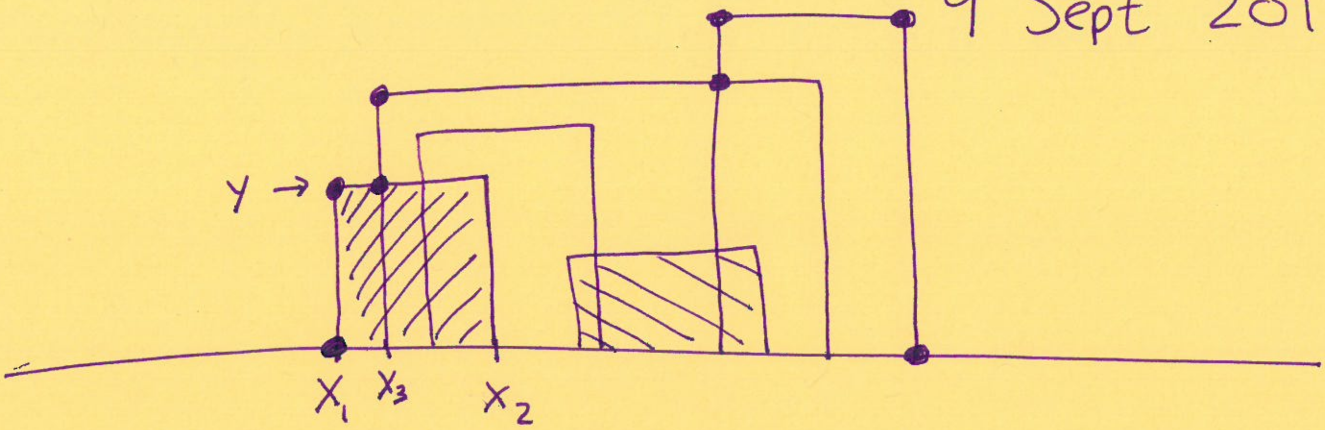
9 Sept 2019



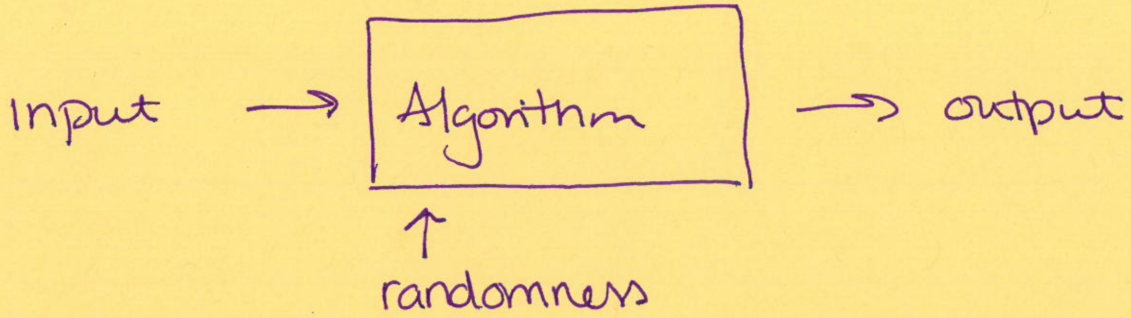
building = triple (x_1, x_2, y)

here: $[(x_1, 0), (x_1, y), (x_3, y), \ldots$

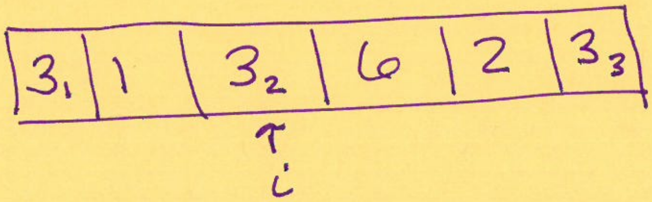
Today: Randomized Algorithms

```
RQUICKSORT (A)
    i ← random int betwn 1 & |A|, inclusive
    (Base case needed))
    before ← all elts less than A[i]
    after ← all elts greater than A[i]
    equal ← all elts, = A[i]
    b ← RQUICKSORT (before)
    a ← RQUICKSORT (after)
    return [b, equal, a]
```

Randomized Algorithms



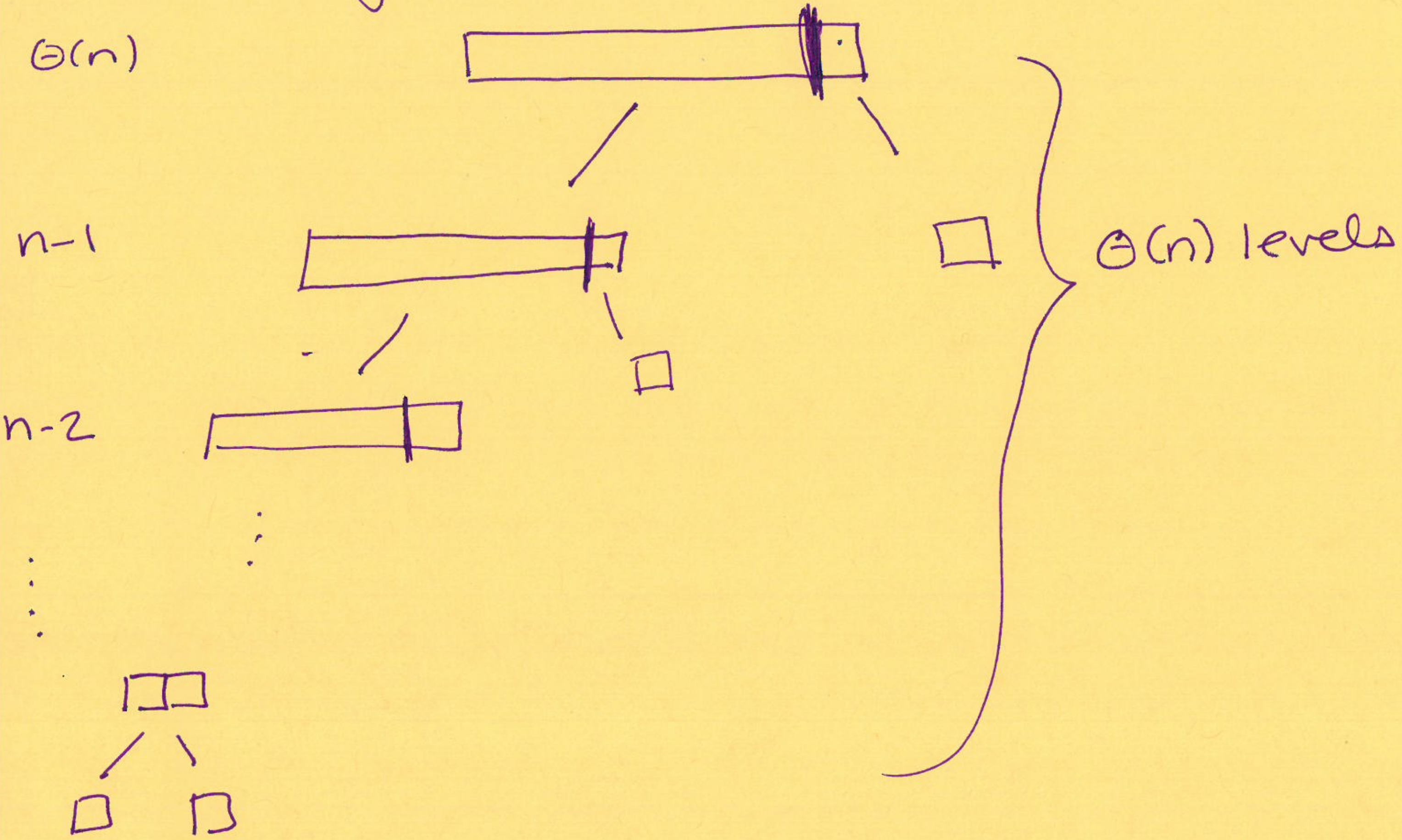
- a deterministic algorithm will produce the same output given the same input, every time.
- Analysis of runtime is often the average case analysis.
- a stable sort is one that does not change the order of equal-valued inputs.



before = [1, 2]
after = [6]
equal = [3_1, 3_2, 3_3]

Worst Case Runtime Of RQUICKSORT:

- occurs when rand. elt. is always the largest elt.



$$\sum_{i=0}^{n} n-k = \Theta(n^2)$$

Recursion:

$$T(n) = \underbrace{T(n-1) + \Theta(1)}_{\text{2 cases}} + \underbrace{\Theta(n)}_{\text{time to split}}$$

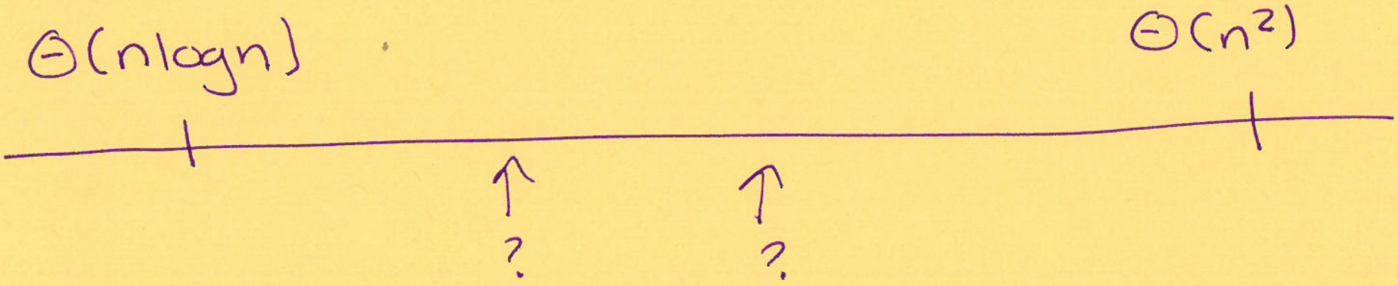
$$\left.\begin{array}{l} T(n) = T(n-1) + \Theta(n) \\ \text{Sol'n: } T(n) \in \Theta(n^2) \end{array}\right\}$$

Best Case:

- Choose the midpoint each time
- Then: $T(n) = 2T(n/2) + \Theta(n)$

$$= \Theta(n \log n)$$

Average Case: What is the expected runtime?



In expectation, the runtime is $\Theta(n \log n)$

How? This is surprising!!

Consider Flipping one $\overset{\wedge}{}$ coin:
fair

$H \leftarrow +\$1$

$T \leftarrow -\$1$

Best Case: $+\$1$

Worst Case: $-\$1$

Expected: $\displaystyle\sum_{v_i} v_i \,\mathbb{P}(v_i) = 1\,\mathbb{P}(H) + (-1)\cdot\mathbb{P}(T)$

$v_i \in$ ~~values~~ outcomes

$= 1\cdot\frac{1}{2} + (-1)(\frac{1}{2})$

$= \$0$

<u>Def</u>: A random variable is a measurable fcn

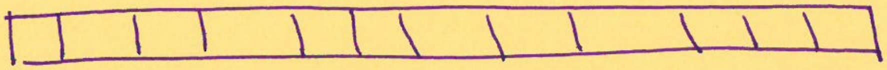
$$X: \Omega \longrightarrow \mathbb{R}$$

↑probability space

= the set of all outcomes

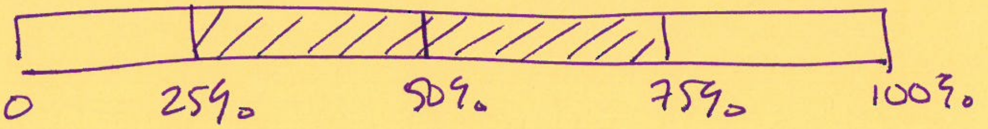
= the set of all input & randomness pairs

Randomized Analysis of Quicksort



$$\mathbb{P}(\text{picked median}) = \frac{\#\text{ occur. of median}}{n}$$

$$= \frac{1}{n}, \text{ if unique values}$$



$\mathbb{P}$(chosen in ▨) $= \frac{1}{2}$

Question: using this info, show
Expect RT of Quicksort is
$\Theta(n \log n)$

- Assume we always pick a pivot in ▨, what is worst case? 25/75

$$T(n) = T(n/4) + T\left(\frac{3n}{4}\right) + \Theta(n)$$

$$2T\left(\frac{n}{4}\right) + \Theta(n) \leq T(n) \leq 2T\left(\frac{3n}{4}\right) + \Theta(n)$$