Homework 6

Elliott Pryor

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Problem 1 7.5.5 Problem 7

If f is C^1 on $[a, b]$ prove that there exists a cubic polynomial P such that $f - P$ and its first derivative vanish at the endpoints of the interval.

Problem 2 7.5.5 Problem 9

If $f(c) = 0$ for some $c \in (a, b)$ prove that the polynomials approximating f on $[a, b]$ may be taken to vanish at c.

Hint: Here $f(x)$ is a continuous function on $[a, b]$. Assume $f_n(x)$ is the sequence of polynomials approximating $f(x)$ uniformly by WTA, consider $g_n(x) = f_n(x) - f_n(c)$

Problem 3 7.5.5 Problem 14

(a) For $c_m = \int_{-1}^{1}(1-x^2)^m dx$ obtain the identity $c_m = c_{m-1} - (1/2m)c_m$ by integration by parts.

(b) Show that

$$c_m = 2\frac{2 \cdot 4 \cdot 6 \cdot ... \cdot 2m}{3 \cdot 5 \cdot 7 \cdot ... \cdot 2m+1} = \frac{2(2^m m!)^2}{(2m+1)!}$$