

Project

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Problem 1**Statement:**

Consider the system

$$G(s) = \frac{50(s+3)}{s(s+2)(s+5)}$$

- Use Bode plot to design a lead compensator $C(s)$ with unity DC gain so that the phase margin of the closed-loop system is no less than 40° .
- Use MATLAB to verify the resulting phase marginally
- What is the bandwidth of the resulting closed-loop system?

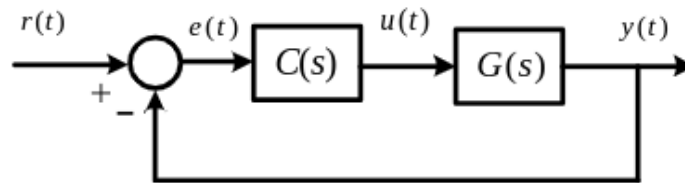
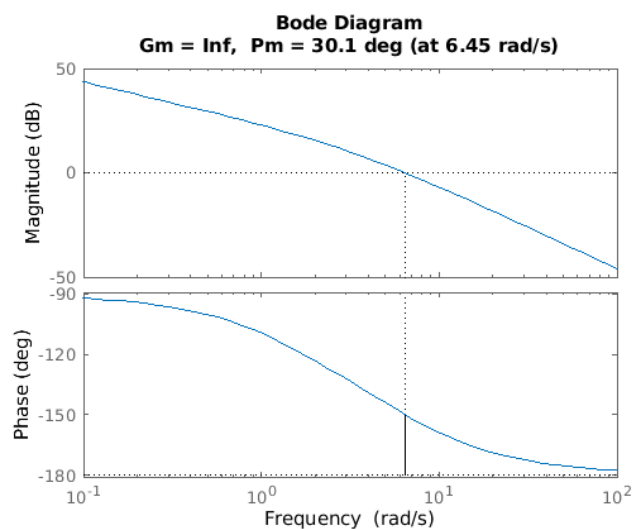


Figure 1: Problem1

Solution

- Unity DC gain means $K = 1$. We see that phase margin is 30° . So we need to add at

Figure 2: Problem 1: Bode plot of $G(s)$

least 10° of phase margin. Choose $\alpha = 1/3$ so $\phi_{max} = 30^\circ$ which is less than 60° . We want $\omega_{max} = \omega_{gc} = 6.45$, so then $T = \frac{1}{\omega_{max}\sqrt{\alpha}} = 0.2684$

So finally $C(s) = \frac{Ts+1}{\alpha Ts+1} = \frac{0.2684s+1}{0.0895s+1}$

b) See figure 3

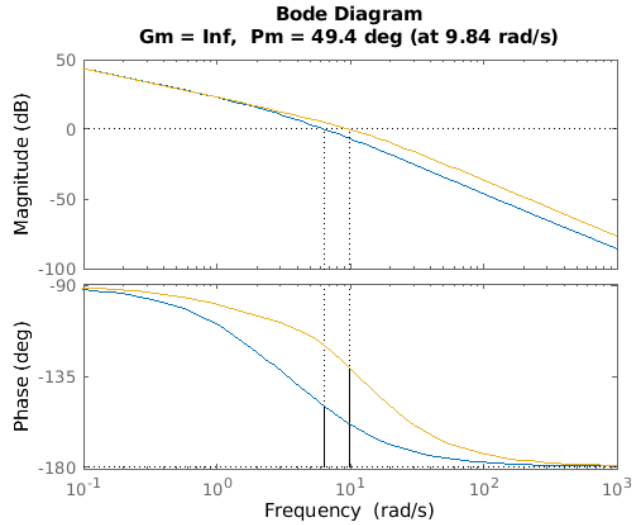


Figure 3: Problem 1: Bode plot of $KC(s)G(s)$ (with lead compensator)

c) The bandwidth is the frequency at which the magnitude is $\sqrt{2}/2$ or -3dB. We find this at 15.981 rad/s. By looking at the Bode plot of the closed loop system ($\frac{KC(s)G(s)}{1+KC(s)G(s)}$).

Problem 2**Statement:**

Consider the system

$$G(s) = \frac{K}{s(s/5 + 1)(s/250 + 1)}$$

- a) Use Bode to design a lag compensator $C(s)$ so that the closed loop system satisfies:
- The steady state error to a unit ramp reference input is less than 0.01
 - The phase margin is no less than 40°
- b) Use MATLAB to verify the resulting phase margin

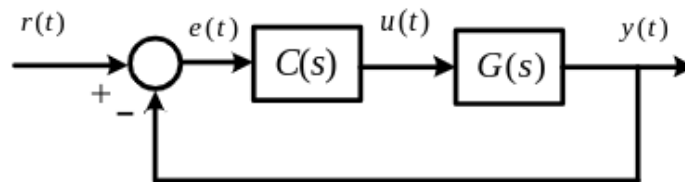


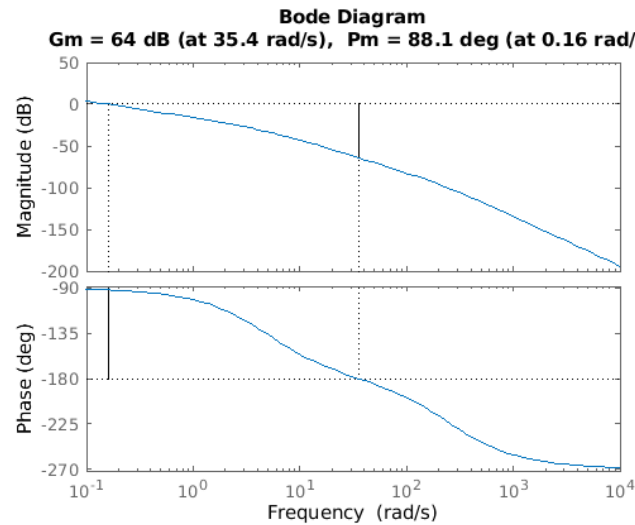
Figure 4: Problem2

Solution

- a) So first we need to find gain to make the steady state error within range. It is a type 1 system with $L_0 = \frac{1250K}{(s+5)(s+250)}$, so the steady state tracking error of unit ramp is $1/L_0(0) = 1/K$, thus $K > 100$. We pick $K = 200$.

We plot the Bode of $KG(s)$ and discover a gain margin of 64dB and phase margin of 88.1° at $\omega_{gc} = 0.16$ rad/s. This already has enough phase margin, so we are done, and we don't need a compensator.

- b) See figure 5

Figure 5: Problem 2: Bode plot of $G(s)$ with $K = 200$ **Problem 3****Statement:**

Given the state space model of a system:

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u \\ y &= (1 \quad 1) x\end{aligned}$$

- Find the state feedback gain F such that the closed-loop system has -1 and -2 at its poles. Compute F without using any state transformation.
- Design an observer to estimate the state of the system. Select the poles for the error dynamics to be $-2 \pm 2i$
- Construct an observer-based output feedback law that stabilizes the system.

Solution

- We want to design feedback F without using any state transformation. So we do this by

inspection of the characteristic polynomial. We know that the characteristic polynomial is:

$$\begin{aligned}
 \det(\lambda I - A + BF) &= \begin{vmatrix} \lambda - 2 + f_1 & -1 + f_2 \\ 1 + 2f_1 & \lambda - 1 + 2f_2 \end{vmatrix} \\
 &= (\lambda - 2 + f_1)(\lambda - 1 + 2f_2) - (-1 + f_2)(1 + 2f_1) \\
 &= \lambda^2 - \lambda + 2f_2\lambda - 2\lambda + 2 - 4f_2 + f_1\lambda - f_1 + 2f_1f_2 + 1 + 2f_1 - f_2 - 2f_1f_2 \\
 &= \lambda^2 + (-1 + 2f_2 - 2 + f_1)\lambda + (2 - 4f_2 - f_1 + 2f_1f_2 + 1 + 2f_1 - f_2 - 2f_1f_2) \\
 &= \lambda^2 + (2f_2 - 3 + f_1)\lambda + (3 - 5f_2 + f_1)
 \end{aligned}$$

So we want the characteristic polynomial to be: $(\lambda+1)(\lambda+2) = \lambda^2+3\lambda+2$, so $2f_2+f_1-3 = 3$ and $f_1-5f_2+3 = 2$, from the first equation we have $f_1 = 6-2f_2$, so $6-2f_2-5f_2+3 = 2$, which means $f_2 = 1$ and $f_1 = 4$. Thus $F = \begin{bmatrix} 4 & 1 \end{bmatrix}$

- b) We want to design an observer with poles at $-2 \pm 2i$. We can do this in the similar way as above. With $\lambda I - A + LC$

$$\begin{aligned}
 \det(\lambda I - A + LC) &= \begin{vmatrix} \lambda - 2 + l_1 & -1 + l_2 \\ 1 + l_1 & \lambda - 1 + l_2 \end{vmatrix} \\
 &= (\lambda - 2 + l_1)(\lambda - 1 + l_2) - (-1 + l_2)(1 + l_1) \\
 &= \lambda^2 - \lambda + l_2\lambda - 2\lambda + 2 - 2l_2 + l_1\lambda - l_1 + l_1l_2 + 1 + l_2 + l_1 - l_1l_2 \\
 &= \lambda^2 + (-1 + l_2 - 2 + l_1)\lambda + (2 - 2l_2 - l_1 + l_1l_2 + 1 + l_2 + l_1 - l_1l_2) \\
 &= \lambda^2 + (l_2 + l_1 - 3)\lambda + (3 - l_2 - 2l_1)
 \end{aligned}$$

We want the characteristic polynomial to be: $\lambda^2+4\lambda+8$. So $l_2+l_1-3 = 4$ and $3-l_2-2l_1 = 8$. From the first we have $l_2 = 7 - l_1$, so $3 - 7 + l_1 - 2l_1 = 8 \rightarrow l_1 = -12$, and thus $l_2 = 19$. So $L = \begin{bmatrix} -12 \\ 19 \end{bmatrix}$

- c) We put this all together:

$$\begin{aligned}
 \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} A - BF & -BF \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r \\
 y &= [C \ 0] \begin{bmatrix} x \\ \hat{x} \end{bmatrix}
 \end{aligned}$$

So we have:

$$\begin{aligned}
 \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} -2 & 0 & -4 & -1 \\ -9 & -1 & -8 & -2 \\ 0 & 0 & 14 & 13 \\ 0 & 0 & -20 & -18 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} r \\
 y &= [1 \ 1 \ 0 \ 0] \begin{bmatrix} x \\ \hat{x} \end{bmatrix}
 \end{aligned}$$

Problem 4

Statement:

Complete the class evaluation:

Solution

Yes

Code Appendix

```

1 clear all;
2 close all;
3 %% Problem 1
4 G = zpk([-3], [0,-2,-5],50)
5 K = 1;
6 sys = K*G;
7
8 figure
9 bode(sys); margin(sys)
10 [Gm, Pm, Wcg, Wcp] = margin(sys);
11
12 % Design C
13 a = 1/3
14 T = 1/(Wcp * sqrt(a))
15 phi_max = rad2deg(asin((1-a)/(1+a)))
16 C = tf([T, 1], [a * T, 1])
17
18 sys2 = K*C*G;
19 figure
20 bode(sys); margin(sys)
21 hold on
22 bode(sys2); margin(sys2)
23
24 [Gm2, Pm2, Wcg2, Wcp2] = margin(sys2);
25 Pm2
26
27 % compute bandwidth, use CL system
28 [mag, phase, wout] = bode(sys2/(1+sys2), 15.9:0.001:16.1);
29 mag = mag(:);
30 err = abs(mag - sqrt(2)/2);
31 [m, idx] = min(err);
32 bandwidth = wout(idx) % bandwidth is omega that comes closest to sqrt(2)/2
33
34 %% Problem 2
35 G = zpk([], [0,-5, -250],1)
36 K = 200; % K > 100 for steady state tracking error
37
38 sys = K*G;
39 figure
40 bode(sys); margin(sys)
41 [Gm, Pm, Wcg, Wcp] = margin(sys);
42 Pm % this is > 40, so we are just done??
43 %% Problem 3
44 % do the matrix multiplication
45 A = [2 1;-1 1];
46 B = [1;2];
47 C = [1 1];
48 F = [4 1];
49 L = [-12; 19];
50
51 result = [A-B*F, -B*F; zeros(2), A-L*C]

```