

## CSCI 338: Assignment 3 (6 points)

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This assignment is due on **Tuesday, March 10, 11:30pm**. It is strongly encouraged that you use Latex to generate a single pdf file and upload it under *Assignment 3* on D2L. But there will NOT be a penalty for not using Latex (to finish the assignment). This is **not** a group-assignment, so you must finish the assignment by yourself.

### Problem 1

Design context-free grammars for the following languages

(1.1)  $A = \{a^n b^m \mid n \neq 2m\}$ .

- $S = aSb \mid A \mid aB \mid B$
- $A = aA \mid a$
- $B = bB \mid b$

(1.2)  $B = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$ .

- $S = XC \mid AY$
- $A = aA \mid \epsilon$
- $C = cC \mid \epsilon$
- $X = aXb \mid \epsilon$
- $Y = bXc \mid \epsilon$

(1.3)  $C = \{a^n b^m \mid n = 3m\}$ .

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- $S = aaaSb|\epsilon$

(1.4)  $D = \{a^n b^m | n \leq m + 3\}.$

- $S = aSb|Sb|A$
- $A = aaa|aa|a|\epsilon$

## Problem 2

Decide whether the following grammar is ambiguous.

$$S \rightarrow AB|aaB$$

$$A \rightarrow a|Aa$$

$$B \rightarrow b$$

Derivation 1	Derivation 2
S	S
AB	aaB
AaB	aab
aaB	
aab	

There are multiple ways to generate the string “aab” with this grammar, so **yes** the grammar is ambiguous.

### Problem 3

Convert the following CFG  $G$  to an equivalent PDA.

$$R \rightarrow XRX|S$$

$$S \rightarrow aTb|bTa$$

$$T \rightarrow XT X|X|\epsilon$$

$$X \rightarrow a|b$$

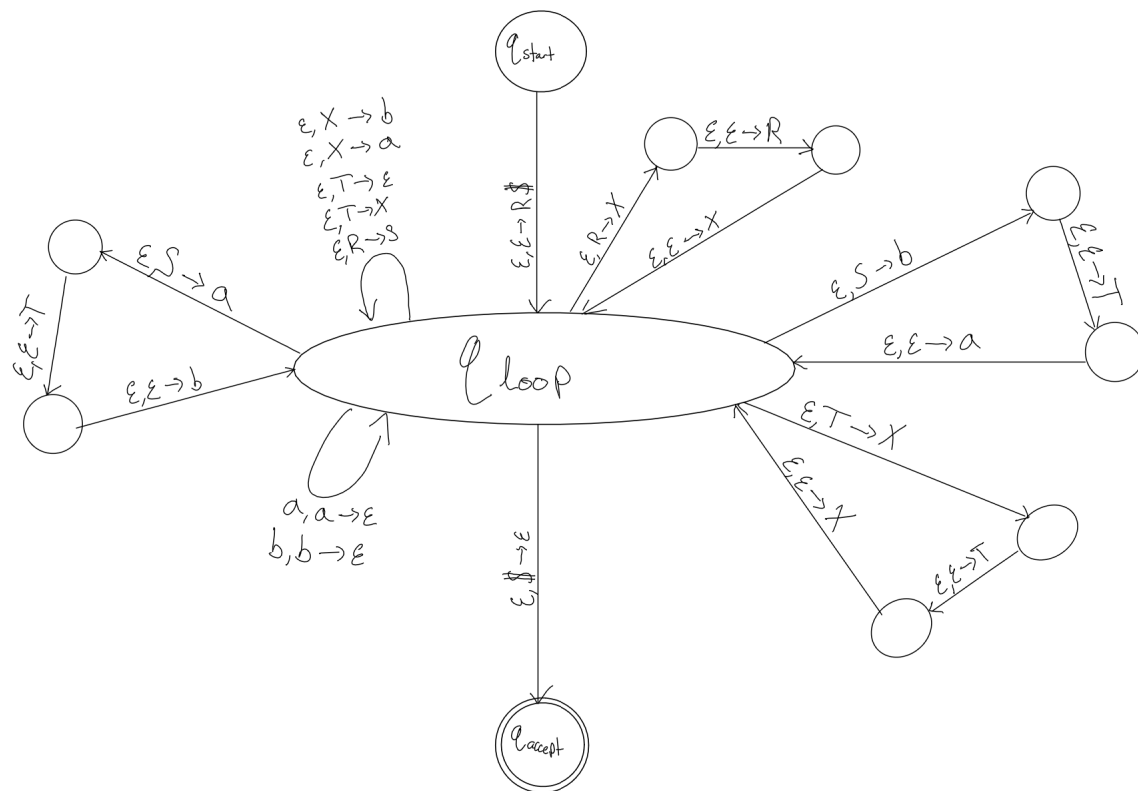


Figure 1: Solution to Problem 3

## Problem 4

Let  $G = (V, \Sigma, R, S)$  be the following grammar.  $V = \{S, T, U\}$ ;  $\Sigma = \{0, \#\}$ ; and  $R$  is the set of rules:

$$S \rightarrow TT|U$$

$$T \rightarrow 0T|T0|\#$$

$$U \rightarrow 0U00|\#$$

**(4.1) Describe  $L(G)$  in English.**

$L(G)$  has two  $\#$ 's within a list of zeros. Or  $L(G)$  a single  $\#$  one third of the way through a list of zeros with a multiple of three length. Ie. it has a list of  $n$  zeros, a single  $\#$  then a list of  $2n$  zeros.

**(4.2) Prove that  $L(G)$  is not regular.**

PROOF. By Contradiction.

Assume that  $L(G)$  is regular. Then we pick  $S = 0^p \# 0^{2p}$

By the pumping lemma,  $S$  can be decomposed into  $xyz$  s.t.

$$(1) \ xy^iz \in A \text{ for } i \geq 0$$

$$(2) \ |y| > 0$$

$$(3) \ |xy| \leq p$$

Let  $i = |y|$ , then by (2)  $i = |y| > 0$ , and by (3)  $|xy| \leq p$  so  $y$  must consist of only zeros. We pump up  $xy^2z = 0^{p+i} \# 0^{2p} \notin L$  because we increased the number of zeros before the  $\#$  without changing those after it, so  $2(p+i) \neq 2p$ .

A contradiction of the pumping lemma, so  $L$  must not be regular.

□

## Problem 5

Convert the following CFG into an equivalent CFG in Chomsky Normal Form

$$A \rightarrow BAB|B|\epsilon$$

$$B \rightarrow 00|\epsilon$$

$$S_0 \rightarrow BC|AB|BA|BB|XX$$

$$A \rightarrow BC|AB|BA|BB|XX$$

$$B \rightarrow XX$$

$$C \rightarrow AB$$

$$X \rightarrow 0$$

Start

$$A \rightarrow BAB|B|\epsilon$$

$$B \rightarrow 00|\epsilon$$


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add  $S_0$

①

$$S_0 \rightarrow A$$

$$A \rightarrow BAB|B|\epsilon$$

$$B \rightarrow 00|\epsilon$$


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Remove  $\epsilon$  rules

②

$$S_0 \rightarrow A$$

$$A \rightarrow BAB|AB|BA|A|B|\epsilon$$

$$B \rightarrow 00$$
  

$$A \rightarrow \epsilon$$

$$S_0 \rightarrow A|\epsilon$$

$$A \rightarrow BAB|AB|BA|BB|A|B$$

$$B \rightarrow 00$$
  

$$S_0 \rightarrow \epsilon$$

$$S_0 \rightarrow A$$

$$A \rightarrow BAB|AB|BA|BB|A|B$$

$$B \rightarrow 00$$


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remove single variable rules

③

$A \rightarrow A$	$A \rightarrow B$
$S_0 \rightarrow A$	$S_0 \rightarrow A$
$A \rightarrow BAB AB BA BB B$	$A \rightarrow BAB AB BA BB 00$
$B \rightarrow 00$	$B \rightarrow 00$

$$S_0 \rightarrow A$$

$$S_0 \rightarrow BAB|AB|BA|BB|00$$

$$A \rightarrow BAB|AB|BA|BB|00$$

$$B \rightarrow 00$$


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Add variables to convert to CNF

④

$$S_0 \rightarrow BC|AB|BA|BB|XX$$

$$A \rightarrow BC|AB|BA|BB|XX$$

$$B \rightarrow XX$$

$$C \rightarrow AB$$

$$X \rightarrow 0$$

Figure 2: Work for Problem 5

## Problem 6

Using pumping lemma to prove that the following languages are not context-free.

**(6.1)**  $L = \{a^n b^j c^k \mid k = nj\}.$

PROOF. By Contradiction

Assume that  $L$  is regular. Then pick  $S = a^p b^p c^{p^2}$  where  $p$  is the pumping length. Then by the pumping lemma  $S$  can be decomposed into  $S = uvxyz$  st.

- (1)  $uv^i xy^i z \in L$  for  $i \geq 0$
- (2)  $|vy| > 0$
- (3)  $|vxy| \leq p$

We examine three cases:

- 1.  $vxy$  contains  $b$ 's and  $c$ 's. Pumping up,  $uv^2 xy^2 z$  is of the form  $a^p b^{p+i} c^{p^2+j}$ . Then  $p(p+i) = p^2 + ip > p^2 + j$  because  $i > 0$  therefore  $j < p$  by (3). Therefore  $uv^2 xy^2 z \notin L$
- 2.  $vxy$  contains no  $c$ 's. Then, we pump up  $uv^2 xy^2 z \notin L$  by (2),  $|vy| > 0$  so there is more  $a$ 's or  $b$ 's without changing the number of  $c$ 's.
- 3.  $vxy$  contains only  $c$ 's. Then, we pump up  $uv^2 xy^2 z \notin L$  by (2),  $|vy| > 0$  so there is more  $c$ 's without changing the number of  $a$ 's or  $b$ 's.

Thus a contradiction in all cases, so  $L$  is not regular.

□

**(6.2)**  $L = \{a^n b^j \mid n \geq (j-1)^3\}.$

PROOF. By Contradiction

Assume that  $L$  is regular. Then pick  $S = a^{(p-1)^3} b^p$  where  $p$  is the pumping length. Then by the pumping lemma  $S$  can be decomposed into  $S = uvxyz$  st.

- (1)  $uv^i xy^i z \in L$  for  $i \geq 0$
- (2)  $|vy| > 0$

$$(3) \quad |vxy| \leq p$$

We examine three cases:

1.  $vxy$  consists only of  $a$ 's. We pump down so  $uv^0xy^0z = uxz$ . By (2)  $i = |vy| > 0$ , so  $uxz$  has the form  $a^{(p-1)^3-i}b^p$ , but  $(p-1)^3 - i < (p-1)^3$ . Therefore  $uxz \notin L$ .
2.  $vxy$  contains both  $a$ 's and  $b$ 's. We pump up,  $uv^2xy^2z$  is of the form  $a^{(p-1)^3+i}b^{p+j}$ . We know that  $j \neq 0$  so  $(p+j-1)^3 \geq p^3$ , and  $i < p$  by (3). Then  $(p-1)^3 + i < (p-1)^3 + 3p^2 - 3p + 1 = p^3$  for  $p > 1$ . Therefore  $(p-1)^3 + i < (p+j-1)^3$ . Therefore  $uxz \notin L$ .
3.  $vxy$  contains only  $b$ 's. We pump up,  $uv^2xy^2z$  is of the form  $a^{(p-1)^3}b^{p+i}$ . By (2),  $i = |vy| > 0$ , so we increase the number of  $b$ 's without altering the number of  $a$ 's. Then  $(p-1)^3 < (p+i-1)^3$ , therefore  $uxz \notin L$ .

Thus a contradiction in all cases, so  $L$  is not regular.

□