CSCI 534
Homework 04
Due 3-18-2019
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Problem 1 (5 points)

Draw a polygon with at least 4 vertices and give each vertex coordinates. Using the duality discussed in class, draw the dual of the polygon. In the dual,

- shade black the duals of the points that are on the vertices in the primal
- shade grey the duals of the points that are on the edges in the primal
- shade red the duals of the points that are inside the polygon in the primal

You should draw the primal and dual to scale. You may draw the figure by hand, but, I suggest using a tool like Ipe or Inkscape.

Problem 2 (20 points)

Explain how to solve each of the following problems in linear (expected) time. Each can be modeled as a linear programming (LP) problem, perhaps with some additional pre- and/or post- processing. In each case, explain how the problem is converted into an LP instance and how the answer to the LP instance is used/interpreted to solve the stated problem.

1. You are given two point sets $P = \{p_1, \ldots, p_n\}$ and $Q = \{q_1, \ldots, q_m\}$ in the plane, and you are told that they are separated by a vertical line $x = x_0$, with P to the left and Q to the right (see Fig. a). Compute the line equations of the two "crossing tangents," that is, the lines ℓ_1 and ℓ_2 that are both supporting lines for conv(P) and conv(Q) such that P lies below ℓ_1 and above ℓ_2 and the reverse holds for Q. (Note that you are not given the hulls, just the point sets.) Your algorithm should run in time $O(n + m)$.

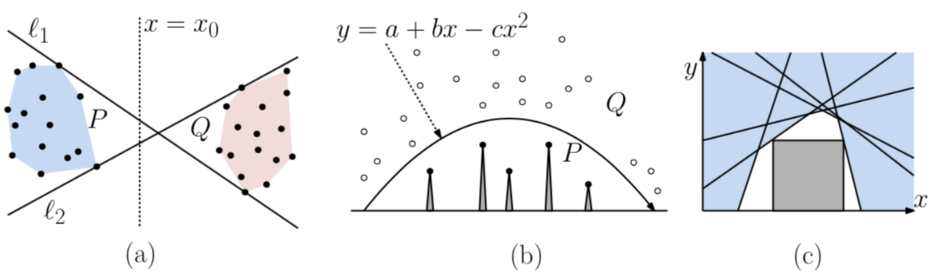

Figure 1: Problem 3: LP applications

2. You have a cannon in $\mathbb{R}^2$. It has three controls labeled "a" "b," and "c". A projectile shot from this cannon travels along the parabolic arc $y = a + bx - cx^2$. You are asked to determine

whether it is possible to adjust the controls so that the projectile travels above a set of n building tops, represented by a point set $P = \{p_1, \ldots, p_n\}$ and beneath a set of m floating balloons, represented by a point set $Q = \{q_1, \ldots, q_m\}$ (see Fig. b)). Your algorithm should run in time $O(n + m)$. (I do not care where the cannon is actually located. If your solution is based on some assumption about the cannon's location, please state this.)

3. You are given a set of n halfplanes $H = \{h_1, \ldots, h_n\}$, where h_i is given as a pair (a_i, b_i) and it consists of all the points of the plane that lie on or beneath the line $y = a_i x + b_i$. Compute the axis-parallel square of the largest side length whose lower edge lies on the x-axis (see Fig. c). If no such square exists, your algorithm should indicate this.

Tips and Acknowledgements

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