UNION-FIND

Quick UNION:

→ each vertex saves "parent" node &
to find the component label, go
up tree until you find the root

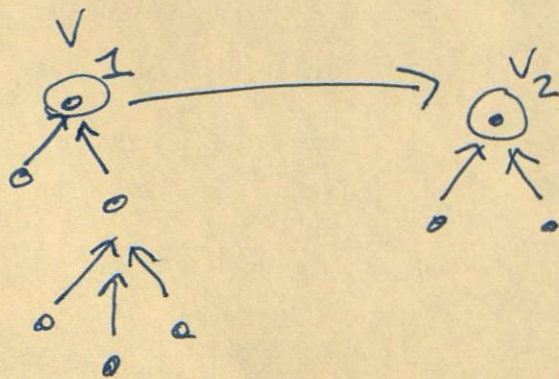UNION (given 2 root nodes): $\Theta(1)$

FIND (to find the root node): $\Theta(n)$
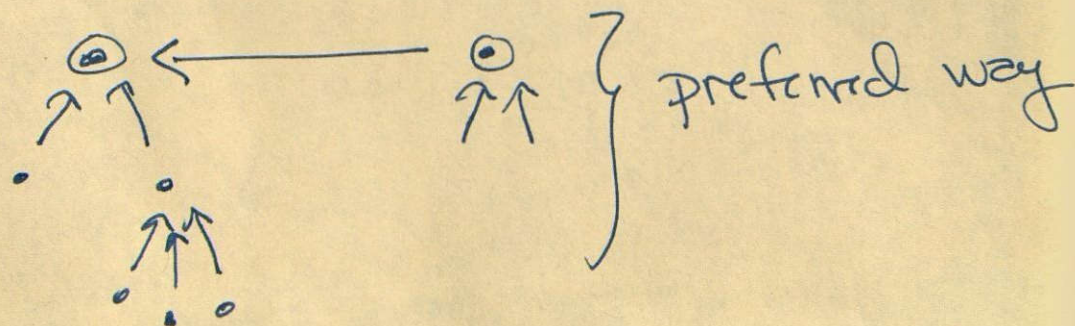
UNION (given 2 arb. nodes)

$\quad = $ 2 FINDS $+$ 1 UNION FROM ROOT NODES

$\quad = \Theta(n) + \Theta(n) + \Theta(1) = \Theta(n)$
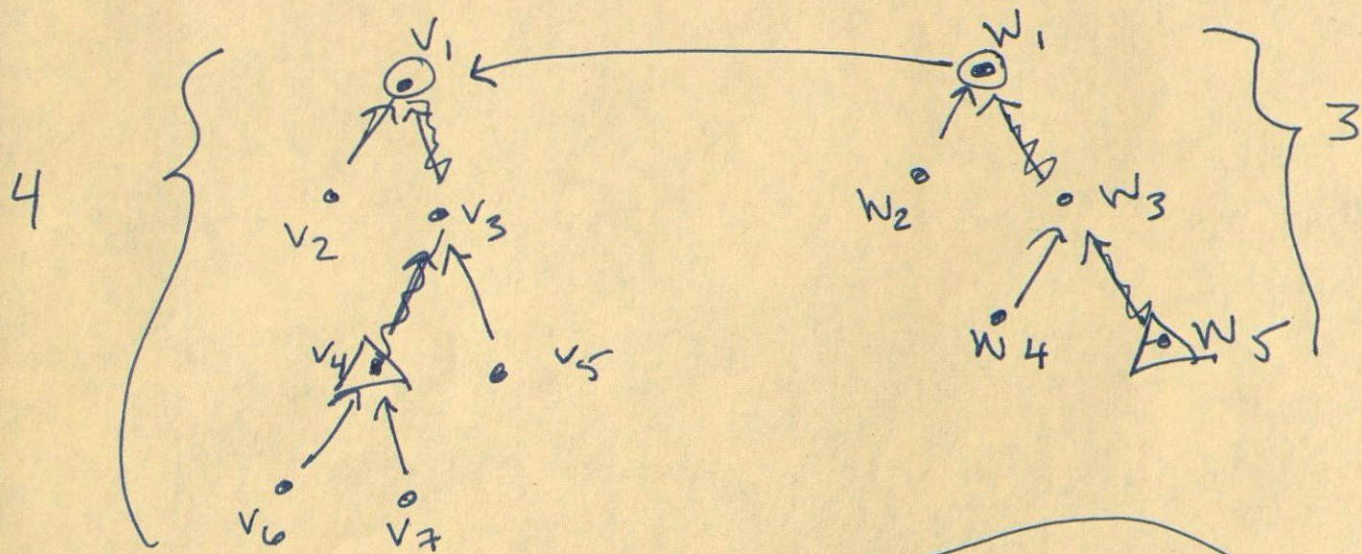
IMprovement:

OR

 } preferred way

depth increases by 1

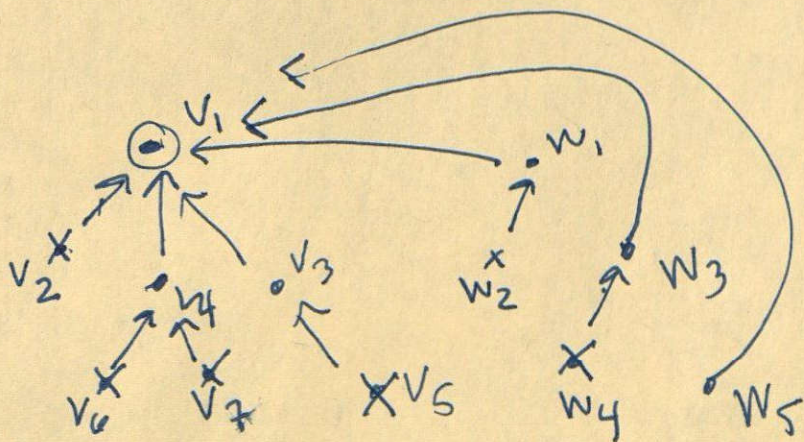o if they aren't, depth does not change.

$\Rightarrow$ FIND $\Theta(\log n)$

UNION $= 2 \cdot$ FIND $+ \Theta(1) = \Theta(\log n)$

IMPROVEMENT #2: Path Compression

4 { ... } 3

depth: $3 \leq \boxed{4}$

pretending
path compression
did not
happen

still merge on "depth potential"

then the runtime

$$\begin{cases} \Theta(\alpha_n \cdot M) \\ \end{cases}$$
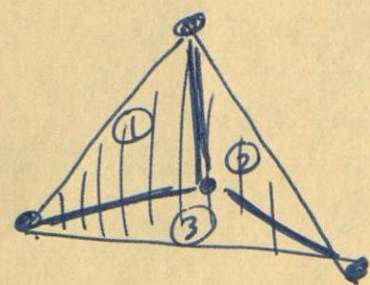
mult.

↑ the inverse Ackermann fcn

⤷ nearly linear in M.

Triangulations (Trianar)
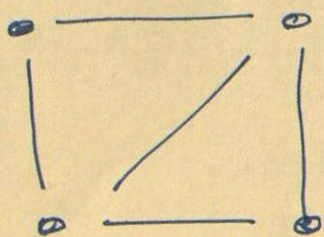
Given $P \subseteq \mathbb{R}^2$, $|P| < \infty$

Want: to use that point set to
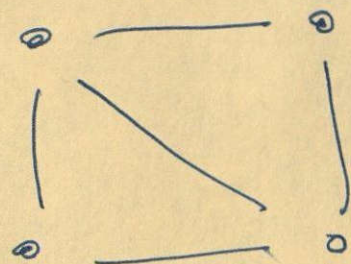~~triangul~~ decompose $CH(P)$
into triangles, edges, & vertices

e.g.

$CH(P) :=$ the intersecti
of all half-planes
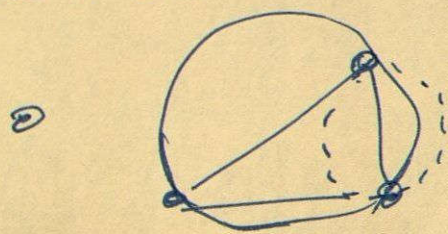containing P
$:=$ the smallest convex
shape containing P

e.g.;

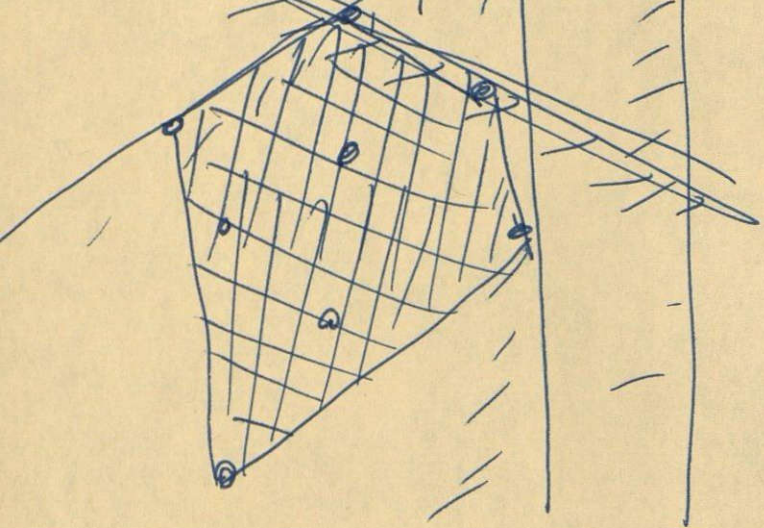OR

Special Aulation: Delaunay Dulation

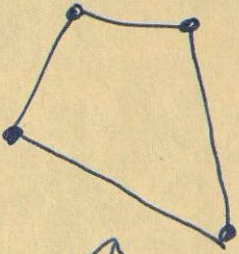Def: • for every triangle, the circumscri
circle is empty of points in P
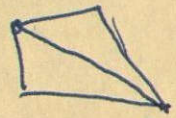Def2
Also, every edge ha
• the empty circle
property too.

Q: What if I am given a
triangulation that is not a
Delaunay Δulation. Can I
fix it in order to obtain
the Delaunay Δulation?

note: there is always a Delaunay
Δulation of any set of pts.
And it is unique, assuming
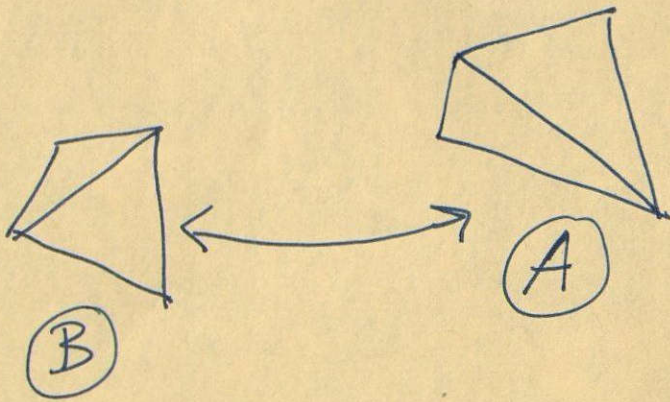general position of the vertices.

2 triangulations.

NOT Del.

the 2 ways that 4 verts can be
configured "in general position"

} the only △ulation

↓Delaunay

Edge flip operation: given an edge
whose 2 triangles
form a convex
shape, flip the
interior edge

(B)   (A)

heuristic: Every time I see (A),
change it to (B).

Algorithm:

while ∃ an edge that
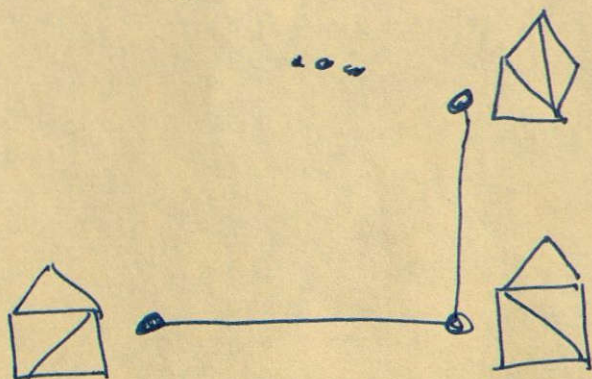can be flipped to
be "locally Delaunay"

   flip it

end while.

Loop invarint:

i = # times loop completed,

L_i = we have (at least) i locally
    Delaunay edges

triangulation of n
points has the
same # of edges

⟹ NOT SO obvious:

① we won't go on
   forever

② we won't wind
   up at a local
   minimum. ✓

Space of all triangulations of P
- vertex = a triangulation
- add edge if exactly 1 edge flip difference

} this space is
(path)-connected!