Project

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5 December 2023

Problem 1
Statement:
Consider the system

$$G(s) = \frac{50(s+3)}{s(s+2)(s+5)}$$

a) Use Bode plot to design a lead compensator $C(s)$ with unity DC gain so that the phase margin of the closed-loop system is no less than $40°$.

b) Use MATLAB to verify the resulting phase marginally

c) What is the bandwidth of the resulting closed-loop system?

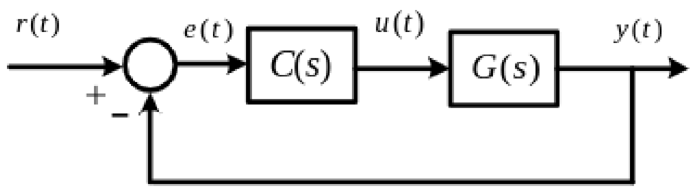

Figure 1: Problem1

Solution

a) Unity DC gain means $K = 1$. We see that phase margin is $30°$. So we need to add at

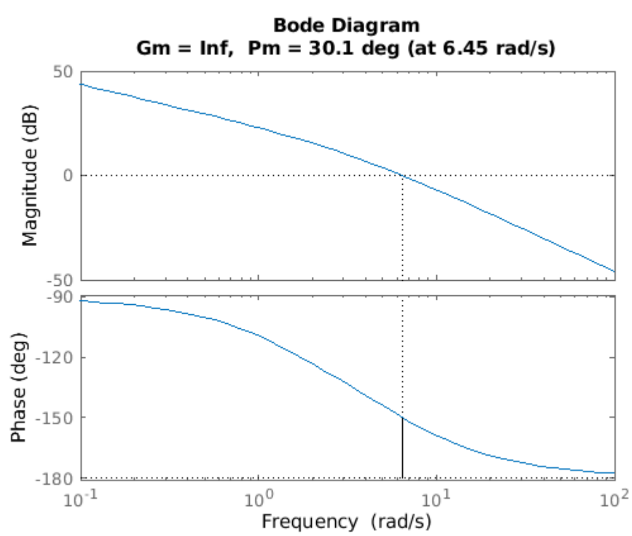

Figure 2: Problem 1: Bode plot of $G(s)$

least 10°of phase margin. Choose $\alpha = 1/3$ so $\phi_{max} = 30°$ which is less than 60. We want $\omega_{max} = \omega_{gc} = 6.45$, so then $T = \frac{1}{\omega_{max}\sqrt{\alpha}} = 0.2684$

So finally $C(s) = \frac{Ts+1}{\alpha Ts+1} = \frac{0.2684s+1}{0.0895s+1}$

b) See figure 3

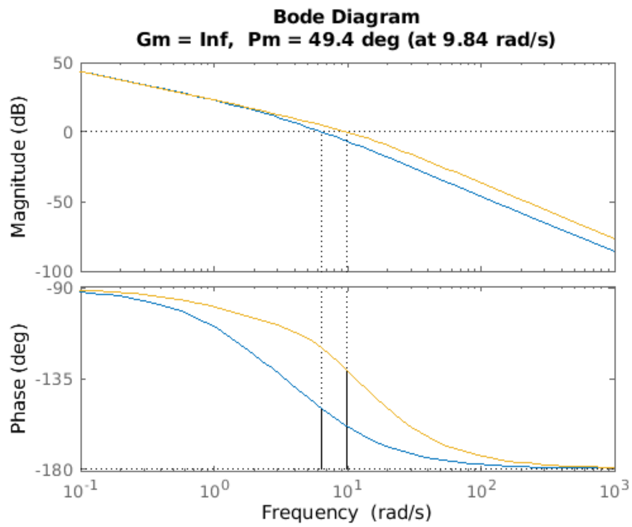

Figure 3: Problem 1: Bode plot of $KC(s)G(s)$ (with lead compensator)

c) The bandwidth is the frequency at which the magnitude is $\sqrt{2}/2$ or -3dB. We find this at 15.981 rad/s. By looking at the Bode plot of the closed loop system ($\frac{KC(s)G(s)}{1+KC(s)G(s)}$).

Problem 2
Statement:
Consider the system

$$G(s) = \frac{K}{s(s/5+1)(s/250+1)}$$

a) Use Bode to design a lag compensator $C(s)$ so that the closed loop system satisfies:

- The steady state error to a unit ramp reference input is less than 0.01
- The phase margin is no less than $40°$

b) Use MATLAB to verify the resulting phase margin

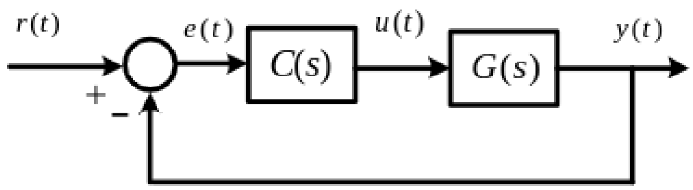

Figure 4: Problem2

Solution

a) So first we need to find gain to make the steady state error within range. It is a type 1 system with $L_0 = \frac{1250K}{(s+5)(s+250)}$, so the steady state tracking error of unit ramp is $1/L_0(0) = 1/K$, thus $K > 100$. We pick $K = 200$.

We plot the Bode of $KG(s)$ and discover a gain margin of 64dB and phase margin of $88.1°$at $\omega_{gc} = 0.16$ rad/s. This already has enough phase margin, so we are done, and we don't need a compensator.

b) See figure 5

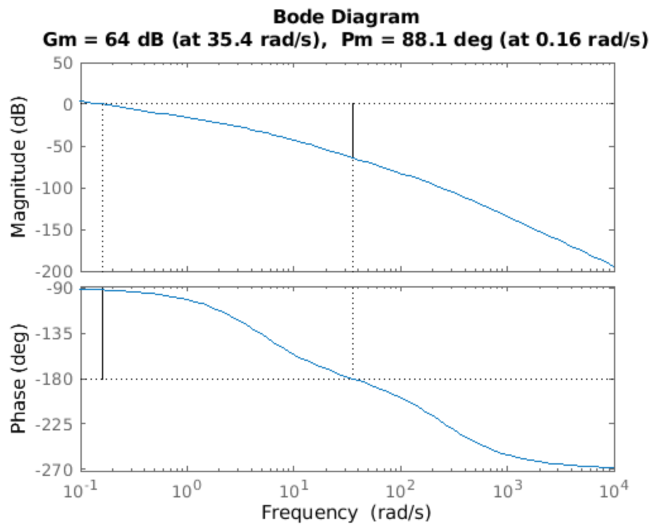

Figure 5: Problem 2: Bode plot of $G(s)$ with $K = 200$

Problem 3
Statement:
Given the state space model of a system:

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

a) Find the state feedback gain F such that the closed-loop system has -1 and -2 at its poles. Compute F without using any state transformation.

b) Design an observer to estimate the state of the system. Select the poles for the error dynamics to be $-2 \pm 2i$

c) Construct an observer-based output feedback law that stabilizes the system.

Solution

a) We want to design feedback F without using any state transformation. So we do this by inspection of the characteristic polynomial. We know that the characteristic polynomial is:

$$\begin{aligned}
\det(\lambda I - A + BF) &= \begin{vmatrix} \lambda - 2 + f_1 & -1 + f_2 \\ 1 + 2f_1 & \lambda - 1 + 2f_2 \end{vmatrix} \\
&= (\lambda - 2 + f_1)(\lambda - 1 + 2f_2) - (-1 + f_2)(1 + 2f_1) \\
&= \lambda^2 - \lambda + 2f_2\lambda - 2\lambda + 2 - 4f_2 + f_1\lambda - f_1 + 2f_1f_2 + 1 + 2f_1 - f_2 - 2f_1f_2 \\
&= \lambda^2 + (-1 + 2f_2 - 2 + f_1)\lambda + (2 - 4f_2 - f_1 + 2f_1f_2 + 1 + 2f_1 - f_2 - 2f_1f_2) \\
&= \lambda^2 + (2f_2 - 3 + f_1)\lambda + (3 - 5f_2 + f_1)
\end{aligned}$$

So we want the characteristic polynomial to be: $(\lambda+1)(\lambda+2) = \lambda^2+3\lambda+2$, so $2f_2+f_1-3 = 3$ and $f_1 - 5f_2 + 3 = 2$, from the first equation we have $f_1 = 6 - 2f_2$, so $6 - 2f_2 - 5f_2 + 3 = 2$, which means $f_2 = 1$ and $f_1 = 4$. Thus $F = \begin{bmatrix} 4 & 1 \end{bmatrix}$

b) We want to design an observer with poles at $-2 \pm 2i$. We can do this in the similar way as above. With $\lambda I - A + LC$

$$\begin{aligned}
\det(\lambda I - A + LC) &= \begin{vmatrix} \lambda - 2 + l_1 & -1 + l_2 \\ 1 + l_1 & \lambda - 1 + l_2 \end{vmatrix} \\
&= (\lambda - 2 + l_1)(\lambda - 1 + l_2) - (-1 + l_2)(1 + l_1) \\
&= \lambda^2 - \lambda + l_2\lambda - 2\lambda + 2 - 2l_2 + l_1\lambda - l_1 + l_1l_2 + 1 + l_2 + l_1 - l_1l_2 \\
&= \lambda^2 + (-1 + l_2 - 2 + l_1)\lambda + (2 - 2l_2 - l_1 + l_1l_2 + 1 + l_2 + l_1 - l_1l_2) \\
&= \lambda^2 + (l_2 + l_1 - 3)\lambda + (3 - l_2 - 2l_1)
\end{aligned}$$

We want the characteristic polynomial to be: $\lambda^2+4\lambda+8$. So $l_2+l_1-3 = 4$ and $3-l_2-2l_1 = 8$. From the first we have $l_2 = 7 - l_1$, so $3 - 7 + l_1 - 2l_1 = 8 \rightarrow l_1 = -12$, and thus $l_2 = 19$ So $L = \begin{bmatrix} -12 \\ 19 \end{bmatrix}$

c) We put this all together:

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A - BF & -BF \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r$$
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

So we have:

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} -2 & 0 & -4 & -1 \\ -9 & -1 & -8 & -2 \\ 0 & 0 & 14 & 13 \\ 0 & 0 & -20 & -18 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Problem 4
Statement:
Complete the class evaluation:

Solution

Yes

Code Appendix

```
clear all;
close all;
%% Problem 1
G = zpk([-3], [0,-2,-5],50)
K = 1;
sys = K*G;

figure
bode(sys); margin(sys)
[Gm, Pm, Wcg, Wcp] = margin(sys);

% Design C
a = 1/3
T = 1/(Wcp * sqrt(a))
phi_max = rad2deg(asin((1-a)/(1+a)))
C = tf([T, 1], [a * T, 1])

sys2 = K*C*G;
figure
bode(sys); margin(sys)
hold on
bode(sys2); margin(sys2)

[Gm2, Pm2, Wcg2, Wcp2] = margin(sys2);
Pm2

% compute bandwidth, use CL system
[mag, phase, wout] = bode(sys2/(1+sys2), 15.9:0.001:16.1);
mag = mag(:);
err = abs(mag - sqrt(2)/2);
[m, idx] = min(err);
bandwidth = wout(idx)  % bandwidth is omega that comes closest to sqrt(2)/2

%% Problem 2
G = zpk([], [0,-5, -250],1)
K = 200;  % K > 100 for steady state tracking error

sys = K*G;
figure
bode(sys); margin(sys)
[Gm, Pm, Wcg, Wcp] = margin(sys);
Pm  % this is > 40, so we are just done??
%% Problem 3
% do the matrix multiplication
A = [2 1;-1 1];
B = [1;2];
C = [1 1];
F = [4 1];
L = [-12; 19];

result = [A-B*F, -B*F; zeros(2), A-L*C]
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