1 Distributions

Uniform: $f(x) = \frac{1}{b-a}; a \leq x \leq b;$
$\bar{X} = \frac{a+b}{2}$; $Var(x) = \frac{(b-a)^2}{12}$
$mgf = \frac{e^{tb} - e^{ta}}{t(b-a)}$

Normal: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-1}{2\sigma^2}(x-\mu)^2;$
$\bar{X} = \mu$; $Var(x) = \sigma^2$;
$mgf = \exp \mu t + \frac{t^2\sigma^2}{2}$;

Exponential: $f(x) = \frac{1}{\beta} e^{\frac{-x}{\beta}}$;
$\bar{X} = \beta$; $Var(x) = \beta^2$;
$mgf = (1 = \beta t)^{-1}$;

Gamma: $f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{\frac{-x}{\beta}}$;
$\bar{X} = \alpha\beta$; $Var(x) = \alpha\beta^2$;
$mgf = (1 = \beta t)^{-\alpha}$; $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$

Beta: $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$;
$\bar{X} = \frac{\alpha}{\alpha+\beta}$;
$Var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$;

2 Misc

Moment Generating Function:
$m_{X_1,\ldots X_n} = \mathbb{E}[e^{tX_1+\cdots+tX_n}] \stackrel{iid}{=} \prod_{i=1}^n \mathbb{E}[e^{tX_i}]$

Jacobian Method: $Y = g(X)$ with $g(X)$ monotone:
$f_Y(y) = f_X(g^{-1}) \left| \frac{d}{dy} g^{-1}(y) \right|$.
If $g(X)$ not monotone, then f_Y is sum of piecewise monotone $g_i(X)$.

3 Order Statistics

Maximum: $F_{X_{(n)}}(x) = F_X(x)^n$,
$f_{X_{(n)}} = n(F_X(x))^{n-1} f_X(x)$

Minimum: $F_{X_{(1)}}(x) = 1 - (1 - F_X(x))^n$,
$f_{X_{(1)}}(x) = n(1 - F_X(x))^{n-1} f_X(x)$

General:
$f_{X_{(j)}} = \frac{n!}{(j-1)!(n-j)!} F_X(x)^{j-1} f_X(x)(1 - F_X(x))^{n-j}$

4 Normal, Chi-Square Distributions

Mean: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.

Given $X \sim N(\mu, \sigma^2)$, $Z_i = \frac{X_i - \mu}{\sigma}$. Then:
$\sum_{i=1}^n Z_i^2 \sim \chi_n^2$

$\chi_\nu^2 = Gamma(\frac{\nu}{2}, \beta = 2)$, $\mathbb{E}[X] = \nu, Var(x) = 2\nu$.

Theorem 7.3: $\bar{x}, S^2$ are independent, and:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2 \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x})^2$$