Homework 4

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Problem 1 3.2.3 Problem 1

Let A be an open set. Show that if a finite number of points are removed from A the remaining set is still open. Is the same true if a countable number of points are removed?

PROOF.

Because A is an open set, we know that A can be represented as the union of disjoint open intervals $\cup^{\infty}(a_i, b_i)$. Furthermore, by the definition of an open set, we know that for any $x \in A$, x is in an open interval that is contained within A. Say that $x \in (a, b)$. Then $x \notin (a, x) \cup (x, b)$. Since every open interval representing A is disjoint, we replace the single interval (a, b) containing x with two more disjoint intervals (a, x) and (x, b). Then $A \setminus \{x\}$ can still be represented by a set of open intervals. So A is open $\square$

No, the same is not true if a countable number of points are removed

Problem 2 3.2.3 Problem 4

Let A be a set and x a number. Show that x is a limit point of A if and only if there exists a sequence $x_1, x_2, ...$ of distinct points in A that converges to x.

Problem 3 3.2.3 Problem 5

Let A be a closed set, x a point in A, and B be the set A with x removed. Under what conditions is B closed.