CSCI 338: Assignment 3 (6 points)

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This assignment is due on **Tuesday, March 10, 11:30pm**. It is strongly encouraged that you use Latex to generate a single pdf file and upload it under *Assignment 3* on D2L. But there will NOT be a penalty for not using Latex (to finish the assignment). This is **not** a group-assignment, so you must finish the assignment by yourself.

Problem 1

Design context-free grammars for the following languages

(1.1) $A = \{a^n b^m | n \neq 2m\}$.

- $S = aaSb|A|aB|B$
- $A = aA|a$
- $B = bB|b$

(1.2) $B = \{a^i b^j c^k | i, j, k \geq 0$ and either $i = j$ or $j = k\}$.

- $S = XC|AY$
- $A = aA|\epsilon$
- $C = cC|\epsilon$
- $X = aXb|\epsilon$
- $Y = bXc|\epsilon$

(1.3) $C = \{a^n b^m | n = 3m\}$.

- $S = aaaSb|\epsilon$

(1.4) $D = \{a^n b^m | n \leq m + 3\}$.

- $S = aSb|Sb|A$
- $A = aaa|aa|a|\epsilon$

Problem 2

Decide whether the following grammar is ambiguous.

$S \rightarrow AB|aaB$

$A \rightarrow a|Aa$

$B \rightarrow b$

Derivation 1	Derivation 2
S	S
AB	aaB
AaB	aab
aaB	
aab	

There are multiple ways to generate the string "aab" with this grammar, so **yes** the grammar is ambiguous.

Problem 3

Convert the following CFG G to an equivalent PDA.

$R \to XRX|S$

$S \to aTb|bTa$

$T \to XTX|X|\epsilon$

$X \to a|b$

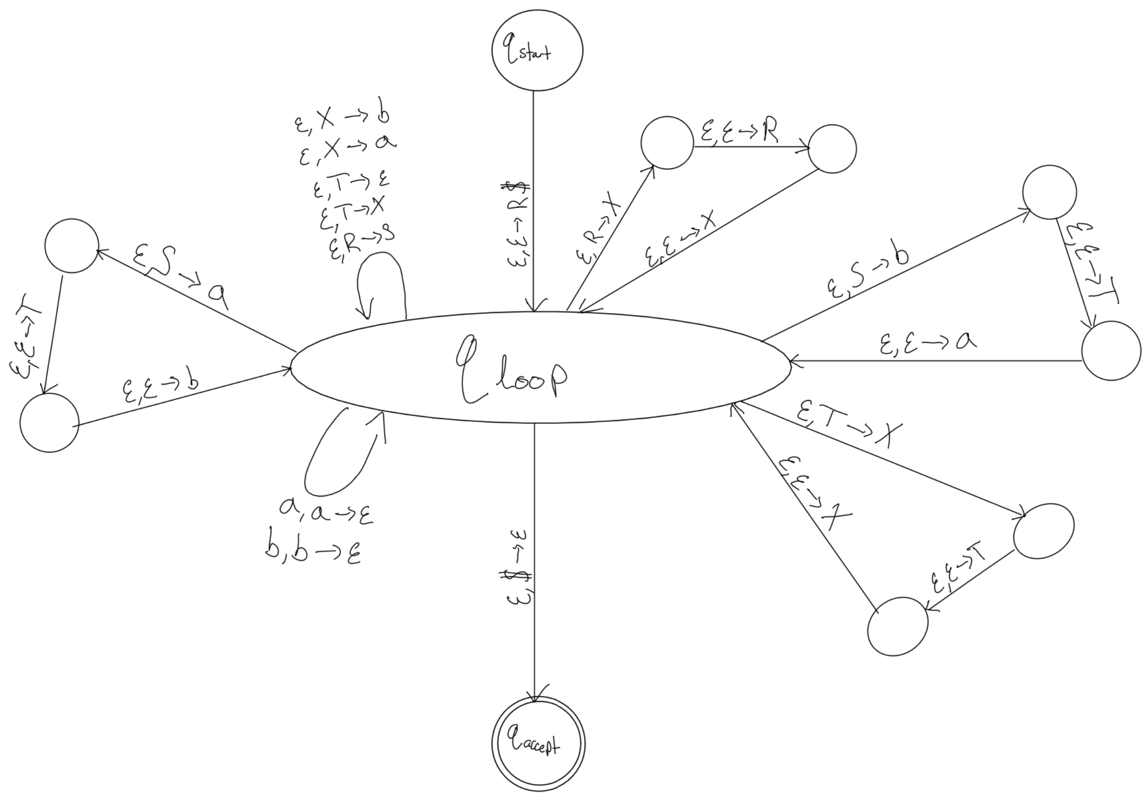

Figure 1: Solution to Problem 3

Problem 4

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

$S \to TT|U$

$T \to 0T|T0|\#$

$U \to 0U00|\#$

(4.1) Describe $L(G)$ in English.

$L(G)$ has two #'s within a list of zeros. Or L(G) a single # one third of the way through a list of zeros with a multiple of three length. Ie. it has a list of n zeros, a single # then a list of $2n$ zeros.

(4.2) Prove that $L(G)$ is not regular.

Proof. By Contradiction.

Assume that $L(G)$ is regular. Then we pick $S = 0^p\#0^{2p}$

By the pumping lemma, S can be decomposed into xyz s.t.

(1) $xy^iz \in A$ for $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$

Let $i = |y|$, then by (2) $i = |y| > 0$, and by (3) $|xy| \leq p$ so y must consist of only zeros. We pump up $xy^2z = 0^{p+i}\#0^{2p} \notin L$ because we increased the number of zeros before the # without changing those after it, so $2(p + i) \neq 2p$.

A contradiction of the pumping lemma, so L must not be regular.

$\square$

Problem 5

Convert the following CFG into an equivalent CFG in Chomsky Normal Form

$A \to BAB|B|\epsilon$

$B \to 00|\epsilon$

$S_0 \to BC|AB|BA|BB|XX$
$A \to BC|AB|BA|BB|XX$
$B \to XX$
$C \to AB$
$X \to 0$

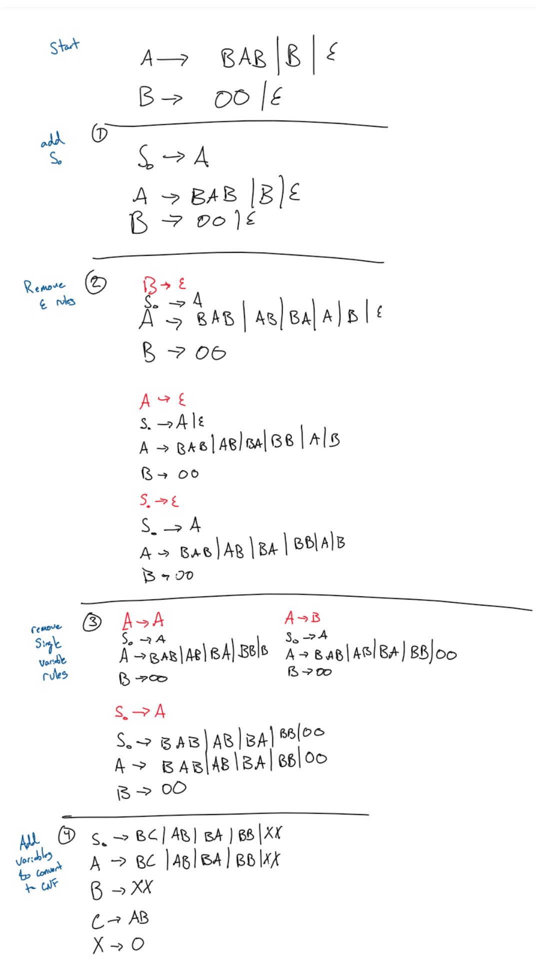

Figure 2: Work for Problem 5

Problem 6

Using pumping lemma to prove that the following languages are not context-free.

(6.1) $L = \{a^n b^j c^k | k = nj\}$.

Proof. By Contradiction

Assume that L is regular. Then pick $S = a^p b^p c^{p^2}$ where p is the pumping length. Then by the pumping lemma S can be decomposed into $S = uvxyz$ st.

(1) $uv^i xy^i z \in L$ for $i \geq 0$

(2) $|vy| > 0$

(3) $|vxy| \leq p$

We examine three cases:

1. vxy contains b's and c's. Pumping up, uv^2xy^2z is of the form $a^p b^{p+i} c^{p^2+j}$. Then $p(p+i) = p^2 + ip > p^2 + j$ because $i > 0$ therefore $j < p$ by (3). Therefore $uv^2xy^2z \notin L$
2. vxy contains no c's. Then, we pump up $uv^2xy^2z \notin L$ by (2), $|vy| > 0$ so there is more a's or b's without changing the number of c's.
3. vxy contains only c's. Then, we pump up $uv^2xy^2z \notin L$ by (2), $|vy| > 0$ so there is more c's without changing the number of a's or b's.

Thus a contradiction in all cases, so L is not regular.

□

(6.2) $L = \{a^n b^j | n \geq (j-1)^3\}$.

Proof. By Contradiction

Assume that L is regular. Then pick $S = a^{(p-1)^3} b^p$ where p is the pumping length. Then by the pumping lemma S can be decomposed into $S = uvxyz$ st.

(1) $uv^i xy^i z \in L$ for $i \geq 0$

(2) $|vy| > 0$

(3) $|vxy| \leq p$

We examine three cases:

1. vxy consists only of a's. We pump down so $uv^0xy^0z = uxz$. By (2) $i = |vy| > 0$, so uxz has the form $a^{(p-1)^3-i}b^p$, but $(p-1)^3 - i < (p-1)^3$. Therefore $uxz \notin L$.
2. vxy contains both a's and b's. We pump up, uv^2xy^2z is of the form $a^{(p-1)^3+i}b^{p+j}$. We know that $j \neq 0$ so $(p+j-1)^3 \leq p^3$, and $i < p$ by (3). Then $(p-1)^3 + i < (p-1)^3 + 3p^2 - 3p + 1 = p^3$ for $p > 1$. Therefore $(p-1)^3 + i < (p+j-1)^3$. Therefore $uxz \notin L$
3. vxy contains only b's. We pump up, uv^2xy^2z is of the form $a^{(p-1)^3}b^{p+i}$. By (2), $i = |vy| > 0$, so we increase the number of b's without altering the number of a's. Then $(p-1)^3 < (p+i-1)^3$, therefore $uxz \notin L$.

Thus a contradiction in all cases, so L is not regular.

$\square$