Homework 5

Elliott Pryor

08 March 2021

Problem 1 7.4.5 Problem 2

If f is analytic in a neighborhood of x_0 and $f(x_0) = 0$, show that $f(x)/(x - x_0)$ is analytic in the same neighborhood.

Hint: Write $f(x)$ as a power series expanded at x_0 and pay attention to what is a_0

Problem 2 7.4.5 Problem 6

Prove that if $f(x)$ is analytic on (a, b), then $F(x) = \int_c^x f(t)dt$ is also analytic onf (a, b), where c is any point in (a, b)

Hint: You need to prove that for any $x_0 \in (a, b)$, $F(x)$ has a power series expansion about x_0 for x close to x_0. Now pick any fixed $x_0 \in (a, b)$ and write $F(x)$ as:

$$F(x) = \int_c^x f(t)dt = \int_c^{x_0} f(t)dt + \int_{x_0}^x f(t)dt = C_0 + \int_{x_0}^x f(t)dt$$

Problem 3 7.4.5 Problem 7 a.

Compute the power-series expansion of $f(x) = \frac{x^2}{1-x^2}$ about $x = 0$

Problem 4

Compute the radius of convergence of the following power series:

a. $\sum(n^4/n!) \cdot x^n$

b. $\sum \sqrt{n}x^n$

c. $(n^2 2^n)x^n$