CSCI 534
Homework 05
Due 3-25-2021
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Problem 1 (10 points)

In Homework 1, we considered a plane-sweep algorithm for determining whether there is any intersection among a collection of n circles in the plane. Here we consider a variant of this problem. The input consists of a collection of n closed circular disks, all having the same radius. (Via scaling, we may assume that they are all unit disks.) Let $C = \{c_1, \ldots, c_n\}$ denote the center points of these disks, and let $\{D_1, \ldots, D_n\}$ denote the actual disks. Thus, D_i consists of the points that lie within unit distance of c_i. Let $U = D_1 \cup \ldots \cup D_n$ denote the union of these disks. The boundary of U may generally consist of multiple parts, each of which consists of a cycle of circular arcs connected by vertices. (In Fig. 4 the boundary consists of three cycles. The vertices are shown as white dots).

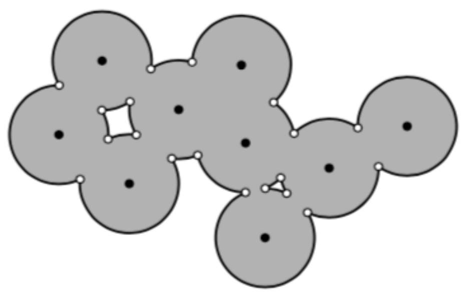

Figure 1: Problem 4: Union of disks

1. Present an algorithm that reports all the vertices on the boundary of U. (Note that circle intersection points in the interior of the union are explicitly excluded.) Your algorithm should run in time $O(nlogn)$. The order in which the vertices are output is arbitrary. (Hint: Don't try to modify the algorithm from Homework 2. A different approach is needed.... think giraffes)
2. Prove that the number of vertices reported by your algorithm is $O(n)$.

Problem 2 (5 points)

Suppose we are given a subdivision of the plane into n convex regions. We suspect that this subdivision is a Voronoi diagram, but we do not know the sites. Develop an algorithm that finds a set of n point sites whose Voronoi diagram is exactly the given subdivision, if such a set exists.

Tips and Acknowledgements

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