Homework 4

Elliott Pryor

25 Sept 2020

Problem 1 True or false

1. The set of **Q** is countable? True
2. Let $A_1, A_2, A_3, ...$ be a sequence of countably many sets, and each $A_i, i = 1, 2, 3, ...$ is countable then their cartesian product $A_1 x \times A_2 \times A_3 \times ...$ i countable.
3. A Cauchy sequence of positive rational numbers cannot be equivalent to a Cauchy sequence of negative rational numbers.
4. Let $x_1, x_2, ...$ be a convergent squence of real numbers, then $limsup x_n = liminf x_n$.
5. The union of any number of closed sets is a closed set.

Problem 2 The statement "A real number x is a limit-point of a sequence of real numbers $x_1, x_2, ...$" can be written explicitly using quantifiers as follows: "For all $n \in \mathbf{N}$, for all $m \in \mathbf{N}$, there exists $j > m$ such that $|x - x_j| < 1/n$". Now write the statement "A real number x is not a limit-point of a sequence of real numbers $x_1, x_2, ...$" explicitly using quantifiers

---

Problem 3 Give an example of a set A that is not closed but such that every point of A is a limit point

---

Problem 4 let $x_1, x_2, ...$ be a sequence of real numbers given by

$$x_n = \frac{\cos(\sqrt{n!}\pi)}{4^n}$$

and define

$$y_n = x_1 + x_2 + ... + x_n$$

prove that the sequence $y_1, y_2, ...$ converges

Problem 5 prove

$$limsup\{x_n + y_n\} \leq limsup\{x_n\} + limsup\{y_n\}$$

if both $limsup\{x_n\}$ and $limsup\{y_n\}$ are finite, and give an example where the equality does not hold.