4 Nov 2019

<u>Theorem</u>: Let f is a flow in flow network $G=(V,E,c)$
Then, the following are equiv:

(1) f is a max flow

(2) Residual graph has no s-t path
($\nexists$ augmenting path)

(3) $\exists$ cut (S,T) such that $|f| = c(S,T)$

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(2) $\Rightarrow$ (3)

f is a flow in $G=(V,E,c)$

$\nexists$ augmenting path. / $\nexists$ an s-t path in residual graph.

Let $S = \{ v \in V \mid \exists$ path from s to v in the residual graph$\}$.

Note: $s \in S$, $t \in T$ by (2)

$$|f| = f(S,T) \quad \text{by a Lemma from the other day}$$

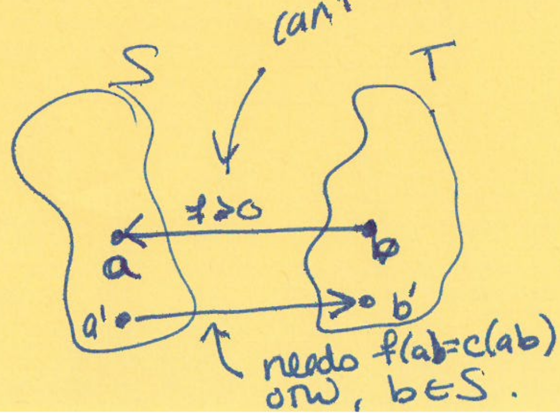
$$= \sum_{a\in S}\sum_{b\in T} f(a,b) - \underline{\sum_{a\in S}\sum_{b\in T} f(b,a)} \quad \text{by def.}$$

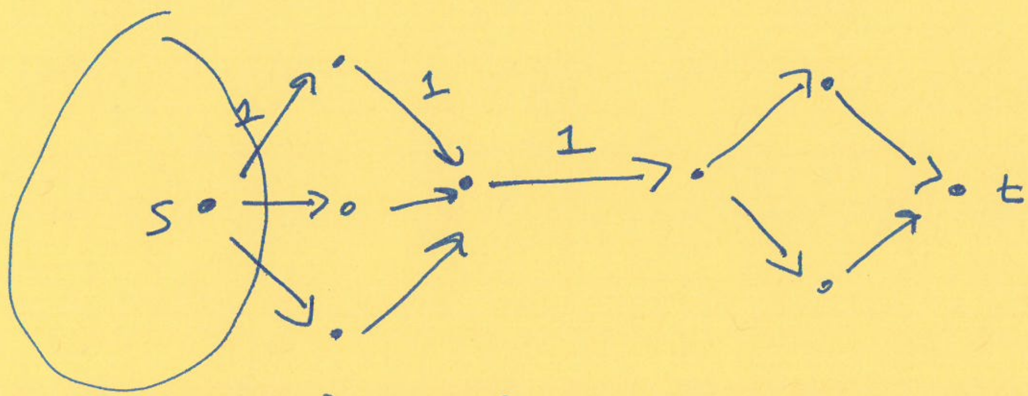
$$= \sum_{a\in S}\sum_{b\in T} f(a,b) - 0$$

$$= \sum_{a\in S}\sum_{b\in T} c(a,b)$$

$$= c(S,T)$$

$\square$





$c(S,T) = 3$

Linear Programming

linear fcn: "Line throug orig." $f(x_1, \dots x_n) = a_1 x_1 + a_2 x_2 + \dots + a_n x_n = \sum_{i=1}^{n} a_i x_i$

affine fcn: "any line" $g(x_1, \dots, x_n) = a_1 x_1 + a_2 x_2 + \dots + a_n x_n + a_0$

Linear equation. Let f be a linear fcn

$f(\vec{x}) = c$

linear inequality:

$f(\vec{x}) \leq c$ or $f(\vec{x}) \geq c$

linear constraints can be either

for some $c \in \mathbb{R}$

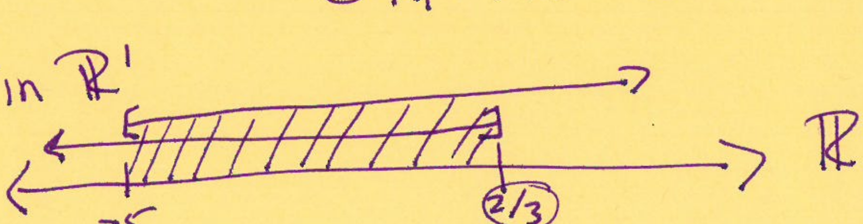
Linear Program (LP) problem

- maximize or minimize a linear fcn subject to a finite set of linear constraints.

Note: $x_i \geq 0$ is a linear constraint where $a_j = 0 \ \forall j \neq i$

$0 \cdot x_1 + \dots + 1 x_i + 0 x_{i+1} \dots + 0 x_n \geq 0$

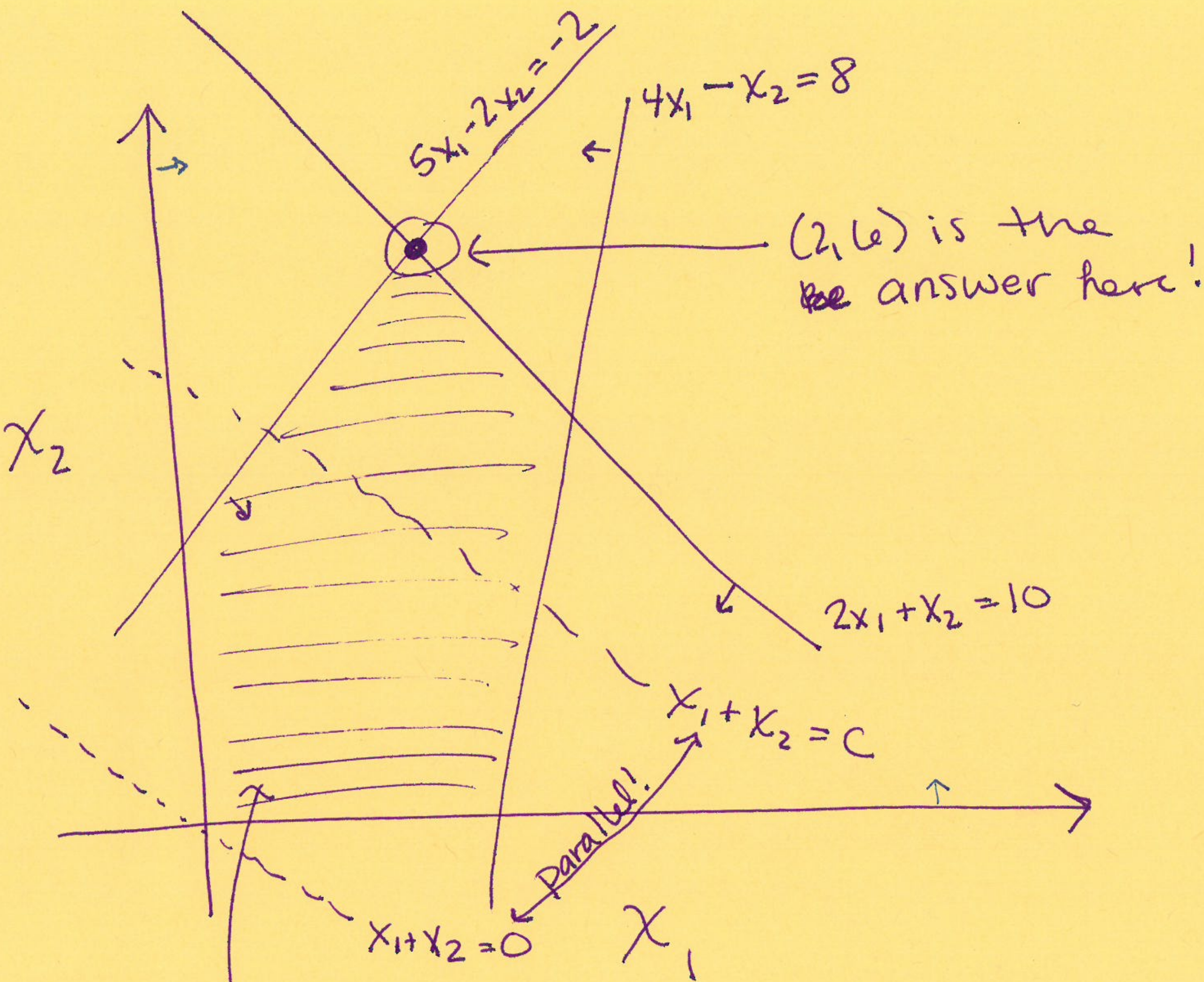
e.g., in $\mathbb{R}^1$



max x subject to $6x \leq 4$

$2x \geq -10$

Ans: $x = 2/3$



feasible solutions to the given linear constraints.

maximize objective: asking for an extreme point in a given direction of the feasible space.

Standard form for LP: (in $\mathbb{R}^n$)

maximize $\sum_{i=1}^{n} a_i x_i$

subject to

$$\sum_{i=1}^{n} c_{ij} x_i \leq b_j \quad \text{for } j = 1 \ldots m$$

$$x_i \geq 0 \quad \text{for } i = 1 \ldots n$$

m linear constraints in $\mathbb{R}^n$

- max not min!
- constraints are $\leq$, not $\geq$ or $=$
- x_i are non-negative.

In-Class Exercise 10

CSCI 432

28 October 2019

Group Number:
Group members present today:

Linear Programming

1. On this graph paper, draw the following lines:

$$4x_1 - x_2 = 8 \quad (1)$$
$$2x_1 + x_2 = 10 \quad (2)$$
$$5x_1 - 2x_2 = -2 \quad (3)$$
$$x_1 = 0 \quad (4)$$
$$x_2 = 0 \quad (5)$$

   HINT: use the bottom left corner as the point $(0, 0)$

2. Shade in the feasible space of points $(x_1, x_2) \in \mathbb{R}^2$) that satisfy the following inequalities:

$$4x_1 - x_2 \leq 8 \quad (6)$$
$$2x_1 + x_2 \leq 10 \quad (7)$$
$$\boxed{5x_1 - 2x_2 \geq -2} \quad (8)$$
$$x_1 \geq 0 \quad (9)$$
$$x_2 \geq 0 \quad (10)$$

3. As dotted lines, draw the lines where $x_1 + x_2 = 0$, $x_1 + x_2 = 2$, and $x_1 + x_2 = 4$ What do you notice about these lines?

4. Suppose we want to maximize $x_1 + x_2$ subject to the constraints above. It turns out that the answer always lies on a vertex of the space you just shaded! Compute $f(x_1, x_2) = x_1 + x_2$ for each of the vertices. Which point attains the maximum value?

5. Write this linear program in standard form.

6. What happens if I change the last constraint to $x_1 \leq 0$?