

4 October 2019

Greedy

Given: set of tasks $T = \{ [b_i, f_i) \}_{i=1}^n$,
find subset of max size s.t.
no 2 tasks overlap

Sol'n:

Schedule (T)

$\Theta(n \log n)$ Sort T from earliest to latest finish time
~~Decision~~
 $\Theta(1)$ $S \leftarrow \{ [b_1, f_1) \}$ // assuming $T \neq \emptyset$
repeat n times
 $last \leftarrow f_1$ // finish time of last ed. of S.
 for $i = 2 \dots n$
 if $b_i \geq last$
 add $[b_i, f_i)$ to S
 $last \leftarrow f_i$
 end if
 end for
return S

$$\Theta(n \log n) + \Theta(1) + n \cdot \Theta(1) = \Theta(n \log n)$$

note: if given sorted array, runtime can
be linear b/c we skip the sort step.

problem :

Given $T = \{ [b_i, f_i] \}_{i=1}^n$

find S such that

① $|S| < \infty$ "S is finite"

② $S \cap t \neq \emptyset \quad \forall t \in T$

③ $|S|$ minimized

Check Tasks (T)

sort by end time

~~times $\leftarrow [b_1, f_1], [b_2, f_2], \dots$~~

~~$[b_1, f_1], [b_2, f_2], \dots$~~

times $\leftarrow \emptyset$

while we haven't reached
the end of T _{finishtime}

→ add $\bullet T.\text{pop}()$ to times

→→ pop off additional
intervals that
overlap that finish
time.

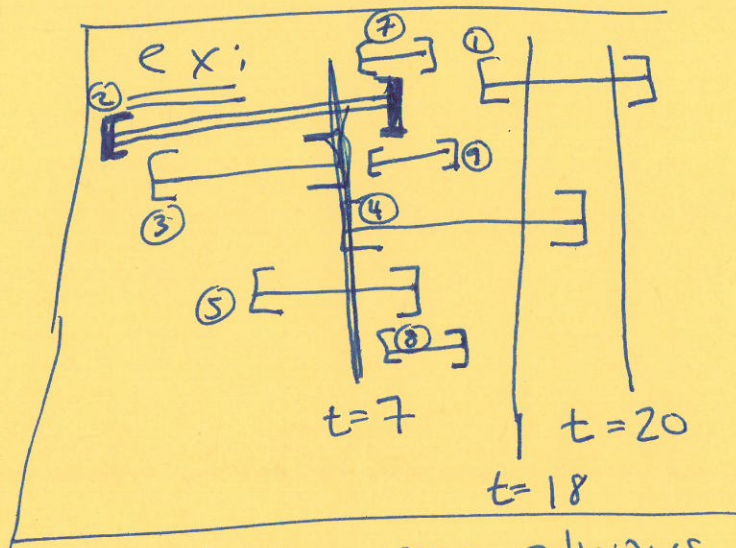
end while

return times

nested
loop!

$i = \#$ items that have been
popped.

$L_i = \text{times}$ is a subset of an optimal solution that
covers all popped intervals
and does not cover unpopped intervals.



a note: you can always
choose a n endpoint
(something to be proven!)

• Consider 1st interval
to end. ~~1~~

① no matter what,
need a pt in that
interval.

② Choosing rightmost
endpoint insures
max coverage of
other intervals.

in groups: Section 17.8: largest rectangle

① The x-coords of the rectangle must be start/end coords of buildings

② ~~Brute Force~~ Brute Force, try every start/end pair is $\Theta(n^3)$. Can you explain why?

③ Another approach can be $\Theta(n^2)$.

also, $\Theta(n)$.

↑
real good!