§4.1 of CLRS

							price

day 1 2 3 . . . . . n

want: to buy stock on day i + sell on day j such that $price[j] - price[i]$ is maximized.

ex:

p:	3	6	1	7	2	9	8	5
	1	2	3	4	5	6	7	8

argmin (p) = 3

min (p) = 1

$i = \text{argmin (price)}$

$j = \text{argmax (price)}$

can't do that! As, we also need $i < j$.

Sol'n 1: Brute force

Try every $\binom{n}{2}$ possibility

$$\binom{n}{2} = \frac{n(n-1)}{2} = \Theta(n^2)$$

Sol'n 2: want $o(n^2)$

$$\boxed{T(n) = \begin{cases} \Theta(1) & \text{if } n \le 1 \\ 2T(n/2) + \Theta(n), & n > 1 \end{cases}} \quad \text{So, } T(n) = \Theta(n \log n)$$

```
          BuySELL (price)
$\Theta(1)$ {   if n ≤ 1
               | return 0
             endif
T(n/2)       a ← BuySELL (price[1 .... ⌊n/2⌋])
T(n/2)       b ← BuySELL (price[⌊n/2⌋+1 .... n])
$\Theta(n)$ {   leftmin ← min value in 1st half
             rightmax ← max value in 2nd half
             return max{a, b, rightmax - leftmin}
```

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Group Exercises:

① Show $\frac{1}{2}n^2 - \frac{1}{2}n$ is $O(n^2)$.

② What is the closed form of $T(n) = T(n/2) + 1$, wher $T(1) = 1$?

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① $f(n) = \frac{1}{2}n^2 - \frac{1}{2}n$ is $O(n^2)$

Let $n_0 = 1$, $c = 1$.

Then, $f(n) = \frac{1}{2}n^2 - \frac{1}{2}n$

$\leq \frac{1}{2}n^2 + \frac{1}{2}n, \forall n \geq 0$

$\leq \frac{1}{2}(\underline{n^2 + n})$

$\leq \frac{1}{2}(2n^2)$, since $n \leq n^2 \ \forall n \geq 1$

$\leq 1 \cdot n^2$

So, $f(n) \leq c \cdot n^2$, as was to be shown.

Scratch:

$\frac{1}{2}n^2 - \frac{1}{2}n \leq c \cdot n^2$

$\frac{1}{2}n^2 + \frac{1}{2}n \leq c \cdot n^2$

$2(\frac{1}{2}n^2) \leq c \cdot n^2$

$n^2 \leq 1 \cdot n^2$

② $T(n)$ is Θ

example: Binary search

1    $T(n)$ 1

1    $T(n/2)$ 1

1    $T(n/4)$

⋮

1

$\log_2 n$ times:

$$\sum_{i=0}^{\log n} 1 = 1 \cdot \log n = \log n$$