# Homework # 5

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## Problem 1

### Statement

Let $X$ be a random variable representing the number of earthquakes in the Mount Saint Helen's region per day.

a. Researchers claim that $X$ can be modeled with a Poisson distribution. Explain why this is (or is not) a reasonable model for $X$.

b. Let the average number of earthquakes in a given day be 20. Using the Poisson model in (a), what is $E(X)$? Include an interpretation of the expectation in the context of this problem

c. Let $Y$ be the number of earthquakes in the next 12 hrs

   i. What is the distribution of Y? *Be sure to specify parameters and their values*

   ii. What is $P(Y \geq 2)$? Write a mathematical expression for this value and evaluate the expression. *You may use R for the calculation, but provide the code you used to find the desired probability.*

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### Solution

a. This is a reasonable model. It does not severely break any of the three Poisson criteria. The independence of non-overlapping intervals is the most dubious. Since they are earthquakes, the likelihood of subsequent earthquakes would depend on previous ones. So independence may be questionable. However, the effect of this would be *very* difficult to measure so a reasonable start would be a Poisson distribution. There is no time correlation in earthquakes, so one is not more likely to occur at 6am than 6pm, so chopping up the day into $n$ intervals the probability of one occurrence would be constant $\lambda/n$. Then we physically can't have 2 earthquakes happen at the same time, so the third Poisson criteria is met. The Poisson distribution is a reasonable model for $X$ with the given information.

b. For a Poisson distribution $E(X) = \lambda$, so in this case $E(X) = 20$. This means that we expect 20 earthquakes to occur on some day near Mount Saint Helen's.

c. i. Y is the number of earthquakes in 1/2 a day. So since Y = Number of successes in $k$ units of effort implies $Y \sim Poisson(k\lambda)$. From the previous problem (b) we have that $\lambda = 20$ so $Y \sim Poisson(1/2 * 2 =) = Poisson(10)$

   ii. We know that $P(Y) \geq 2 = 1 - P(Y < 2) = 1 - (P(Y = 0) + P(Y = 1))$. We have the $P(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!}$. So we have

$$P(Y) \geq 2 = 1 - (P(Y = 0) + P(Y = 1)) = 1 - (e^{-10} + 10e^{-10}) = 1 - 11e^{-10} = 0.9995006$$

   Where the final step using R as below

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1 - 11 * exp(-10)
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## [1] 0.9995006
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# Problem 2

## Statement

Suppose scientists expose 100 individual fish eggs from the same fish species to the same amount of a potentially harmful substance and then record how many fish have defects at age 6 weeks. The researchers are interested in estimating the probability of this species of fish developing defects after 6 weeks if they are exposed to this chemical.

a. Let Y be the number of fish at age 6 weeks that develop defects due to this experiment. Specify a reasonable distribution for the random variable Y.

b. Give at least one reason why your choice of distribution in (a) might be inadequate

c. Suppose the researchers inspect the 6-week-old fish one at a time until they observe the fourth fish with defects. Let Z be the number of fish inspected until the fourth fish with defects is identified. One of the researchers claims that it would be reasonable to model Z with a negative binomial distribution.

   i. Specify the appropriate value of r for the proposed model (i.e., write $Z \sim NegativeBinomial(r = ?, p)$

   ii. Explain to the researcher why this model is not appropriate for this situation.

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## Solution

# Problem 3

## Statement

A hospital receives 15% of its flu vaccine shipments from Company A and the remainder of its shipments from Company B. Suppose each shipment contains exactly 20 vaccine vials.

- For each of Company A's shipments, suppose 20% of the vials are ineffective
- For each of Company B's shipments, 10% of the vials are ineffective.
- The hospital plans to randomly select a shipment (of exactly 20 vaccine vials) and test 3 randomly selected vials from the shipment.

a. Assuming the 20 vials in the shipment are independently made (i.e., they do not come from the same batch) what is the probability that a shipment from Company A would have exactly 2 ineffective vials out of the 3 vials that are randomly selected and tested? *Be sure to clearly define events and random variables*

b. Suppose another shipment is randomly selected and it is missing its label. That is, you do not know which company the shipment came from.

   i. What is the probability 2 of 3 randomly selected vials from a randomly selected shipment would be ineffective? That is, what is the overall probability that a random selection of 3 vials from a shipment will be ineffective. *Again, be sure to clearly define events and random variables.*

   ii. If 2 of 3 randomly selected vials from a randomly selected shipment are found to be ineffective, what is the probability the shipment came from Company A? *Again, be sure to clearly define events and random variables.*

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## Solution

# Problem 4

## Statement

Consider the function: $p(y) = 1/5$ for $y \in \{3, 4, 5, 6, 7\}$

a. What is missing from p(y) to make it a legitimate probability function?

b. Create a clearly labeled plot for the pmf of Y

c. Create a clearly labeled plot for the CDF of Y

d. What is $Var(Y)$?

e. **Extra Credit** Find $E(\frac{1}{Y^2})$

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## Solution