Homework 6

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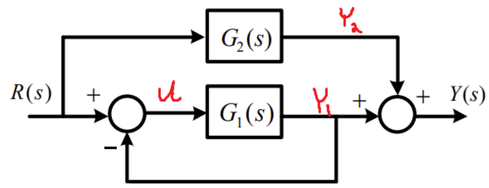
16 November 2023

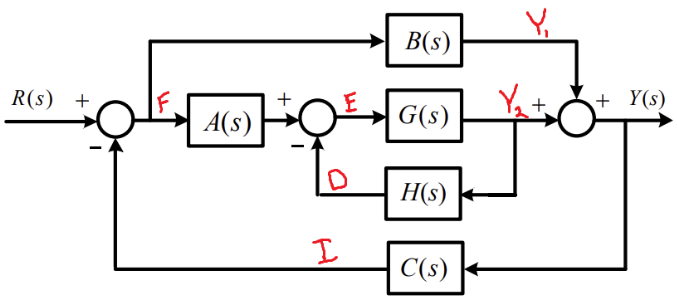
Problem 1

Statement:

Find the transfer function for the block diagrams:



(a)



(b)

Figure 1: Problem 1

Solution

a) $Y(s) = Y_1(s) + Y_2(s) = G_1(s)U(s) + G_2(s)R(s)$,
$U(s) = R(s) - G_1(s)U(s) \implies (1 + G_1(s))U(s) = R(s) \implies U(s) = \frac{R(s)}{1+G_1(s)}$
So, $Y(s) = \left[\frac{G_1(s)}{1+G_1(s)} + G_2(s)\right] R(s)$

Problem 2
Statement:
Determine the time constant of the system:

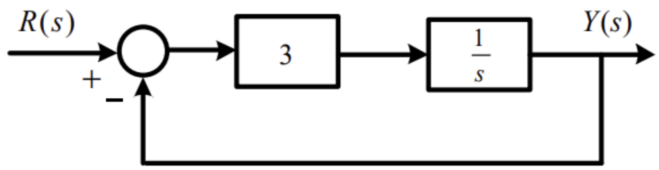


Figure 2: Problem 2

Solution

Problem 3
Statement:
Specify the gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more then 10% in response to a unit step

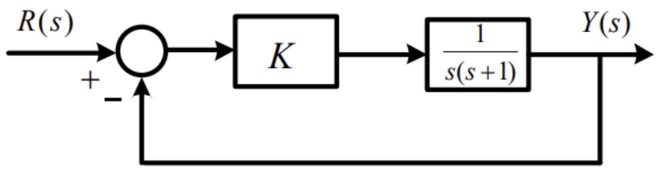


Figure 3: Problem 3

Solution

Problem 4
Statement:

Consider the system

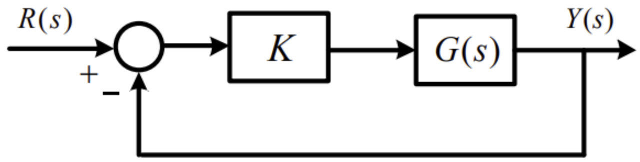


Figure 4: Problem 4

with

a) $KG(s) = \frac{4(s+2)}{s(s^3+2s^2+3s+4)}$

b) $KG(s) = \frac{2(s+4)}{s^2(s+1)}$

Use Routh's stability criterion to determine whether the each of the resulting closed-loop system will be asymptotically stable.

Solution

Problem 5
Statement:
Using Routh's stability criterion to determine how many roots with positive real parts the following equations have:

a) $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$

b) $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0$

Solution

Problem 6
Statement:

Consider the closed-loop system:

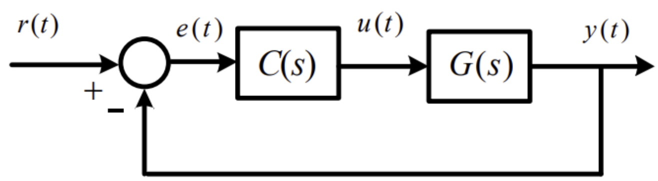


Figure 5: Problem 6

$G(s) = \frac{1}{s^2}$, and $C(s) = \frac{10(s+2)}{s+5}$.

Find the system type and determine the steady state tracking errors for:

a) $r(t) = 1(t)$

b) $r(t) = t1(t)$

c) $r(t) = 1/2t^2 1(t)$

Solution

Problem 7
Statement:
Sketch the Nyquist plot for an open-loop system with transfer function:

a) $G(s) = \frac{1}{s^2}$

b) $G(s) = \frac{1}{s^2+4}$

Solution

Problem 8
Statement:
Consider the system with loop gain

$$L(s) = KG(s) = \frac{K(s+2)}{s+10}$$

Use Matlab command nyquist to plot nyquist plot for $G(s)$, and based on the nyquist plot, determine the range of K for which the closed-loop system is asymptotically stable

Solution

Problem 9
Statement:
Is the following system controllable? Observable?

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

Solution