Variational Inference: Introduction

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https://github.com/philschulz/VITutorial

Problems

Supervised problems: "learn a mapping from this to that"

- e.g. machine translation, syntactic parsing, semantic role labelling, image captioning, ...

Unsupervised problems: "learn a distribution that generates the data with high probability"

- sentences in natural language, images, videos, ...

Maximum likelihood estimation

We have data $x^{(1)}, \ldots, x^{(N)}$ e.g.

- sentences, images, ...

generated by some **unknown** procedure

Maximum likelihood estimation

We have data $x^{(1)}, \ldots, x^{(N)}$ e.g.

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generated by some **unknown** procedure
which we assume can be captured by a probabilistic model

- with **known** probability (mass/density) function e.g.

$$X \sim \mathrm{Cat}(\pi_1, \ldots, \pi_K) \qquad \text{or} \qquad X \sim \mathcal{N}(\mu, \sigma^2)$$

and proceed to estimate parameters that assign maximum likelihood to observations

Where does deep learning kick in?

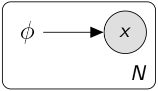
Let ϕ be all side information available
e.g. deterministic *inputs/features*

Have neural networks predict parameters of our probabilistic model

$$X|\phi \sim \mathrm{Cat}(\pi_w(\phi)) \quad \text{or} \quad X|\phi \sim \mathcal{N}(\mu_w(\phi), \sigma_w(\phi)^2)$$

and proceed to estimate parameters w of the NNs

Multiple problems, same language



(Conditional) Density estimation

	ϕ	x
Parsing	a sentence	its syntactic/semantic parse tree/graph
Translation	a sentence	its translation
Captioning	an image	caption in English
Entailment	a text and hypothesis	entailment relation

Task-driven feature extraction

Often our side information ϕ is itself some high dimensional data

- ϕ is a sentence and x a tree
- ϕ is the source sentence and x is the target
- ϕ is an image and x is a caption

and part of the job of the NNs that parametrise our models is to also deterministically encode that input in a low-dimensional space

NN as efficient parametrisation

From the statistical point of view NNs do not generate data

- they parametrise distributions that *by assumption* govern data

Compact and efficient way to map from complex side information to parameter space

Prediction is done by a decision rule outside the statistical model

- e.g. beam search

MLE via gradient-based optimisation

The probability of an observation $X = x$ is given by some differentiable probability function

- the parameters of which are predicted by f_w *(also differentiable)*

Example: K classes

$$\mathrm{Cat}(X = x | \underbrace{f_1^K := f_w(\phi)}_{\text{class probabilities}}) = \prod_{i=1}^{K} f_i^{[x=i]}$$

Given a dataset of i.i.d. observations, SGD gives us a local optimum of the log-likelihood

DL in NLP recipe

Maximum likelihood estimation

- tells you which loss to optimise
  (i.e. negative log-likelihood)

Automatic differentiation (*backprop*)

- "give me a tractable forward pass and I will give you gradients"

Stochastic optimisation powered by backprop

- general purpose gradient-based optimisers

Tractability is central

Likelihood gives us a differentiable objective to optimise for

- but we need to stick with tractable likelihood functions

When do we have intractable likelihood?

Latent variables: assessing the likelihood requires marginalisation

- too many forward passes

$$P_X(x|\phi) = \sum_{c=1}^{K} \mathrm{Cat}(c|\pi_1, \ldots, \pi_K) \underbrace{\mathcal{N}(x|\mu_w(c), \sigma_w(c)^2)}_{\text{forward pass}}$$

- even infinitely many

$$P_X(x|\phi) = \int \mathcal{N}(z|0, I) \underbrace{\mathrm{Cat}(x|\pi_w(z))}_{\text{forward pass}} \mathrm{d}z$$

But I know approximations!

Beam-search

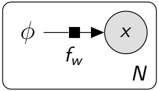
- biased gradient estimates
  bye bye stochastic optimisation!

Monte Carlo sampling

- breaks differentiability
  bye bye backprop!

What do we do then?

Vast majority of papers published at ACL



encode more and more inductive bias through the design of the architecture alone

- what if we want to learn clusters?
- or segmentation?
- or sparse models?
- or latent factors?
- or learn from incomplete supervision?
- or Bayesian NNs?

Deep Generative Models

Probabilistic models parametrised by neural networks

- better modelling assumptions
  one of the reasons why there's so much interest
- but requires efficient inference
  which is the reason why we are here today

Literature I