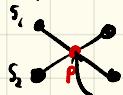


# Line Seg Intersections

Line segment intersection problem:

Given a set  $S$  of  $n$  line segments in the plane  
output all of the points where segments intersect



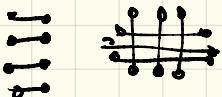
$$\text{rep } p = (p_x, p_y)$$

or

$(s_1, s_2) \leftarrow$  we prefer this rep

# of seg intersections:

$$O - O(n^2)$$



Let's try an output sensitive algo

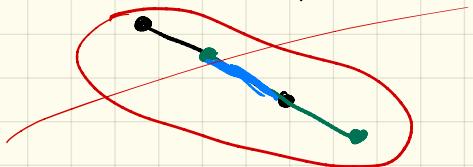
$$I(S) = I \# \text{ of intersections in } S$$

General position assumptions:

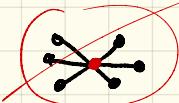
- x-coord of endpoints & intersections are distinct



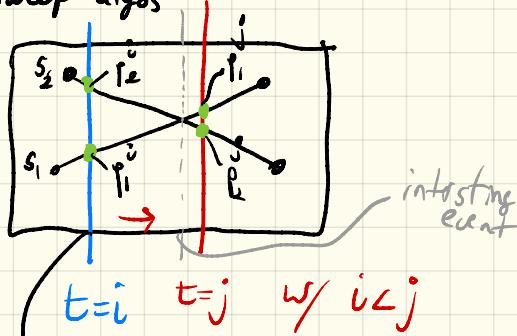
- if segments intersect they do so at a single point



- no 3 segments intersect at a common point
- no vertical segments



## Plane sweep algs

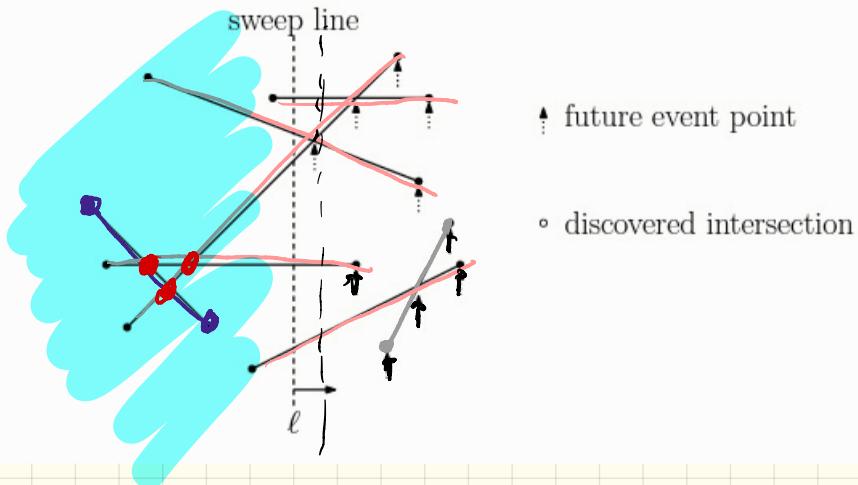


want to keep sweepline states

event points - discrete events where interesting things happen (or may happen)

3 parts of Sweepline algos:

1. partial solution to the problem "left" of sweepline
2. current status of objects on the sweepline
3. events to process in the future



## Sweepline States

- sweep from left to right ( $x$ -coord - a t coord)
- an ordered list of line segments intersecting  $l$  (sorted top to bottom)

Ops for the datastructure

- search (sort) [find intersection]
- insert (sort) [start segment]
- delete (sort) [end segment]



Ordered dictionary - (balanced binary search tree  
 $\hookrightarrow$  red/black tree)  
 - insert in  $O(\log n)$   
 - delete in  $O(\log n)$   
 - search in  $O(\log n)$

What we need is

given two segments  $s_1, s_2$   
 intersecting vertical line  $l$   
 is  $s_1$  above  $s_2$  on  $l$

$$y_2(x) = a_2x + b_2 \quad (a_2, b_2)$$

$$y_1(x) = a_1x + b_1 \quad (a_1, b_1)$$

## Events:

1. left endpoints of segments
  2. right endpoints of segments
  3. intersections of segments
- } we know  
Seg Endpoints  
before  
we start
- } Sort  
by x-coord  
before  
Starting sweep line
- } find during  
the sweepline

How to find type 3 events  
before they occur

- For two lines adj on the sweepline  
check for an intersection  
if yes we found a type 3 event  
create an event w/ x-coord as event time

Does this find all intersections

Can we have 2 segs intersect w/o being adj. on the sweepline?