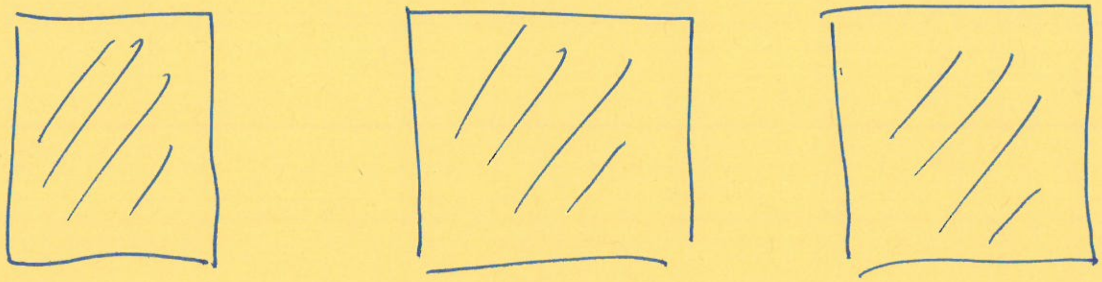
13 Sept 2019

Monty Hall Problem

A
F
F



Choice

F

Choice → to switch

$\mathbb{P}(F) = 1/3$

$\mathbb{P}(A) = 2/2$

In-Class Exercise 04

CSCI 432

September 11, 2019

Disc Circle

Group Number:
Group members present today:

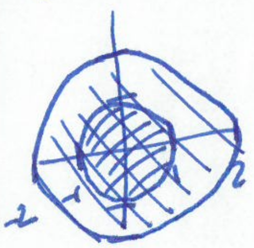
Minimum Enclosing Disc (MED)

Definitions:

center radius

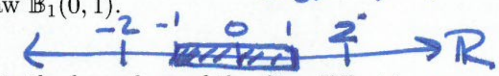
1. Given $c \in \mathbb{R}^2$ and $r \in \mathbb{R}$ such that $r \geq 0$, we define the disc $D(c, r) := \{x \in \mathbb{R}^2 \mid ||x - c|| \leq r\}$.

$\mathbb{R}^2$



   - Draw $D(0, 1)$ and $D(0, 2)$. Note: $0 \in \mathbb{R}^2$ is the origin $(0, 0)$.
   - Let $d > 1$. The generalization of a two-dimensional disc is a the *Euclidean metric ball.* Give a general definition of a ball in $\mathbb{R}^d$. Denote this ball by $\mathbb{B}_d(c, r)$.

     $\mathbb{B}_d(c,r) = \{x \in \mathbb{R}^d \mid ||x-c|| \leq r\}$
   - Draw $\mathbb{B}_1(0, 1)$.



2. A *circle* is the boundary of the disc. What is an equation that defines the circle $C(c, r)$?

   $C(c,r) := \{x \in \mathbb{R}^2 \mid ||x-c|| = r\} = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = r^2\}$

Problem Statement: Let $P \subset \mathbb{R}^2$, with $|P| = n \in \mathbb{N}$. We wish to find the smallest radius r such that there exists a $c \in \mathbb{R}^2$, where $P \subset D(c, r)$.

unknowns/degrees of freedom: 3
$c = (c_x, c_y)$
r

1. If $n = 1$, what is the minimum enclosing disc? Is it unique?
2. If $n = 2$, what is the minimum enclosing disc? Is it unique?
3. If $n = 3$, what is the minimum enclosing disc? Is it unique?
4. If $n = 4$, what are the possible cases that could arise? How do we decide what the MED is?
5. Use the following to consider the general case: consider the following: choose a point p at random. Remove p from P to obtain P' and compute ~~SEB~~ MED of P'. What are the two cases that can happen when we add p back in? What is the probability of each?
6. For the expected time analysis, what is the recursion that we have? What is the closed form?
7. Challenge: In $\mathbb{R}^d$, how many points are needed in order to uniquely define a ball whose boundary contains those points?

point set to cover

must be on the boundary

MED (P, S)

Base Case(s)

p ← random pt in P

tryme ← MED (P \ {p}, S)

if p ∈ tryme

    return tryme

else

    return MED (P \ {p}, S ∪ {p})