Exam 2

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Problem 1

Statement

Let the random variable X be the time in minutes a hungry customer waits to recieve their tacos at Taco Montes. Suppose X follows an exponential distribution.

a. The *median* time a hungry patron waits to receive their tacos is claimed to be 12 minutes (i.e., the probability that a patron waits 12 minutes or less is 0.5, or $\phi_{0.5} = 12$ mins). What is the mean amount of time a hungry patron waits for their tacos at Taco Montes?

b. Use the MGF ($m_X(t)$), to find the mean wait time a customer waits to receive a taco — verify that this is the same mean you found in part (a). *Note: You do not need to derive the MGF of X, you may use the appropriate mgf provided in notes or text*

c. Consider 15 independent Taco Montes customers. Let the RV Y be the number of hungry customers who wait more than 12 minutes to receive their tacos.

   i. Provide an appropriate probability distribution for modeling Y (i.e., specify $Y \sim$ Distribution-Name(appropriate parameter(s) and their values)).

   ii. Write a probability statement, and a mathematical expression for the probability that more than 12 of the customers wait longer than 12 minutes to receive their tacos. *You do not have to evaluate the expression.*

   iii. Use R to calculte the probability in ii. and share the code you used to answer the question

   iv. **Extra Credit** Imagine an infinite line of people waiting for tacos, and define the customer place in line by 1 (first in line), 2 (second in line), 3 (third in line), etc. Let Z be the place in line for which this particular customer is the 5th who waits more than 12 minutes to get their tacos. Provide a reasonable distribution for modeling Z, and find $P(Z > 10)$ — you may use R, but share your code.

Solution

Problem 2

Statement

Suppose two people are waiting in the same line at the grocery store (i.e., they will be checked out by the same cashier). Let X be the time at which the first person in line pays for their items and let Y be the time at which the second person in line pays for their items. Note that these random variables represent the time of payment, not the time spent in line. The joint distribution of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & ? \\ 0 & otherwise \end{cases}$$

Without doing any calculations, choose the support that you belive would be reasonable for this situation and *thouroughly explain your reasoning.*

(a) $0 < x < \infty,\ 0 < y < \infty$ (b) $0 < x < y < \infty$ (c) $0 < y < x < \infty$ (d) $0 < y < \infty$

Solution

Problem 3

Statement

Suppose the daily number of hours, X, a teenager watches television, and the daily number of hours, Y, they spend on homework can be modeled with the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} xye^{-(x+y)} & x > 0, y > 0 \\ 0 & otherwise \end{cases}$$

a. Are X, Y independent? Why or why not?
b. Given what X and Y represent, does your answer in (a) seem realistic? Explain
c. What is the probability that a teenager spends less than half the time doing homework than they do watching television, but less than six hours watching television? Provide the final integral for finding this probability, but DO NOT actually evaluate the integral. *For full credit, Include a well-labeled picture of the support and the region of integration of interest!*

Solution

Problem 4

Statement

Let X represent the time in years until a randomly selected watch quits working and let Y represent the amount of measurement error for a watch in minutes (i.e., the amount of time, in minutes, a watch is off from the exact time). Assume the two random variables are statistically independent (even though this may not be realistic). Suppose $X \sim Exponential(\beta = 2)$ and $Y \sim Uniform(-1, 1)$

a. Give the joint distribution $f_{X,Y}(x, y)$
b. Find, $F_{X,Y}(x, y)$ *Be sure to include ALL regions of interest*
c. **Extra Credit** Find $E(Y^2X^3)$

Solution

Problem 5

Statement

A three parameter gamma distribution can be defined by incorporating a 'location' parameter shift to the distribution with pdf

$$f(x) = \frac{1}{\Gamma(\alpha)\beta}\left(\frac{x-r}{\beta}\right)^{\alpha-1} e^{-(x-r)/\beta} I_{r,\infty}(x)$$

What are the restrictions on the paramters, α, β, and r? Clearly explain your reasoning for each parameter