Final Exam

Elliott Pryor

11/25/2020

Problem 1

Statement

Choose one of the distribution videos of your peers to watch and critique. The only restriction is that this should be a group different from yours and from the project group you peer reviewed. You can find the videos here https://www.youtube.com/ playlist?list=PLMd58R18w9hcUaMGJGXRV0GFqOZ27Kt5w and https://spark.adobe.com/ video/GcMjER5KZAZLp. Answer the following questions.

a. For each distribution discussed: State the distribution that was used in the problem. Was the distributions used correctly with respect to their support? That is, was the random variable defined in a way that it could be reasonably modeled with the distribution? Briefly explain why or why not. **Make sure you answer these questions for both distributions!**

b. Choose the problem you found the most interesting out of the problems presented by the group. What made this particular problem more interesting than the other(s)?

c. Briefly reflect on what you learned with respect to how probability distributions (pmf/pdfs) are used for modeling random events in the real world as a result of participating in the distributions project. Specifically, can a probability model perfectly represent a random event that arises in the real world? Why is communicating model assumptions (i.e., assumptions that are required if a particular probability distribution is used) so important when applying probability models to real world problems? **There is no right answer here, please provide an honest and thoughtful reflection in ≈ 5-10 sentences.**

Solution

Problem 2

Statement

Let X be a random variable with *pdf* $f_X(x) = kX^2 I_{(0,2)}(x)$ where k is a constant

a. Find k

b. Let $Y = \frac{1}{X}$

   i. Draw a well-labeled graph showing the transformation from $\mathcal{X}$ to $\mathcal{Y}$ and state $\mathcal{Y}$

   ii. Find pdf $f_Y(y)$

   iii. *Extra Credit* What is the name of the distribution assumed for X.

Solution

Problem 3

Statement

A submarine is missing and is presumed to have equal probability of being sunk in one of three Regions off of the Gulf Coast. If the submarine is actually down in Region i, let the probability of a successful search in that region be equal to $1 - \alpha_i$ for i = 1, 2, 3. For example, the probability a search of Region 1 will recover the sub, given that it sunk in Region 1 is $1 - \alpha_1$. What is the conditional probability the submarine is sunk in Region 1, given that the search of Region 1 was unsuccessful? Be sure to clearly define events and probabilities!

Solution

Problem 4

Statement

Let X and Y be random variables, where X is the number of workplace injuries occurring in a factory on any given day and Y is the level of activity in the factory. The joint distribution of X and Y is given by

$$f_{XY}(x, y) = \begin{cases} \frac{e^{-y}y^x}{6x!} & 0 < y < 6, \quad x = 0, 1, 2, ... \\ 0 & otherwise \end{cases}$$

a. Are X, Y statistically independent? Explain.
b. One can show that $Y \sim Uniform(0, 6)$ and $X|Y \sim Poisson(\lambda = y)$ if $y \in (0, 6)$ Use that information to find:
   i. E(X|Y = y)
   ii. Var(X|Y = y)
   iii. E(Y)
   iv. Var(Y)
c. Show $E[XY] = 12$ show each step clearly
d. Show $Var(X) = 6$ show each step clearly.

Solution

Problem 5

Statement

A coal-operated power plant has two smokestacks: one with a cleaning device known as a **scrubber** and another without a scrubber. An environmental engineer measures the amount of pollutant in each smokestack by weighing the particulate pollution in a random sample of a fixed volume of air. Let X be the amount of pollutant in a sample from the smokestack **without the scrubber**, and Y be the amount of pollutant in a sample of the same volume from the smokestack **with the scrubber**. The joint distribution can be written as,

$$f_{XY}(x, y) = \begin{cases} 1 & 0 \le x \le 2, \quad 0 \le y \le 1, \quad 2y \le x \\ 0 & otherwise \end{cases}$$

Find the probability that the sample taken from the smokestack **with the scrubber** shows a reduction in the amount of pollutant by one-third or more (as compared to the sample taken from the smokestack **without the scrubber**).

a. Make a clearly labeled graph to show the joint support of $f_{XY}(x, y)$ and the region of interest for finding the probability

b. Find the probability of interest *using integration.* **Make sure you have written a clear probability statement using proper notation along with its corresponding integral**.

c. *Extra Credit.* Explain how/why this particular problem can be solved without using integration, and find the probability using geometry.

Solution