Homework 7

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19 Oct 2020

Problem 1 4.1.5 Problem 2

Let A be the set defined by the equations $f_1(x) = 0$, $f_2(x) = 0$, ... $f_n(x) = 0$. Where $f_1, ...f_n$ are continuous functions defined on the whole line. Show that A is closed. Must A be compact?
Hint: you can use result of 4.1.5 Problem 1 without proof

So if $\mathcal{D}_i$ is the set of x satisfying $f_i(x) = 0$ then $A = \cup_{i=1}^{n} \mathcal{D}_i$. We know that $\mathcal{D}_i = f^{-1}(0)$. From 4.1.5 Problem 1 we know that a function f is continuous iff the inverse image of every closed set is a closed set. Since f_i is continuous, $\mathcal{D}_i$ must be closed. Then A is the finite union of a finite number of closed sets which is closed by Theorem 3.2.3. So then A must be closed.

Problem 2 4.1.5 Problem 4

Give a definition of $\lim_{x\to\infty} f(x) = y$. Show that this is true iff for every sequence $x_1, x_2, \ldots$ of point in the domain of f such that $\lim_{x\to\infty} x_n = \infty$ we have $\lim_{n\to\infty} f(x_n) = y$. *Hint:* For the proof of the 2nd part of the problem, refer to the proof of Theorem 4.1.1.

Problem 3 4.1.5 Problem 7

Give an example of a continuous function with domain $\mathbb{R}$ such that the inverse image of a compact set is not compact

Problem 4 4.1.5 Problem 10

Show that a function that satisfies a Lipschitz condition is uniformly continuous.