STAT 422: HW #6

Due: April 4, 2022, 11:59pm

Problem 1

Statement

Let $X_1, ..., X_n$ be a random sample from the distribution with the following pdf:

$$f(x;\theta) = \theta x^{\theta-1} I_{(0,1)}(x), \text{ where } \theta > 0$$

a. Find a complete sufficient statistic for θ

b. Using your answer in part (a), explain why $\prod_{i=1}^{n} X_i$ is also a sufficient statistic for θ.

Solution

Problem 2

Statement

A real estate firm wants to estimate the rate of new houses sold in a week in Bozeman. Assume $X_1, ..., X_n$ is a random sample of weekly house sales in Bozeman, where each X_i is a Poisson random variable with mean μ. The observed number of new houses sold per week for 5 randomly chosen weeks, were 2, 3, 3, 4, and 6. Find the best unbiased estimator (i.e., the UMVUE) of μ and its estimate. Show all work, and carefully explain why it is the UMVUE.

Solution

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