

Welcome and Introduction

Philip Schulz and Wilker Aziz

<https://github.com/philschulz/VITutorial>

About us . . .

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- ▶ Research associate at UvA
- ▶ VI, Sampling methods, Machine Translation

Philip Schulz

- ▶ Applied Scientist at Amazon
- ▶ VI, Machine Translation, Bayesian Models

Problems

Supervised problems: “learn a distribution over observed data”

- ▶ sentences in natural language, images, videos,
...

Unsupervised problems: “learn a distribution over observed and unobserved data”

- ▶ sentences in natural language + parse trees,
images + bounding boxes ...

Supervised problems

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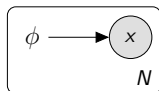
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and proceed to **estimate parameters** that assign maximum likelihood to observations

Multiple problems, same language



(Conditional) Density estimation

Parsing	Side information (ϕ) a sentence	Observation (x) its syntactic/semantic parse tree/graph
Translation	a sentence	its translation
Captioning	an image	caption in English
Entailment	a text and hypothesis	entailment relation

Where does deep learning kick in?

Let ϕ be all side information available
e.g. deterministic *inputs/features*

Have neural networks predict parameters of our probabilistic model

$$X|\phi \sim \text{Cat}(\pi_{\theta}(\phi)) \quad \text{or} \quad X|\phi \sim \mathcal{N}(\mu_{\theta}(\phi), \sigma_{\theta}(\phi)^2)$$

and proceed to **estimate parameters** θ of the NNs

Task-driven feature extraction

Often our side information ϕ is itself some high dimensional data

- ▶ ϕ is a sentence and x a tree
- ▶ ϕ is the source sentence and x is the target
- ▶ ϕ is an image and x is a caption

and part of the job of the NNs that parametrise our models is to also **deterministically** encode that input in a low-dimensional space

NN as efficient parametrisation

From the statistical point of view NNs do not generate data

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- ▶ compact and efficient way to map from complex side information to parameter space

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Prediction is done by a decision rule outside the statistical model

- ▶ e.g. beam search

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MLE via gradient-based optimisation

If the log-likelihood is **differentiable** and **tractable** then backpropagation can give us the gradient

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and we can update θ in the direction

$$\gamma \nabla_{\theta} \mathcal{L}(\theta | x^{(1:N)})$$

to attain a local maximum of the likelihood function

Big Data

For large N , computing the gradient is inconvenient

$$\nabla_{\theta} \mathcal{L}(\theta | x^{(1:N)}) = \underbrace{\sum_{s=1}^N \nabla_{\theta} \log p(x^{(s)} | \theta)}_{\text{too many terms}}$$

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 &= \mathbb{E}_{S \sim \mathcal{U}(1/N)} [N \nabla_{\theta} \log p(x^{(S)} | \theta)]
 \end{aligned}$$

S selects data points uniformly at random

Stochastic optimisation

For large N , we can use a gradient estimate

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and take a step in the direction

$$\gamma \frac{N}{M} \nabla_{\theta} \mathcal{L}(\theta | x^{(s_1:s_M)})$$

where $x^{(s_1:s_M)}$ is a random mini-batch of size M

DL in NLP recipe

Maximum likelihood estimation

- ▶ tells you which **loss** to optimise (i.e. negative log-likelihood)

Automatic differentiation (*backprop*)

- ▶ “give me a tractable forward pass and I will give you **gradients**”

Stochastic optimisation powered by backprop

- ▶ general purpose gradient-based optimisers

Tractability is central

Likelihood gives us a differentiable objective to optimise for

- ▶ but we need to stick with **tractable** likelihood functions

When do we have intractable likelihood?

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thus assessing the marginal likelihood requires
marginalisation of latent variables

$$p(x|\theta) = \int p(x, z|\theta) \, dz = \int p(z)p(x|z, \theta) \, dz$$

Examples of latent variable models

Discrete latent variable, continuous observation

$$p(x|\theta) = \underbrace{\sum_{c=1}^K \text{Cat}(c|\pi_1, \dots, \pi_K)}_{\text{too many forward passes}} \underbrace{\mathcal{N}(x|\mu_\theta(c), \sigma_\theta(c)^2)}_{\text{forward pass}}$$

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Continuous latent variable, discrete observation

$$p(x|\theta) = \underbrace{\int \mathcal{N}(z|0, I) \text{Cat}(x|\pi_\theta(z)) \, dz}_{\text{infinitely many forward passes}}$$

forward pass

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Exact gradient is intractable

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- ▶ uncertainty quantification
e.g. Bayesian NNs

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which is the reason why we are here today