Edexcel AS and A Level Modular Mathematics
Core 3

Elliott Whyman

December 28, 2016

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2 Functions

2.1 Function Definition

A function is a mapping such that every element of set A is mapped to exactly one element of set B.
The mapping set is considered to be the domain of the function, and the mapped set is considered to be the range.

Consider the set, A, $\{1, 2, 3, 4, 5\}$ being mapped to set B, $\{1, 4, 16, 25\}$ through the operation of *square*. Set A is the domain and set B is the range.

The function can be defined by:

$$f(x) = x^2,\ \{1 \leq x \leq 5\} \qquad\qquad f : x \to x^2,\ \{1 \leq x \leq 5\}$$

It's range is subsequently defined by

$$1 \leq f(x) \leq 25$$

Consider the function $f : x \to \sqrt{x}$. It can only be considered a function if it's domain is greater than zero. Else it would have values less than zero that are not mapped anywhere as they do not have a real solution.

$$f : x \to \sqrt{x},\ \{x \in \mathbb{R}, x \geq 0\}$$

2.2 Composite Functions

Multiple functions can be combined to make a composite function.

$$\begin{array}{rcl} fg(x) & \Rightarrow & f(g(x)) \\ gf(x) & \Rightarrow & g(f(x)) \\ f^2(x) & \Rightarrow & f(f(x)) \end{array}$$

To solve these composite functions, take the inner most function and input it as an parameter into the its nested function and so on.

2.3 Inverse Functions

The inverse function performs the opposite operation in reverse order to get the original input of the function. When graphed, the inverse function is a reflection of the original function in the line $y = x$

An inverse function is noted by $f^{-1}(x)$

$$\begin{array}{rcl} f(x) = x^2 & \Rightarrow & f^{-1}(x) = \sqrt{x} \\ f(x) = 5x + 2 & \Rightarrow & f^{-1}(x) = \dfrac{x-2}{5} \\ f(x) = 2x^2 - 9 & \Rightarrow & f^{-1}(x) = \sqrt{\dfrac{x+9}{2}} \end{array}$$

3 Exponentials and Logarithmic Functions

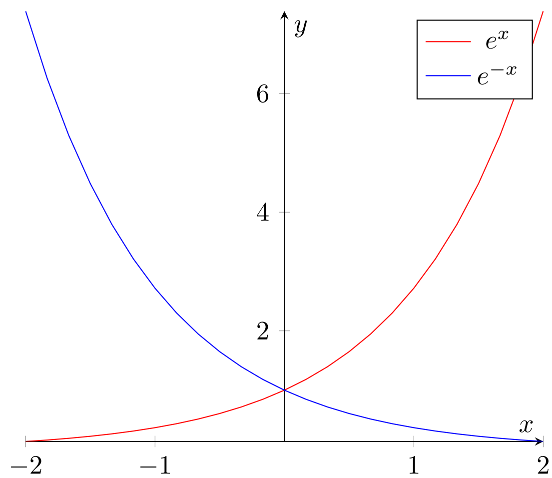
Exponential functions are in the form $y = a^x$ and categories any functions containing exponents.

3.1 The Exponential Function

The exponential function is $y = e^x$, $e \approx 2.718$. At any point, the function is equal to it's gradient, and is therefore refereed to as **the** exponential function. It is used to represent exponential growth, which is how population growth is modeled

$$\frac{d[e^x]}{dy} = e^x$$

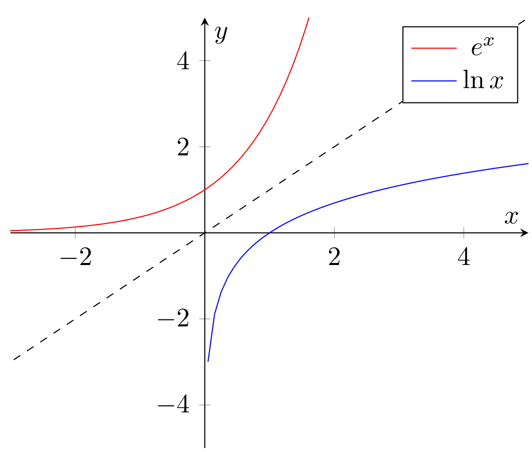
3.2 Graphing Exponential Functions



3.3 The Inverse of the Exponential Function

To understand the inverse of the exponential function, it is necessary to read section 2.3.

The inverse of e^x is $\log_e x$. A log with base e is referred to as the *natural log*, $\ln x$.



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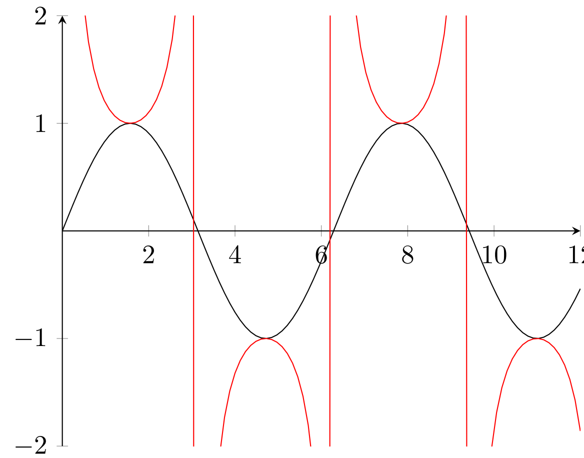
6 Trigonometry

6.1 Introduction of Secant, Co-Secant and Co-Tangent Functions

$$
\begin{aligned}
\text{The Secant function is the reciprocal of the Sine function} \quad & \sec\theta &=& \frac{1}{\cos\theta} \\
\text{The Co-Secant function is the reciprocal of the Cosine function} \quad & \csc\theta &=& \frac{1}{\sin\theta} \\
\text{The Co-Tangent function is the reciprocal of the Sine Tangent function} \quad & \cot\theta &=& \frac{1}{\tan\theta}
\end{aligned}
$$

An easy way to identify each function from its abbreviated notation is to compare its 3rd character to the begging of each standard Trigonometric Function

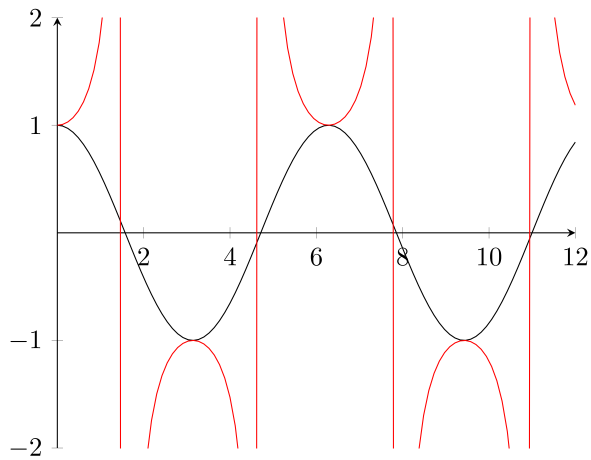


Above, the Sine function is graphed in black. As defined earlier it is that the Co-Secant function is reciprocal of the Sine function. *Not to be confused with the inverse Sine function,* arcsin, $\sin^{-1}\theta$.

$$\csc\theta = \frac{1}{\sin\theta}$$

From analysing the plot above, asymptotes are present whenever the Sine function intersects with the x-axis, this is caused as at the intersection $\sin\theta = 0$ and as $\csc\theta \equiv \frac{1}{\sin\theta}$ therefore $\csc\theta = \frac{1}{0}$ is undefined - causing an asymptote. This causes $y = \csc\theta$ to tend to infinity as it approaches $\sin\theta = 0$.

$$\sin\theta \to 0 \Rightarrow \csc\theta \to \infty$$

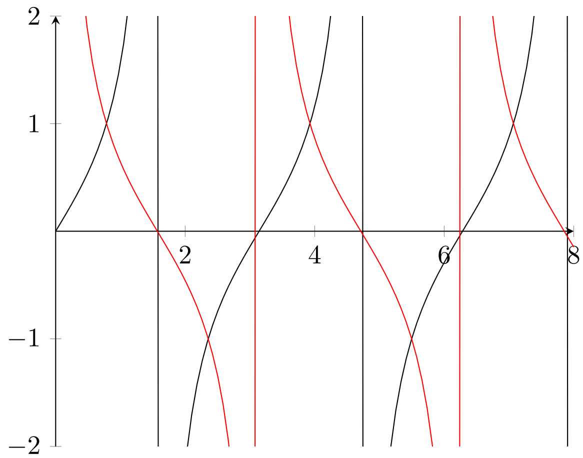


The Cosine function above is also graphed in black, and we know it to be a transformation of the Sine function. As defined earlier it is that the Secant function is reciprocal of the Cosine function. *Not to be confused with the inverse Cosine function,* arccos, $\cos^{-1}\theta$).

$$\sec\theta = \frac{1}{\cos\theta}$$

From analysing the plot above, asymptotes are present whenever the Cosine function intersects with the x-axis, this is caused as at the intersection $\cos\theta = 0$ and as $\sec\theta \equiv \frac{1}{\cos\theta}$ therefore $\sec\theta = \frac{1}{0}$ is undefined - causing an asymptote. This causes $y = \sec\theta$ to tend to infinity as it approaches $\cos\theta = 0$.

$$\cos\theta \to 0 \Rightarrow \sec\theta \to \infty$$



Above, the Tangent function is graphed in black. As defined earlier it is that the Co-Tangent function is reciprocal of the Tan function. *Not to be confused with the inverse Tangent function,* arctan, $\tan^{-1}(x)$.

$$\cot\theta = \frac{1}{\tan\theta}$$

From analysing the plot above, asymptotes are present whenever the Tan function intersects with the x-axis, as graphed on the normal Tangent function. As like before, there are also asymptotes whenever the Tangent function intersecs with the x-axis, this is caused as at the intersection $\tan\theta = 0$ and as $\cot\theta \equiv \frac{1}{\tan\theta}$ therefore $\cot\theta = \frac{1}{0}$ is undefined - causing an asymptote. This causes $y = \cot\theta$ to tend to infinity as it approaches $\tan\theta = 0$.

$$\tan\theta \to 0 \Rightarrow \cot\theta \to \infty$$

6.2 Proving Identities

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