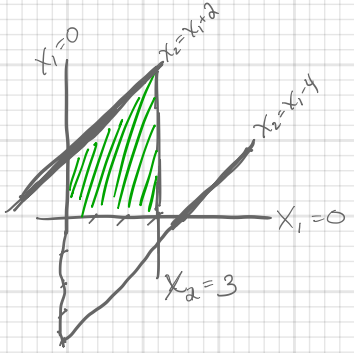
# SIMPLEX EXAMPLE

maximize $x_1 + 2x_2$

subject to:
$-x_1 + x_2 \leq 2$
$-x_1 + x_2 \geq -4$
$x_1 \leq 3$
$x_1, x_2 \geq 0$




**Slack Form:** max $x_1 + 2x_2$

subject to

$$x_3 = 2 + x_1 - x_2$$
$$x_4 = 4 - x_1 + x_2$$
$$x_5 = 3 - x_1$$
$$x_1, x_2 \geq 0$$
$$x_3, x_4, x_5 \geq 0$$

**sol'n 1:** $(0, 0, 2, 4, 3)$ ($x_1, x_2, x_3, x_4, x_5$)

value: $x_1 + 2x_2 = 0 + 2 \cdot 0 = 0$

$c_1 = 1$, $c_2 = 2$ Pivoting on $x_2$.

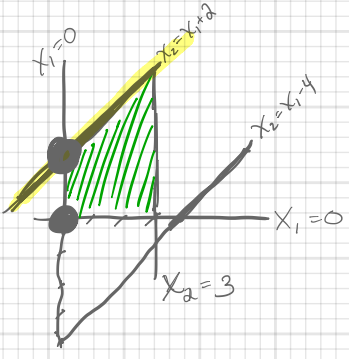
$x_3 = 2 + x_1 - x_2$ : limiting by $x_2 = 2$

$x_4 = 4 - x_1 + x_2$ $x_2 = \infty$

$x_5 = 3 - x_1$ $x_2 = \infty$

→ solve for $x_2$:

$x_2 = 2 + x_1 - x_3$




**NEXT LP!**

max $x_1 + 2x_2$

$= x_1 + 2(2 + x_1 - x_3)$

$= x_1 + 4 + 2x_1 - 2x_3$

$= 4 + 3x_1 - 2x_3$

subject to

$$x_2 = 2 + x_1 - x_3$$
$$x_4 = 4 - x_1 + 2 + x_1 - x_3$$
$$= 6 - x_3$$
$$x_5 = 3 - x_1$$

Basic: $x_1, x_3$ ⇒ next sol'n: $(0, 2, 0, 6, 3)$

NEXT LP:

max $4 + 3x_1 - 2x_3$

subject to

$x_2 = 2 + x_1 - x_3$ — How big can $x_1$ go? $\infty$

$x_4 = 6 - x_3$ — $\infty$

$x_5 = 3 - x_1$ — 3

Sol'n 2: $(0, 2, 0, 6, 3)$ value: $4 + 3x_1 - 2x_3 = 4 = x_1 + 2 \cdot x_2$

$c_1 = 3$, $c_2 = 0$, $c_3 = -2$

Pivot on $x_1$

solve for $x_1$

$x_1 = 3 - x_5$ & substitute

Next LP:

max $4 + 3(3 - x_5) - 2x_3$

$= 4 + 9 - 3x_5 - 2x_3$

$= 13 - 3x_5 - 2x_3$

subject to: $x_2 = 2 + (3 - x_5) - x_3$

$= 5 - x_3 - x_5$

$x_4 = 6 - x_3$

$x_1 = 3 - x_5$

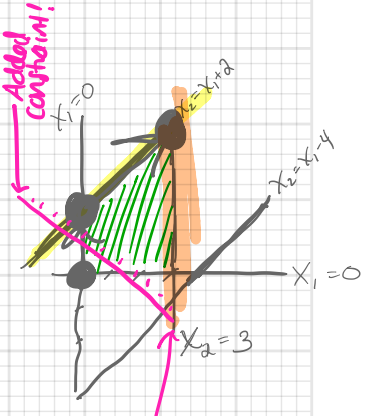
$x_i \geq 0$

Basic variables: $x_3, x_5$

Non basics: $x_1, x_2, x_4$

Sol'n 3: $(3, 5, 0, 6, 0)$

value: $13 - 3x_5 - 2x_3 = 13 = 3 + 2 \cdot 5$




Done ☺

try it again, with this constraint.