# STAT 422: Homework #4

Due: March 02, 2022 (midnight)

# Problem 1

## Statement

Assume $X_1, X_2 \ldots X_n$ are independent, normal random variables with $E(X_i) = \mu_i$ and $Var(X_i) = \sigma_i^2$, $i = 1, 2, \ldots, n$. Recall that the mgf of the $N(\mu, \sigma^2)$ distribution is $e^{\mu t + \frac{t^2 \sigma^2}{2}}$.

a. Is $X_1, X_2 \ldots X_n$ a random sample, why or why not?

b. Find the distribution (including parameters) of $Y = \sum_{i=1}^{n} X_i$ using mgfs. Explain, in words, what this result indicates and how you think it is useful.

c. THM 6.3 in your book is a generalization of the result you found in part (b). Explain, in words, how this builds upon the result you found in (b).

d. Give the distribution (including parameters) of each of the following random variables. Justify/explain each step in your path to determining each constructed RVs corresponding distribution. *You may use the theorems you proved in parts a-c and THM 6.3.*

- $3X_1 + 5X_2$
- $Z_1^2 + Z_2^2$, where $Z_i = \dfrac{X_i - \mu_i}{\sigma_i}$ for $i = 1, 2, ..., n$
- $\dfrac{Z_2 - Z_1}{\sqrt{2}}$, where $Z_i = \dfrac{X_i - \mu_i}{\sigma_i}$ for $i = 1, 2, ..., n$
- $\dfrac{(Z_1 + Z_2)^2}{(Z_3 - Z_4)^2}$, where $Z_i = \dfrac{X_i - \mu_i}{\sigma_i}$ for $i = 1, 2, ..., n$

## Solution

# Problem 2

## Statement

Let $X_i$, $i = 1, 2, 3$ be independent with $N(i, i^2)$ distributions. That is, $X_1 \sim N(1, 1^2)$, $X_2 \sim N(2, 2^2)$, and $X_3 \sim N(3, 3^2)$. For each of the following situations, use any subset of the

$X_i$s to construct a random variable with the stated distribution. Justify why each constructed random variable follows the corresponding distribution of interest.

a. Standard normal distribution
b. Chi-squared distribution with 3 degrees of freedom.
c. $t$ distribution with 2 degrees of freedom
d. $F$ distribution with 1 numerator and 2 denominator degrees of freedom

## Solution