Homework 4

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Problem 1 3.1.3 problem 1a)

Compute the sup, inf limsup, liminf and all the limit points of $x_n = 1 + (-1)^n/n$

$x_n = 0, 3/2, 2/3, 5/4, 4/5, 5/6, 6/7, 9/8, ...$

Clearly the sup is 3/2 and inf is 0. We then show that the sequence is convergent to 1. We need to show $\forall n \in \mathbf{N}\ \exists m \in \mathbf{N}\ s.t. \forall j \geq m\ |x_j - 1| \leq 1/n$.

$$|x_j - 1| = |1 + (-1)^j/j - 1| = |(-1)^j/j| = 1/j$$

If we choose $m = n$ then $1/j \leq 1/n\ \forall j \geq n$ as required. Since it is a convergent sequence, by theorem 3.1.5 $limsup = liminf = 1$.

Problem 2 3.1.3 problem 2

If a bounded sequence is the sum of a monotone increasing and monotone decreasing sequence ($x_n = y_n + z_n$ where $\{y_n\}$ is monotone increasing and $\{z_n\}$ is monotone decreasing) does it follow that the sequence converges? What if $\{y_n\}$ and $\{z_n\}$ are bounded?

Problem 3 3.1.3 problem 4

Prove $sup(A \cup B) \geq sup(A)$ and $sup(A \cap B) \leq sup(A)$

Problem 4 3.1.3 problem 6

Is every subsequence of a subsequence of a sequence also a subsequence of the sequence?c