Homework 8

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Problem 1 5.1.3 Problem 1

Show that $f(x) = O(|x - x_0|^2)$ as $x \to x_0$ implies $f(x) = o(x - x_0)$ as $x \to x_0$ but give an example to show the converse is not true.

Problem 2 5.2.4 Problem 1 Let f and g be continuous functions on $[a, b]$ and differentiable at every point in the interior, with $g(a) \neq g(b)$. Prove that there exists a point in x_0 in (a, b) such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(x_0)}{g'(x_0)}$$

This is also called second mean value theorem

Problem 3 5.2.4 Problem 2

if f is a function satisfying $f(x) - f(y) \leq M|x-y|^\alpha$ for all x, y and some fixed M and $\alpha > 1$, prove that f is constant. *Hint: what is* f'. It is rumored that a graduate student once wrote a whole thesis on the class of functions satisfying this condition!

Problem 4 5.2.4 problem 3

Suppose f is defined on $[a, b]$ and g is defined on $[b, c]$ with $f(b) = g(b)$ then define:

$$h(x) = \begin{cases} f(x) & \text{if } a \leq x \leq b \\ g(x) & \text{if } b \leq x \leq c \end{cases}$$

give an exaple where f and g are differentiable but h is not. Give a definition of one-sided deriviatives $f'(b)$ $g'(b)$ and show that the equality of these is a necessary and sufficient condition for h to be differntiable. Given that $f, g$are differentiable.