Homework # 3

Elliott Pryor

9/11/2020

Problem 1

Statement

The amount of time (in hours), X, needed by a local repair shop, Rusts 'R Us, to repair a randomly selected piece of equipment is assumed to be an exponential random variable with pdf

$$f_X(x) = \begin{cases} 1/\lambda e^{-x/\lambda} & x > 0; \lambda > 0 \\ 0 & otherwise \end{cases}$$

and mgf $m_X(t) = (1 - \lambda t)^{-1}$.

1. Explain in words what the random variable, X, represents in this scenario. Why is X considered a random variable?
2. Identify the unknown parameter and explain in words what the parameter represents in this scenario
3. In a random sample of 5 repair times, Rusts 'R Us observes the following values (in hours): 14, 6, 3, 4, 1.5. Explain in words what n, $X_1 \dots X_n$, λ, and $x_1, \dots, x_n$ are in this situation, and, if applicable, give the numerical values of each.
4. Find the joint distribution of the random sample in part (c). List all assumptions made when finding this joint distribution and explain whether or not each is reasonable in this scenario.

Solution

Problem 2

Statement

Suppose the length of time, X, it takes a worker at the UPS store to assemble a box has the following pdf :

$$f_X(x) = \begin{cases} e^{-x-\Theta} & x > 0; \Theta > 0 \\ 0 & otherwise \end{cases}$$

Where Θ represents the minimum time until task completion. Let $X_1, \dots, X_n$ be a random sample of completion times from this distribution.

1. Derive (i.e, go through the steps, don't use the shortcut THM) the CDF of the smallest order statistic, $X_{(1)}$
2. Find the pdf of the smallest order statistic, $X_{(1)}$
3. Give an example of how the pdf of $X_{(1)}$ might be used by a UPS store manager and explain why it would be useful.