine, contrasting it to the performance of the legendary Pratt & Whitney J58 engine. Through inlet optimization using the Oswatitsch principle and a parametric cycle analysis, the fictional engine demonstrated better performance under sea-level conditions across key parameters, while having the ability to operate efficiently at Mach 3.2. The fictional engine is also easily capable of completing the required mission from New York to London in under 3 hours.

References

[1] Oswatitsch, K. (1947).*Pressure recovery for missiles with reaction propulsion at high supersonic speeds. NACA TM No 1140.*

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[3] Hill, C.R. and Peterson, C.R., *Mechanics and Thermodynamics of Propulsion*, 2nd Edition, Addison-Wesley, 1992.

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[7] J. D. Mattingly, *Elements of Propulsion: Gas Turbines and Rockets*, Reston, VA: American Institute of Aeronautics and Astronautics, 2006.

Appendix A

Sea-level TSFC and Thrust to mass flow rate for fictional engine w/ compressor pressure ratio of **9.5**: