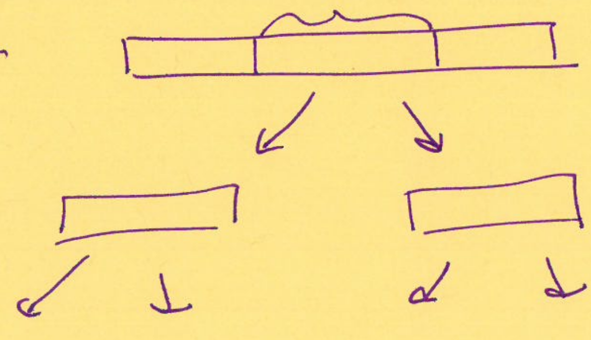
11 Sept 2019

rand var. X	Ω	$\mathbb{P}(X = *)$	rand var Y = # times X until reach X = *
result of coin flip H=1, T=0	H=1 T=0	$\mathbb{P}(X = H) = \frac{1}{2}$ if fair	$\mathbb{E}(Y) = \frac{1}{1/2} = 2$
roll die	1 2 3 4 5 6	$\mathbb{P}(X = 3) = \frac{1}{6}$ if fair	$\mathbb{E}(Y) = \frac{1}{1/6} = 6$
index into an array of size n	1 . . . . n	$\mathbb{P}\left(X \in \left[\frac{n}{4}, \frac{3n}{4}\right]\right) = \frac{1}{2}$	$\mathbb{E}(Y) = 2$

Analysis of RQuicksort I

expected # of layers $= 2\log n$



← each layer has $\leq n$ entries in the arrays in total.

$\Rightarrow RT(QS) = \Theta(n \log n)$

Analysis of RQS - 2

$T(n) :=$ RT of RQS on input of size n. = total # comparisons

recursion:

$$T(n) = \begin{cases} 0 & , n \leq 1 \\ (n-1) + \frac{1}{n}\sum_{i=0}^{n-1} T(i) + T(n-i-1) \end{cases}$$

$(n-1)$ ↑ comparisons against pivot

Claim: $T(n)$ is $\Theta(n\log n)$, in ~~average~~ expected runtime.

<u>Proof</u>: ($T(n)$ is $O(n\log n)$)

Let $n_0 = 1$, $c = 2$.

<u>Base case</u>: $T(1) = 0 \leq 2 \cdot 1\log 1 = 0$

Let $n > 2$.

Let our ind. assumption be

$T(i) \leq 2 \cdot i\log i \quad \forall i < n$ and $2 \leq i$

(WTS: $T(n) \leq 2 \cdot n\log n$.)

Facts about asymptotics

- $\Theta(n) = \Theta(2n)$
- $\Theta(\log_2 n) = \Theta(\log_{10} n) = \Theta(\log_e n) \ldots$
- $O(1) \subset O(\log n) \subset O(n) \subset O(n \log n)$

constant — lograthmic

$$O(n \log n) \subset O(n^2) \subset O(n^3)$$

quadratic ($O(n^2)$), cubic ($O(n^3)$)

$n = 3$

$$\sum_{i=0}^{n-1} (T(i) - T(n-i-1)) = \begin{array}{l} T(0) + T(2) \\ T(1) + T(1) \\ \underline{T(2) + T(0)} \\ 2T(0) + 2T(1) + 2T(2) \end{array}$$

Proof (cont.)

$$T(n) = n-1 + \frac{1}{n}\sum_{i=0}^{n-1}\big(T(i) + T(n-i-1)\big)$$

by rearranging.

$$= n-1 + \frac{2}{n}\sum_{\substack{\cancel{i=0}\\ i=1}}^{n-1} T(i)$$

by ind. assumpt.

$$\le n-1 + \frac{2}{n}\sum_{i=1}^{n-1} 2\cdot i\log i$$

$$\le n-1 + \frac{2}{n}\int_{i=1}^{n} 2x\log x\,dx$$

since $2x\log x$ is increasing

$$= n-1 + \frac{2}{n}\left[x^2\log x - \frac{1}{2}x^2\right]_1^n$$

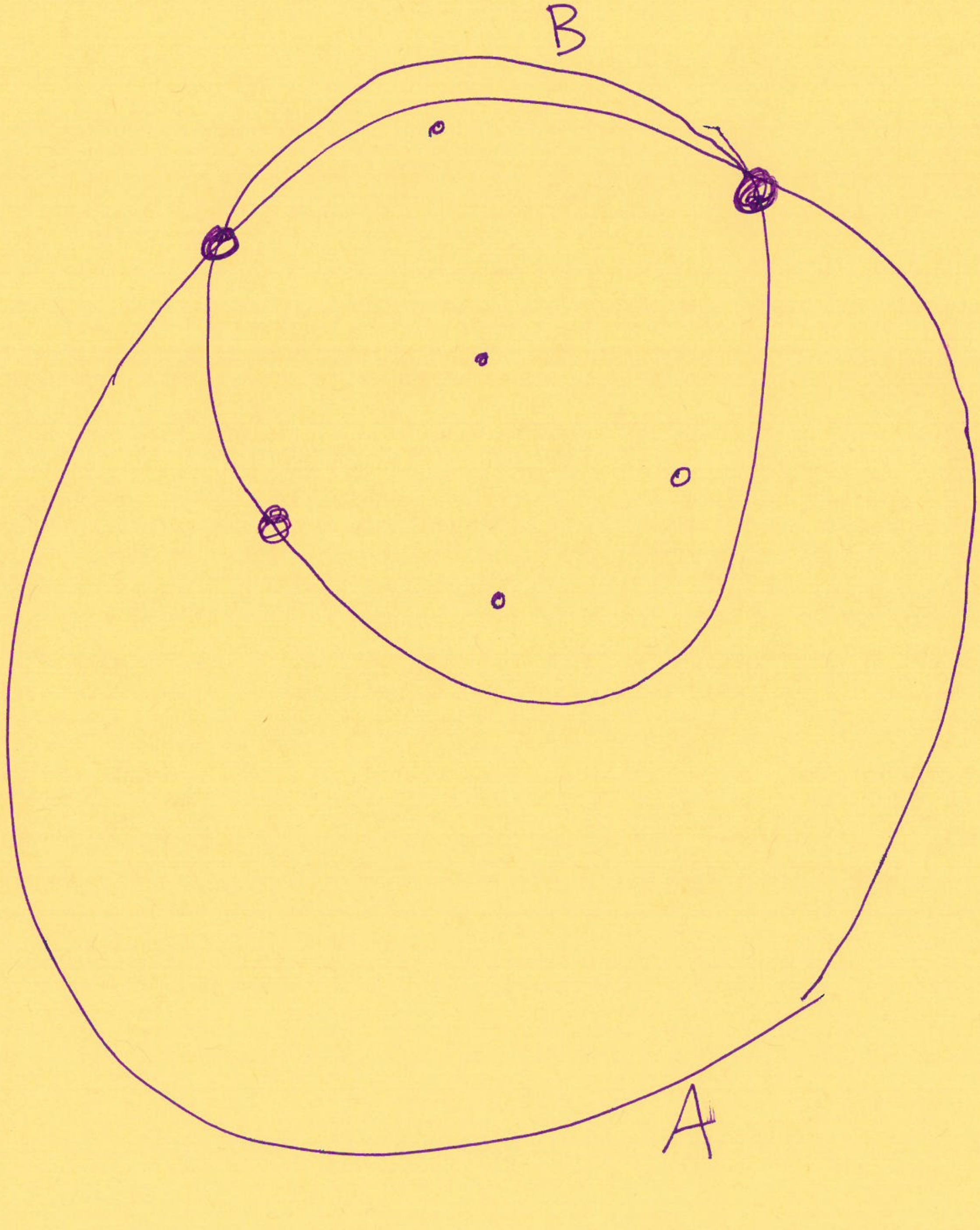
$$= n-1 + \frac{2}{n}\left(\left(n^2\log n - \frac{1}{2}n^2\right) - \left(0 - \frac{1}{2}\right)\right)$$

$$= \cancel{n} - 1 + 2n\log n - \cancel{n} + \frac{1}{n}$$

$$\le 2n\log n \qquad \square$$

Given: $P \subseteq \mathbb{R}^2$, $|P| < \infty$

What is the radius of the smallest enclosing $\binom{\text{ball}}{\text{disc}}$?



In-Class Exercise 04

CSCI 432

September 11, 2019

Group Number:
Group members present today:

Minimum Enclosing Disc (MED)

Definitions:

1. Given $c \in \mathbb{R}^2$ and $r \in \mathbb{R}$ such that $r \geq 0$, we define the disc $D(c, r) := \{x \in \mathbb{R}^2 \mid ||x - c|| \leq r\}$.
   - Draw $D(0, 1)$ and $D(0, 2)$.
     ↳ $\vec{0} = (0,0)$
   - Let $d > 1$. The generalization of a two-dimensional disc is a the *Euclidean metric ball.* Give a general definition of a ball in $\mathbb{R}^d$. Denote this ball by $\mathbb{B}_d(c, r)$.
   - Draw $\mathbb{B}_1(0, 1)$.
2. A *circle* is the boundary of the disc. What is an equation that defines the circle $C(c, r)$?

Problem Statement: Let $P \subset \mathbb{R}^2$, with $|P| = n \in \mathbb{N}$. We wish to find the smallest radius r such that there exists a $c \in \mathbb{R}^2$, where $P \subset D(c, r)$.

1. If $n = 1$, what is the minimum enclosing disc? Is it unique?
2. If $n = 2$, what is the minimum enclosing disc? Is it unique?
3. If $n = 3$, what is the minimum enclosing disc? Is it unique?
4. If $n = 4$, what are the possible cases that could arise? How do we decide what the MED is?
5. Use the following to consider the general case: consider the following: choose a point p at random. Remove p from P to obtain P' and compute SEB of P'. What are the two cases that can happen when we add p back in? What is the probability of each?
6. For the expected time analysis, what is the recursion that we have? What is the closed form?
7. Challenge: In $\mathbb{R}^d$, how many points are needed in order to uniquely define a ball whose boundary contains those points?