Homework 5

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Problem 1
Statement:
Find the laplace transform of:

$$f(t) = \begin{cases} 0, & t < 0 \\ te^{-3t}, & t \geq 0 \end{cases}$$

Solution

This is frequency shift ($e^{-\alpha t} f(t)$), which is $F(s+\alpha)$, so $f(t) = t, \implies F(s) = 1/s^2$. So our answer is $\mathcal{L}[f(t)] = \frac{1}{(s+3)^2}$

Problem 2
Statement:
Find the laplace transform of:

$$f(t) = \begin{cases} 0, & t < 0 \\ \sin(\omega t + \theta), & t \geq 0 \end{cases}$$

Solution

We know from trig rules that $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$. So we re-write $f(t) = \sin(\omega t)\cos(\theta) + \cos(\omega t)\sin(\theta)$ Then we know the laplace:

$$F(s) = \frac{\cos(\theta)\omega}{s^2 + \omega^2} + \frac{\sin(\theta)s}{s^2 + w^2} = \frac{\cos(\theta)\omega + \sin(\theta)s}{s^2 + \omega^2}$$

Problem 3
Statement:
Given:

$$F(s) = \frac{2(s+2)}{s(s+1)(s+3)}$$

use the Initial Value theorem to determine $f(0)$

Solution

Initial value theorem is $f(0) = \lim_{s\to\infty} sF(s)$, so we get $\lim_{s\to\infty} \frac{2s(s+2)}{s(s+1)(s+3)} = 0$ since it is quadratic on top and cubic on bottom.

Problem 4
Statement:
Given

$$F(s) = \frac{5(s+2)}{s(s+1)(s+3)}$$

use the Final Value theorem to determine $f(\infty)$

Solution

Final value theorem is $f(\infty) = \lim_{s\to 0} sF(s)$, so we get
$\lim_{s\to 0} \frac{5s(s+2)}{s(s+1)(s+3)} = \lim_{s\to 0} \frac{5s+10}{s^2+4s+3} = 10/3$

Problem 5
Statement:
Find the inverse laplace transform of:

$$F(s) = \frac{s+3}{(s+1)(s+2)}$$

Solution

Problem 6
Statement:
Find the inverse laplace transform of:

$$F(s) = \frac{1}{s(s^2 + 2s + 2)}$$

Solution

Problem 7
Statement:
A LTI system is described by:

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 2 \end{pmatrix} x$$

Find the transfer function of the system.

Solution

Problem 8
Statement:
Find a state space realization of the transfer function:

$$G(s) = \frac{s^3}{s^3 + 3s^2 + 2s + 1}$$

Solution