Homework 1

Elliott Pryor

31 Jan 2021

Problem 1 6.2.4 Problem 6

Prove that if f is Riemann integrable on $[a, b]$ and $g(x) = f(x)$ for every x except for a finite number, then g is Riemann integrable.

Hint: Mimic the proof of Theorem 6.2.3

Problem 2 6.2.4 Problem 9, part b

If f is Riemann integrable on $[a, b]$, prove it satisfies a Lipschitz condition

Problem 3 6.2.4 Problem 10

If f is Riemann integrable on $[a,b]$ and continuous at x_0, prove that $F(x) = \int_a^x f(t)dt$ is differentiable at x_0 and $F'(x_0) = f(x_0)$. Show that if f has a jump discontinuity at x_0, then F is not differentiable at x_0.

Hint: Refer to the proof of Theorem 6.1.2, note that $f(x)$ being continuous at x_0 can be written as $\forall 1/m, \quad \exists 1/n \quad st. \quad \forall x \in [a,b], \quad |x - x_0| < 1/n$, we have $|f(x) - f(x_0)| < 1/m$ or $f(x_0) - 1/m < f(x) < f(x_0) + 1/m$