1 Dynamics

Generally: $\dot{x} = f(x,u), y = h(x,u),\ x \in \mathbb{R}^n, u \in \mathbb{R}^m$.
Jacobian Linearization: given $u = u_0, x = x_0$ is a stationary point $(f(x_0, u_0) = 0)$, then $\dot{x} \approx J_{f,x}x + J_{f,u}u, y \approx J_{h,x}x + J_{h,u}u$

2 Matrices

Linear dependence $\exists \alpha_1, \dots \alpha_m$ not all zero, $\alpha_1 x_1 + \cdots + \alpha_m x_m = 0$.
Similarity: B is similar to A if $\exists Q, st. B = QAQ^{-1}$.

Cayley-Hamilton: let $\Delta(\lambda) = det(A - \lambda I)$, then $\Delta(A) = 0$.
For any $f(A)$: $f(A) = \beta_0 I + \cdots + \beta_{n-1}A^{n-1}$. If f polynomial, let $h(\lambda) = \beta_0 + \beta_1\lambda + \cdots + \beta_{n-1}\lambda^{n-1}$, solved from $f^{(l)}(\lambda_i) = h^{(l)}(\lambda_i)$.

Matrix exponential: $e^{At} = \sum_{k=0}^{\infty} \frac{1}{k!} t^k A^k$
$e^{A(t_1+t_2)} = e^{At_1}e^{At_2}$, $(e^{At})^{-1} = e^{-At}$, $\frac{d}{dt}e^{At} = e^{At}A = Ae^{At}$, $e^{At} = \mathcal{L}^{-1}((sI - A)^{-1})$

Compute A^{100} with $A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$

$\Rightarrow f(\lambda) = \lambda^{100}$

$\Delta(\lambda) = \lambda^2 + 2\lambda + 1 = (\lambda+1)^2 \Rightarrow \lambda_1 = -1,\ n_1 = 2$

Let $h(\lambda) = \beta_0 + \beta_1\lambda$

$\Rightarrow \begin{cases} f(-1) = h(-1): & (-1)^{100} = \beta_0 - \beta_1 \\ f'(-1) = h'(-1): & 100 \times (-1)^{99} = \beta_1 \end{cases} \Rightarrow \begin{cases} 1 = \beta_0 - \beta_1 \\ -100 = \beta_1 \end{cases}$

$\Rightarrow \begin{cases} \beta_0 = -99 \\ \beta_1 = -100 \end{cases} \Rightarrow h(\lambda) = -99 - 100\lambda$

$\Rightarrow A^{100} = h(A) = -99I - 100A = \begin{pmatrix} -199 & -100 \\ 100 & 101 \end{pmatrix}$

3 Other

$rank(A) = \#$ of independent columns $=$ size of largest-square submatrix with nonzero det. $rank(A) + nullity(A) = \#$ of columns.

Capacitor - $i = C\frac{dv}{dt}$, Inductor: $v = L\frac{di}{dt}$, Friction $F = -kv$