Homework 8

Elliott Pryor

9 April 2021

Problem 1 10.1.5 Problem 3

If f is differentiable at y, show that $d_u f(y)$ is linear in u, meaning $d_{(au+bv)} f(y) = a d_u(y) + b_v f(y)$.

Hint: Apply Theorem 10.1.1

Problem 2 10.1.5 Problem 10

Let $g : [a,b] \to \mathbb{R}^n$ be differentiable. If $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable, what is the derivative $(d/dt)f(g(t))$

Hint: Use notation $g(t) = (g_1(t), \ldots, g_n(t)), t \in [a,b]$ and $f(z) = f(z_1, \ldots, z_n), z = (z_1, \ldots z_n) \in \mathbb{R}^n$ and Apply the chain rule.

Problem 3 10.1.5 Problem 13

Compute df of

1. $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x_1, x_2) = x_1 e^{x_2}$
2. $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x_1, x_2, x_3) = (x_3, x_2)$
3. $f : \mathbb{R}^2 \to \mathbb{R}^3, \quad f(x_1, x_2) = (x_1, x_2, x_1 \cdot x_2)$

Problem 4 10.1.5 Problem 15

If $f : D \to \mathbb{R}$ is C^1 with $D \subseteq \mathbb{R}^n$ and D contains the line segment joining x and y, show that $f(y) = f(x) + \nabla(z) \cdot (y - x)$ for some point z on the line segment. Explain why this is an n-dimensional analog of the mean value theorem