

## CSCI 338: Assignment 3 (6 points)

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This assignment is due on **Tuesday, March 10, 11:30pm**. It is strongly encouraged that you use Latex to generate a single pdf file and upload it under *Assignment 3* on D2L. But there will NOT be a penalty for not using Latex (to finish the assignment). This is **not** a group-assignment, so you must finish the assignment by yourself.

### Problem 1

Design context-free grammars for the following languages

(1.1)  $A = \{a^n b^m \mid n \neq 2m\}$ .

- $S = aaSb \mid A \mid B$
- $A = aA \mid a$
- $B = bB \mid b$

(1.2)  $B = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$ .

- $S = XC \mid AY$
- $A = aA \mid \epsilon$
- $C = cC \mid \epsilon$
- $X = aXb \mid \epsilon$
- $Y = bXc \mid \epsilon$

(1.3)  $C = \{a^n b^m \mid n = 3m\}$ .

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- $S = aaaSb|\epsilon$

(1.4)  $D = \{a^n b^m | n \leq m + 3\}.$

- $S = aSb|Sb|A$
- $A = aaa|aa|a|\epsilon$

## Problem 2

Decide whether the following grammar is ambiguous.

$$S \rightarrow AB|aaB$$

$$A \rightarrow a|Aa$$

$$B \rightarrow b$$

S	S
AB	aaB
AaB	aaB
aaB	aaB
aab	aab

There are multiple ways to generate the string “aab” with this grammar, so **yes** the grammar is ambiguous.

### Problem 3

Convert the following CFG  $G$  to an equivalent PDA.

$$R \rightarrow XRX|S$$

$$S \rightarrow aTb|bTa$$

$$T \rightarrow XT X|X|\epsilon$$

$$X \rightarrow a|b$$

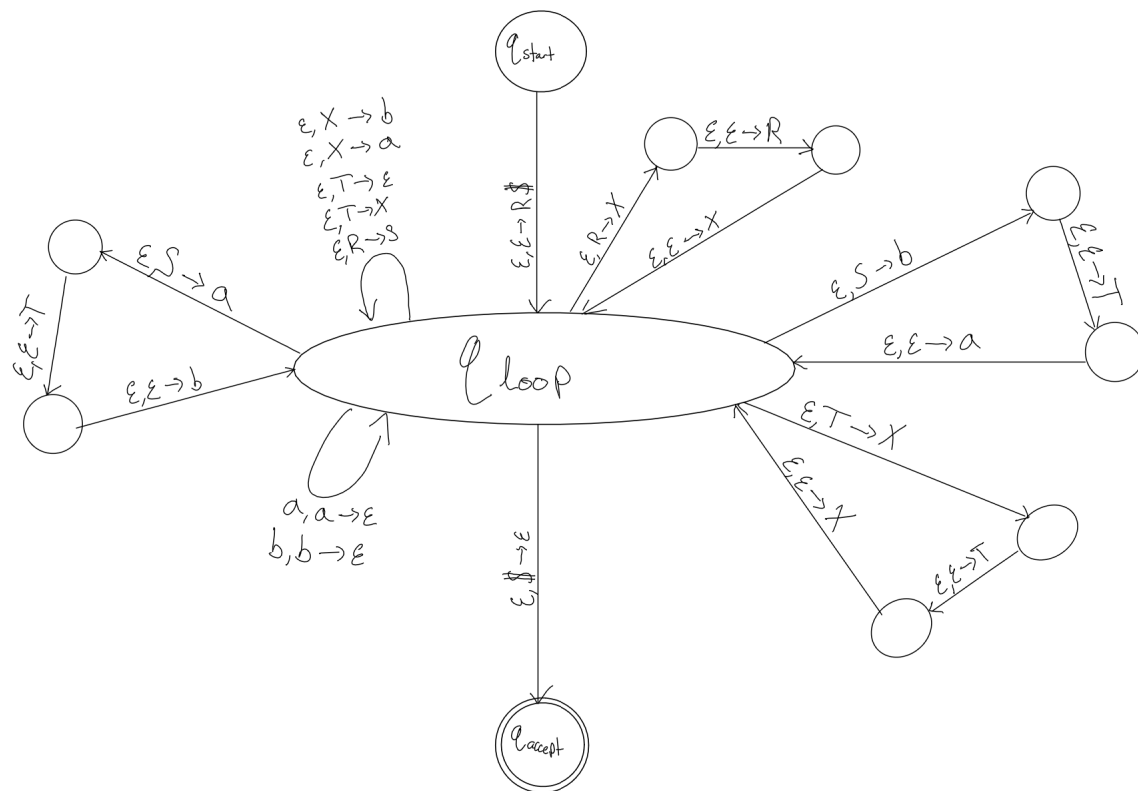


Figure 1: Solution to Problem 3

**Problem 4**

Let  $G = (V, \Sigma, R, S)$  be the following grammar.  $V = \{S, T, U\}$ ;  $\Sigma = \{0, \#\}$ ; and  $R$  is the set of rules:

$$S \rightarrow TT|U$$

$$T \rightarrow 0T|T0|\#$$

$$U \rightarrow 0U00|\#$$

(4.1) Describe  $L(G)$  in English.

$L(G)$  has two  $\#$ 's within a list of zeros. Or  $L(G)$  a single  $\#$  one third of the way through a list of zeros with a multiple of three length.

(4.2) Prove that  $L(G)$  is not regular.

PROOF. By Contradiction.

Assume that  $L(G)$  is regular. Then we pick  $S =$

□

**Problem 5**

Convert the following CFG into an equivalent CFG in Chomsky Normal Form

$$A \rightarrow BAB|B|\epsilon$$

$$B \rightarrow 00|\epsilon$$

$$S_0 \rightarrow BD|AB|BA|CC$$

$$A \rightarrow BD|AB|BA|CC$$

$$B \rightarrow CC$$

$$C \rightarrow 0$$

$$D \rightarrow AB$$

## Problem 6

Using pumping lemma to prove that the following languages are not context-free.

**(6.1)**  $L = \{a^n b^j c^k \mid k = nj\}.$

PROOF. By Contradiction

Assume that  $L$  is regular. Then pick  $S = a^p b^p c^{p^2}$  where  $p$  is the pumping length. Then by the pumping lemma  $S$  can be decomposed into  $S = uvxyz$  st.

- (1)  $uv^i xy^i z \in L$  for  $i \geq 0$
- (2)  $|vy| > 0$
- (3)  $|vxy| \leq p$

We examine three cases:

- 1.  $vxy$  contains  $b$ 's and  $c$ 's. Pumping up,  $uv^2xy^2z$  is of the form  $a^p b^{p+i} c^{p^2+j}$ . Then  $p(p+i) = p^2 + ip > p^2 + j$  because  $i > 0$  therefore  $j < p$  by (3). Therefore  $uv^2xy^2z \notin L$
- 2.  $vxy$  contains no  $c$ 's. Then, we pump up  $uv^2xy^2z \notin L$  by (2),  $|vy| > 0$  so there is more  $a$ 's or  $b$ 's without changing the number of  $c$ 's.
- 3.  $vxy$  contains only  $c$ 's. Then, we pump up  $uv^2xy^2z \notin L$  by (2),  $|vy| > 0$  so there is more  $c$ 's without changing the number of  $a$ 's or  $b$ 's.

Thus a contradiction in all cases, so  $L$  is not regular.

□

**(6.2)**  $L = \{a^n b^j \mid n \geq (j-1)^3\}.$

PROOF. By Contradiction

Assume that  $L$  is regular. Then pick  $S = a^{(p-1)^3} b^p$  where  $p$  is the pumping length. Then by the pumping lemma  $S$  can be decomposed into  $S = uvxyz$  st.

- (1)  $uv^i xy^i z \in L$  for  $i \geq 0$
- (2)  $|vy| > 0$

$$(3) \quad |vxy| \leq p$$

We examine three cases:

1.  $vxy$  consists only of  $a$ 's. We pump down so  $uv^0xy^0z = uxz$ . By (2)  $i = |vy| > 0$ , so  $uxz$  has the form  $a^{(p-1)^3-i}b^p$ , but  $(p-1)^3 - i < (p-1)^3$ . Therefore  $uxz \notin L$ .
2.  $vxy$  contains both  $a$ 's and  $b$ 's. We pump up,  $uv^2xy^2z$  is of the form  $a^{(p-1)^3+i}b^{p+j}$ . We know that  $j \neq 0$  so  $(p+j-1)^3 \leq p^3$ , and  $i < p$  by (3). Then  $(p-1)^3 + i < (p-1)^3 + 3p^2 - 3p + 1 = p^3$  for  $p > 1$ . Therefore  $(p-1)^3 + i < (p+j-1)^3$ . Therefore  $uxz \notin L$ .
3.  $vxy$  contains only  $b$ 's. We pump up,  $uv^2xy^2z$  is of the form  $a^{(p-1)^3}b^{p+i}$ . By (2),  $i = |vy| > 0$ , so we increase the number of  $b$ 's without altering the number of  $a$ 's. Then  $(p-1)^3 < (p+i-1)^3$ , therefore  $uxz \notin L$ .

Thus a contradiction in all cases, so  $L$  is not regular.

□