CSCI 338: Assignment 2 (6 points)

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This assignment is due on **Tuesday, Feb 18, 11:30pm**. It is strongly encouraged that you use Latex to generate a single pdf file and upload it under *Assignment 2* on D2L. But there will NOT be a penalty for not using Latex (to finish the assignment). This is **not** a group-assignment, so you must finish the assignment by yourself.

Problem 1

(1.1) Problem 1.6.c, 1.6.f (page 84— all the questions with only numbers given are referred to the 3rd edition of the textbook).

1.6.c

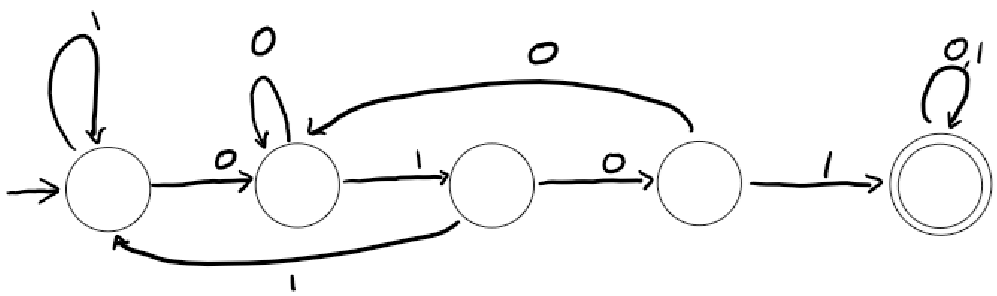

Figure 1: Solution to 1.6.c

1.6.f

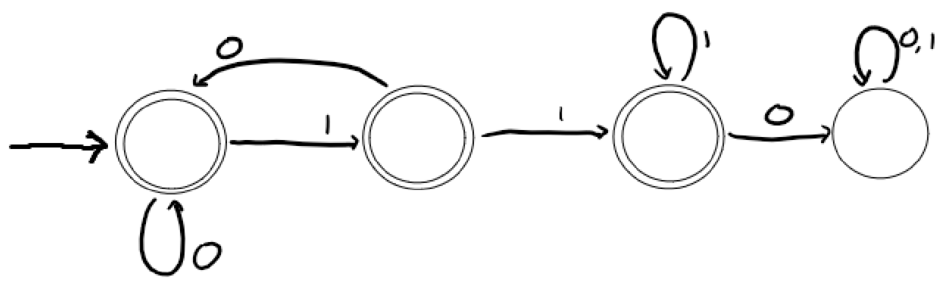

Figure 2: Solution to 1.6.f

(1.2) Problem 1.7.b, 1.7.c (page 84).

1.7.b

The solution for 1.6.c (Figure 1) is also a valid NFA and has 5 states.

1.7.c

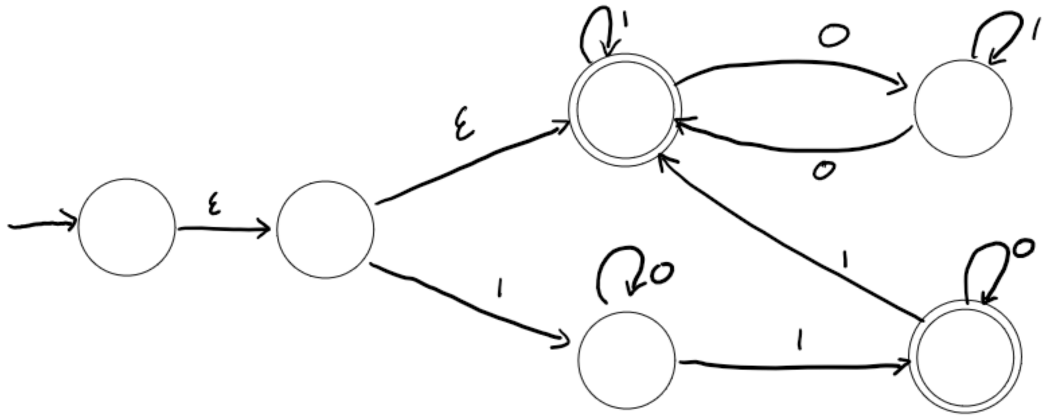

Figure 3: Solution to 1.7.c

Problem 2

Problem 1.16.a, Problem 1.16.b (page 86).

1.16.a

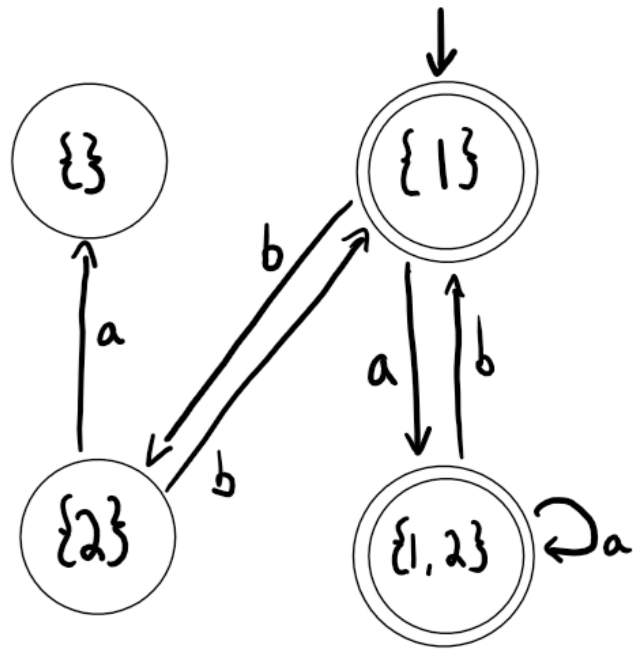

Figure 4: Solution to 1.16.a

1.16.b

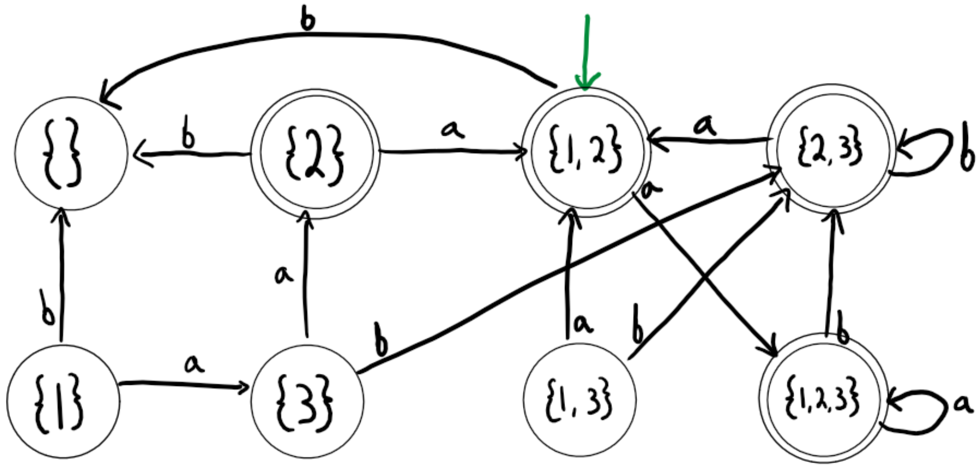

Figure 5: Solution to 1.16.b

Problem 3

Problem 1.19.a, 1.19.b (page 86).

Note in solutions some of the internal epsilon transitions are ommitted for concision.

1.19.a

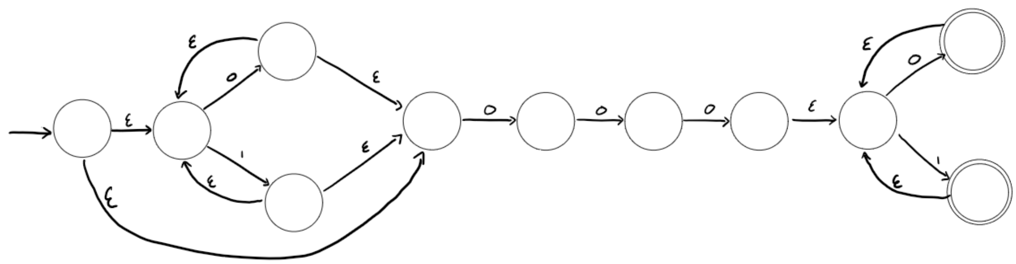

Figure 6: Solution to 1.19.a

1.19.b

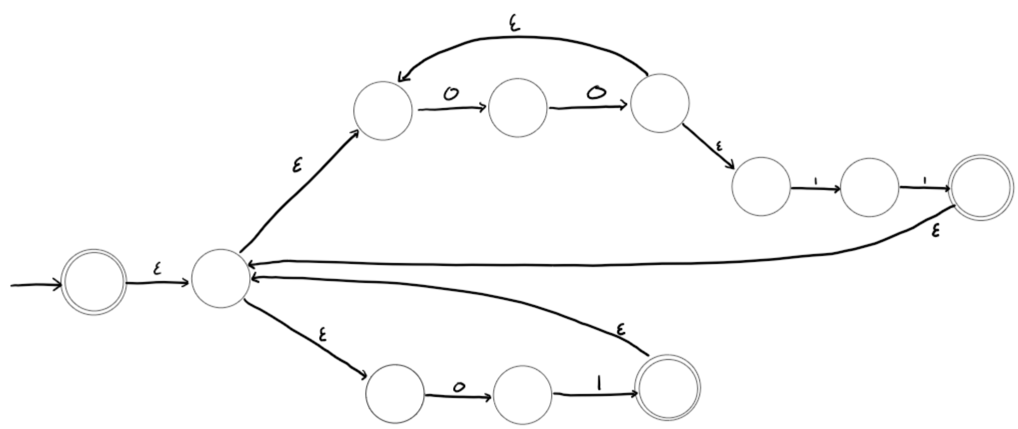

Figure 7: Solution to 1.19.b

Problem 4

Problem 1.21.a (page 86).

1.21.a

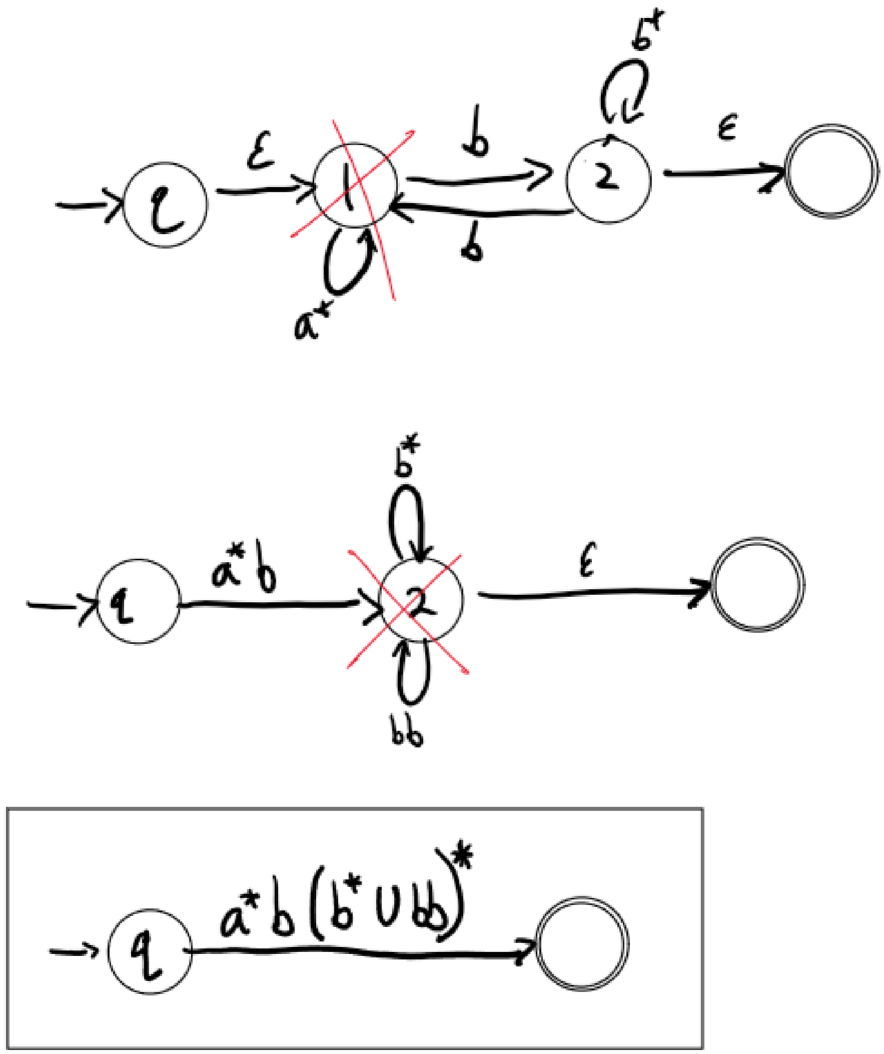

Figure 8: Solution to 1.21.a

Problem 5

Prove the following languages are not regular.

(5.1) $A = \{a^{n^3} | n \geq 0\}$. Here a^x means a string of x a's.

PROOF. Assume A is regular. Then let $S = a^{p^3}$ where p is the pumping length.

By the pumping lemma, S can be decomposed into xyz s.t.

(1) $xy^i z \in A$ for $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$

By (3), $|y| \leq p$. Pumping up, $|xy^2z| \leq p^3 + p < p^3 + 3p^2 + 3p + 1 = (p+1)^3$. By (2), $|y| > 0$ hence $p^3 < |xy^2z| < (p+1)^3$. Thus $xy^2z \notin A$, a contradiction of the pumping lemma. Therefore A is not regular.

$\square$

(5.2) $B = \{0^n 1^m 0^n | m, n \geq 0\}$.

PROOF. Assume B is regular. Then let $S = 0^p 1\, 0^p$ where p is the pumping length.

By the pumping lemma, S can be decomposed into xyz s.t.

(1) $xy^i z \in A$ for $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$

By (3), y must consist only of 0's. Let $\delta = |y|$ then by (2) $\delta > 0$. Then pumping up, $xy^2z = 0^{p+\delta} 1\, 0^p \notin B$ because there are not the same number of 0's before and after the 1. A contradiction of the pumping lemma, therefore B is not regular.

$\square$