Examples of Arrangements

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Sweep an arrangement
- size for n lines $\Rightarrow O(n^2)$ verts, edges or faces
- Start sweepline at $x = -\infty$
  - start w/ lines ordered by slope
  - events are verts of the arrangement
  - sweepline status:
    * ordered by lines intersection w/ sweepline
      - size is always n
      - no need for searching
      - swap lines at intersections

      $\rightarrow$ an array of size n

How long for event processing?
(1) linear in n
(2) linear in # of intersection n^2
(3) $\boxed{O(n^2 \lg n)}$
(4) $O(n^3)$

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Sweep takes $O(n^2 \lg n)$ and $O(n)$ space

Topological sweep paper (Edels & Guibas '89) $O(n^2)$-time and $O(n)$ space

General position test: Given a set of n points in the plane, determine whether any three are collinear.

Minimum area triangle: Given a set of n points in the plane, determine the minimum area triangle whose vertices are selected from these points.

Minimum k-corridor: Given a set of n points, and an integer k, determine the narrowest pair of parallel lines that enclose at least k points of the set. The distance between the lines can be defined either as the vertical distance between the lines or the perpendicular distance between the lines (see Fig. 72(a)).

Visibility graph: Given line segments in the plane, we say that two points are *visible* if the interior of the line segment joining them intersects none of the segments. Given a set of n non-intersecting line segments, compute the *visibility graph*, whose vertices are the endpoints of the segments, and whose edges a pairs of visible endpoints (see Fig. 72(b)).

Maximum stabbing line: Given a set of n line segments in the plane, compute the line ℓ that stabs (intersects) the maximum number of these line segments (see Fig. 72(c)).

Ham Sandwich Cut: Given n red points and m blue points, find a single line ℓ that simultaneously bisects these point sets. It is a famous fact from mathematics, called the *Ham-Sandwich Theorem*, that such a line always exists. If the two point sets are separable by a line (that is, the red convex hull and the blue convex hull do not intersect), then this can be solved in time $O(n + m)$ (see Fig. 72(d)).

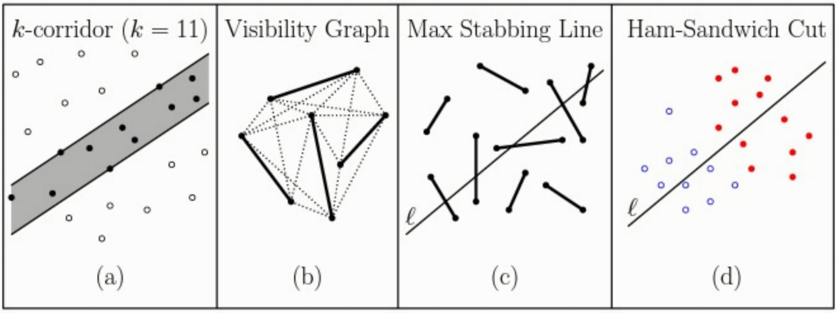
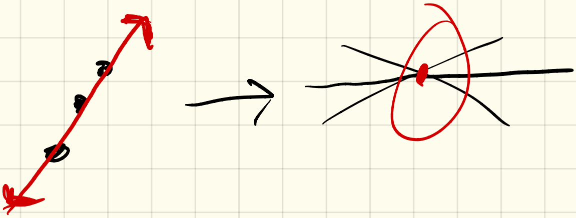


Fig. 72: Applications of arrangements.

Colinear



Half plane discrepancy

given n points P in $[0,1]^2 = \mathcal{U}$

unit square

want

for any halfplane h

def $\mu(h)$ as the area of $h \cap \mathcal{U}$

$$\mu_P(h) = \frac{|P \cap h|}{|P|}$$ ← proportion of points from P in h

want to find

$$\mu(h) \simeq \mu_P(h)$$

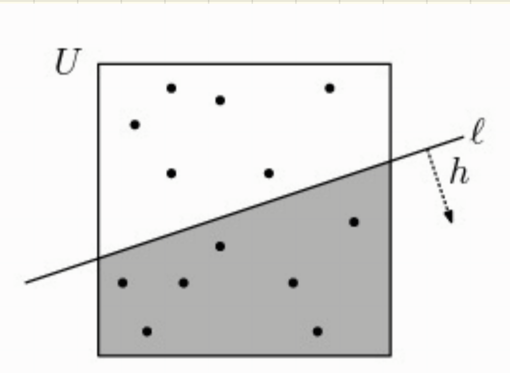
discrepancy of P w.r.t. h is

$$\Delta_P(h) = |\mu(h) - \mu_P(h)|$$

halfplane discrepancy of P:

$$\Delta(P) = \sup_h \Delta_P(h)$$

Eg



$\mu(h) = .625$ (area of shaded region)

$$\mu_P(h) = \frac{|P \cap h|}{|P|} = \frac{7}{13} = .538$$

$$\Delta_P(h) = |\mu(h) - \mu_P(h)|$$

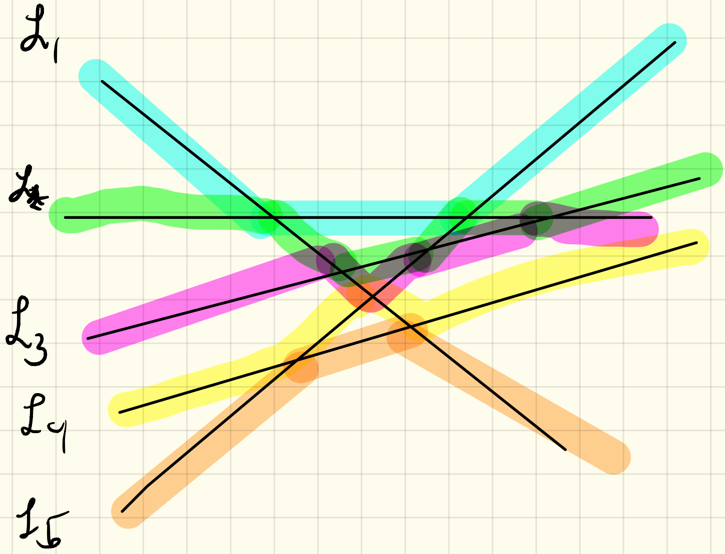
$$= |.625 - .538| = .087$$

how to compute?

Lemma: Let h be halfplane w/ max discrepancy w.r.t P
let ℓ be the line bounding h
either:
(type i) ℓ passes through one point of P and it is midpoint of linesegment $\ell \cap U$
(type ii) ℓ passes through at least 2 points of P

(type i) it can be shown that
there are a const # of lines through p that are midpoint of line segment $\ell \cap U$
$\Rightarrow O(n)$ segs to consider for type i
w/ brute force compute disc for each seg in $O(n)$ time
$\Rightarrow O(n^2)$ time for all type i segs

Type 2
Compute disc for h w/ boundry line ℓ



def level of arrangement

a point at level k if at most k-1 lines are above n-k lines below and denote as $\mathcal{L}_k$

- compute level at each vertex w/ plane sweep
  ⇒ discrepancy at each line in $O(n^2)$ time

narrowest k-corridor

Given P points in the plane and int k
find narrowest pair of ∥ lines enclosing k-points

k = 3

GP assump:
- no 2 pts have same x-coord
- no 3 pts collinear