

CSCI 338: Assignment 3 (6 points)

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This assignment is due on **Tuesday, March 10, 11:30pm**. It is strongly encouraged that you use Latex to generate a single pdf file and upload it under *Assignment 3* on D2L. But there will NOT be a penalty for not using Latex (to finish the assignment). This is **not** a group-assignment, so you must finish the assignment by yourself.

Problem 1

Design context-free grammars for the following languages

(1.1) $A = \{a^n b^m \mid n \neq 2m\}$.

- $S = aaSb \mid A \mid B$
- $A = aA \mid a$
- $B = bB \mid b$

(1.2) $B = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$.

- $S = XC \mid AY$
- $A = aA \mid \epsilon$
- $C = cC \mid \epsilon$
- $X = aXb \mid \epsilon$
- $Y = bXc \mid \epsilon$

(1.3) $C = \{a^n b^m \mid n = 3m\}$.

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- $S = aaaSb|\epsilon$

(1.4) $D = \{a^n b^m | n \leq m + 3\}.$

- $S = aSb|Sb|A$
- $A = aaa|aa|a|\epsilon$

Problem 2

Decide whether the following grammar is ambiguous.

$$S \rightarrow AB|aaB$$

$$A \rightarrow a|Aa$$

$$B \rightarrow b$$

| | |
|-----|-----|
| S | S |
| AB | aaB |
| AaB | aaB |
| aaB | aaB |
| aab | aab |

There are multiple ways to generate the string “aab” with this grammar, so **yes** the grammar is ambiguous.

Problem 3

Convert the following CFG G to an equivalent PDA.

$$R \rightarrow XRX|S$$

$$S \rightarrow aTb|bTa$$

$$T \rightarrow XT X|X|\epsilon$$

$$X \rightarrow a|b$$

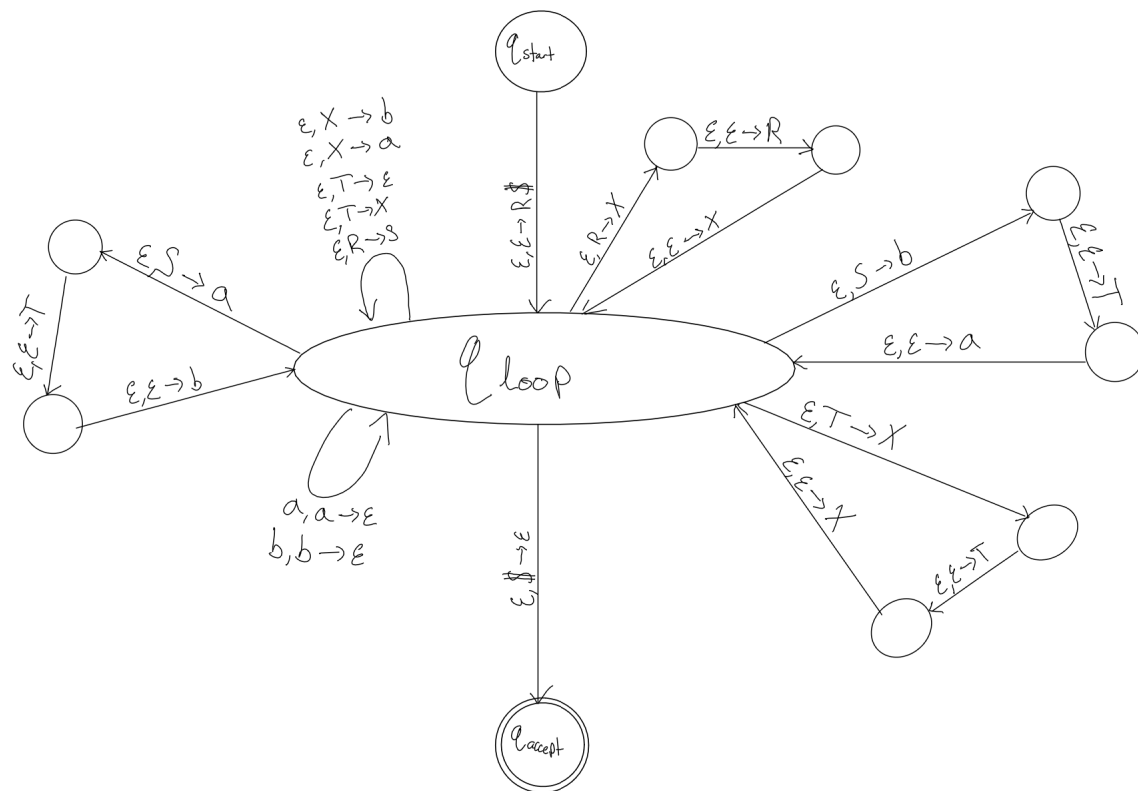


Figure 1: Solution to Problem 3

Problem 4

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

$$S \rightarrow TT|U$$

$$T \rightarrow 0T|T0|\#$$

$$U \rightarrow 0U00|\#$$

(4.1) Describe $L(G)$ in English.

$L(G)$ has two $\#$'s within a list of an even number of zeros. Or $L(G)$ a single $\#$ one third of the way through a list of zeros with a multiple of three length.

(4.2) Prove that $L(G)$ is not regular.

Problem 5

Convert the following CFG into an equivalent CFG in Chomsky Normal Form

$$A \rightarrow BAB|B|\epsilon$$

$$B \rightarrow 00|\epsilon$$

$$S_0 \rightarrow BD|AB|BA|CC$$

$$A \rightarrow BD|AB|BA|CC$$

$$B \rightarrow CC$$

$$C \rightarrow 0$$

$$D \rightarrow AB$$

Problem 6

Using pumping lemma to prove that the following languages are not context-free.

$$(6.1) \ L = \{a^n b^j c^k \mid k = nj\}.$$

$$(6.2) \ L = \{a^n b^j \mid n \geq (j - 1)^3\}.$$