

1 Distributions

Uniform: $f(x) = \frac{1}{b-a}; a \leq x \leq b;$
 $\bar{X} = \frac{a+b}{2}; \text{Var}(x) = \frac{(b-a)^2}{12}$
 $mgf = \frac{e^{tb} - e^{ta}}{t(b-a)}$

Normal: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-1}{2\sigma^2}(x - \mu)^2;$
 $\bar{X} = \mu; \text{Var}(x) = \sigma^2;$
 $mgf = \exp \mu t + \frac{t^2 \sigma^2}{2};$

Exponential: $f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}};$
 $\bar{X} = \beta; \text{Var}(x) = \beta^2;$
 $mgf = (1 - \beta t)^{-1};$

Gamma: $f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}};$
 $\bar{X} = \alpha\beta; \text{Var}(x) = \alpha\beta^2;$
 $mgf = (1 - \beta t)^{-\alpha}; \Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$

Beta: $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1};$
 $\bar{X} = \frac{\alpha}{\alpha+\beta};$
 $\text{Var}(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)};$

2 Misc

Moment Generating Function:

$$m_{X_1, \dots, X_n} = \mathbb{E}[e^{tX_1 + \dots + tX_n}] \stackrel{iid}{=} \prod_{i=1}^n \mathbb{E}[e^{tX_i}]$$

Jacobian Method: $Y = g(X)$ with $g(X)$ monotone:

$$f_Y(y) = f_X(g^{-1}) \left| \frac{d}{dy} g^{-1}(y) \right|.$$

If $g(X)$ not monotone, then f_Y is sum of piecewise monotone $g_i(X)$.

3 Order Statistics

Maximum: $F_{X_{(n)}}(x) = F_X(x)^n,$
 $f_{X_{(n)}} = n(F_X(x))^{n-1} f_X(x)$

Minimum: $F_{X_{(1)}}(x) = 1 - (1 - F_X(x))^n,$
 $f_{X_{(1)}}(x) = n(1 - F_X(x))^{n-1} f_X(x)$

General:

$$f_{X_{(j)}} = \frac{n!}{(j-1)!(n-j)!} F_X(x)^{j-1} f_X(x) (1 - F_X(x))^{n-j}$$

4 Normal, Chi-Square Distributions

Mean: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n}).$

Given $X \sim N(\mu, \sigma^2), Z_i = \frac{X_i - \mu}{\sigma}.$ Then:
 $\sum_{i=1}^n Z_i^2 \sim \chi_n^2$

$$\chi_\nu^2 = \text{Gamma}(\frac{\nu}{2}, \beta = 2), \mathbb{E}[X] = \nu, \text{Var}(x) = 2\nu.$$

Theorem 7.3: \bar{x}, S^2 are independent, and:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2 \quad S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

Student T Distribution: if $Z \sim N(0, 1)$ and $W \sim \chi_\nu^2,$ and Z, W independent. Then RV: $T = \frac{Z}{\sqrt{W/\nu}}$ follows the student T distribution. Where:

$$f_T(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}} (1 + \frac{t^2}{\nu})^{-(\nu+1)/2}$$