Homework 1

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Problem 1 6.1.5 Problem 3

Derive the integration of the derivative theorem from the differentiation of the integral theorem. Can you prove the converse implication?

You **don't** need to prove the converse case. Here, assume the differentiation of the integral theorem is true, namely,

$$\frac{d}{dx}\int_a^x g(t)dt = g(x)$$

for any continuous function $g(x)$ on $[a, b]$ and $a \leq x \leq b$, you need to use this to prove:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

for any C^1 function f on $[a, b]$.

Hint: Consider function

$$G(x) = \int_a^x f'(t)dt,^a \leq x \leq b$$

and see how $G(x)$ and $f(x)$ are related.

Problem 2 6.1.5 Problem 4

Prove the integral mean value theorem: if f is continuous on $[a, b]$ then there exists y in (a, b) such that $\int_a^b f(x)dx = (b - a)f(y)$

Problem 3 6.1.5 Problem 8

Let f be a C^1 function on the line, and let $g(x) = \int_0^1 f(xy)y^2 dy$. Prove that g is a C^1 function and establish a formula for $g'(x)$ in terms of f

Hint: Use theorem 6.1.7

Problem 4 6.1.5 Problem 10

For a continuous, positive function $w(x)$ on $[a, b]$, define the weighted average operator A_w to be:

$$A_w(f) = \frac{\int_a^b f(x)w(x)dx}{\int_a^b w(x)dx}$$

for continuous functions f. Prove that A_w is linear and lies between the maximum and mininum values of f.

Hint: A_w is linear if

$$A_w(c_1 f + c_2 g) = c_1 A_w(f) + c_2 A_w(g)$$

for any constants c_1, c_2 and continuous functions f, g on $[a, b]$