Deep Generative Models: Discrete Latent Variables

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https://github.com/philschulz/VITutorial

What we know so far

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- Because the ELBO cannot be computed exactly, we need to sample latent values
- Main problem: the MC estimator is not differentiable
- Solution: reparametrisation gradient

Reparametrisation Gradient

Model Gradient

$$\frac{\partial}{\partial\theta}\mathbb{E}_{q(z|\lambda)}\left[\log p(x|z,\theta)\right] - \frac{\partial}{\partial\theta}\mathrm{KL}\left(q(z|\lambda)\;||\;p(z|\theta)\right)$$

Reparametrisation Gradient

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Inference Network Gradient

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Reparametrisation Gradient

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Reparametrisation Gradient

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$$\mathbb{E}_{\phi(\epsilon)}\left[\frac{\partial}{\partial z}\log p(x|\overbrace{h^{-1}(\epsilon,\lambda)}^{z},\theta) \times \frac{\partial}{\partial \lambda}\overbrace{h^{-1}(\epsilon,\lambda)}^{z}\right]$$

Reparametrisation

In order to tranform variables, we need to compute the Jacobian (matrix of derivatives).

$$p(z) = \phi(h(z)) \left| \frac{d}{dz} h(z) \right|$$

The Jacobian is generally not available for discrete variables.

Continuity

The outcome space of discrete variables is non-continuous. Thus, we cannot take derivatives with respect to real variables.

$$\frac{\partial}{\partial \lambda} \mathbb{E}_{q(z|\lambda)} \left[\log p(x|z, \theta) \right] =$$

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Back to Basic Calculus

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Consequence

$$\frac{d}{d\lambda} f(\lambda) = \frac{d}{d\lambda} \log f(\lambda) \times f(\lambda)$$

Score Function Estimator

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$$\mathbb{E}_{q(z|\lambda)} \left[\frac{\partial}{\partial \lambda} \log q(z|\lambda) \times \log p(x|z,\theta) \right]$$

Comparison Between Estimators

- Score function gradient

$$\mathbb{E}_{q(z|\lambda)}\left[\frac{\partial}{\partial\lambda}\log q(z|\lambda) \times \log p(x|z,\theta)\right]$$

- Reparametrisation gradient

$$\mathbb{E}_{\phi(\epsilon)}\left[\frac{\partial}{\partial\lambda}\log p(x|h^{-1}(\epsilon,\lambda),\theta)\right]$$

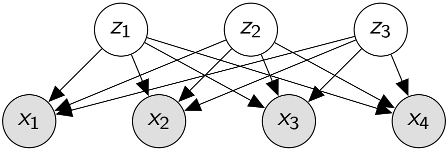
Example Model

Let us consider a latent factor model for topic modelling. Each document x consists of n i.i.d. categorical draws from that model. The categorical distribution in turn depends on the binary latent factors $z = (z_1, \ldots, z_k)$ which are also i.i.d.

$$\begin{aligned} z_j &\sim \text{Bernoulli}\,(\phi) & (1 \leq j \leq k) \\ x_i &\sim \text{Categorical}\,(g(z)) & (1 \leq i \leq n) \end{aligned}$$

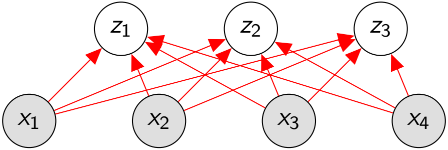
Here $g(\cdot)$ is a function computed by neural network with softmax output.

Example Model



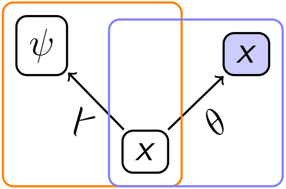
At inference time the latent variables are marginally dependent. For our variational distribution we are going to assume that they are not (recall: mean field assumption).

Inference Network



The inference network needs to predict k Bernoulli parameters ψ. Any neural network with sigmoid output will do that job.

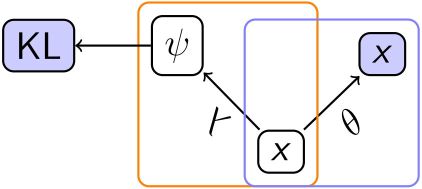
Computation Graph



inference model

generation model

Computation Graph



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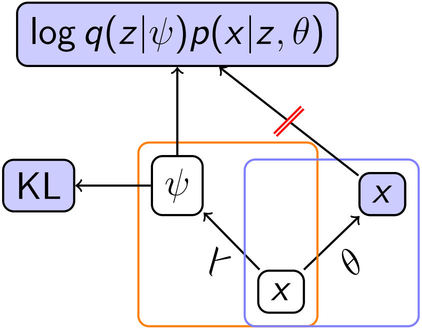

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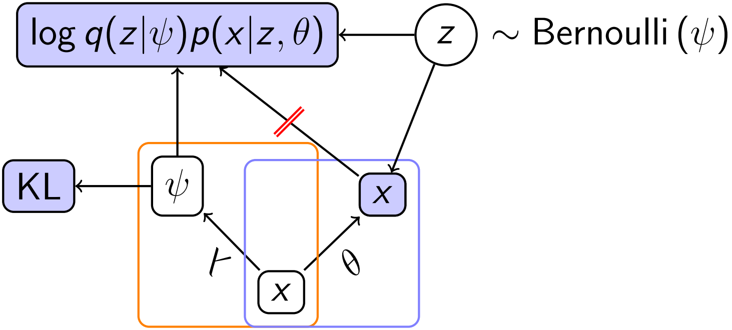

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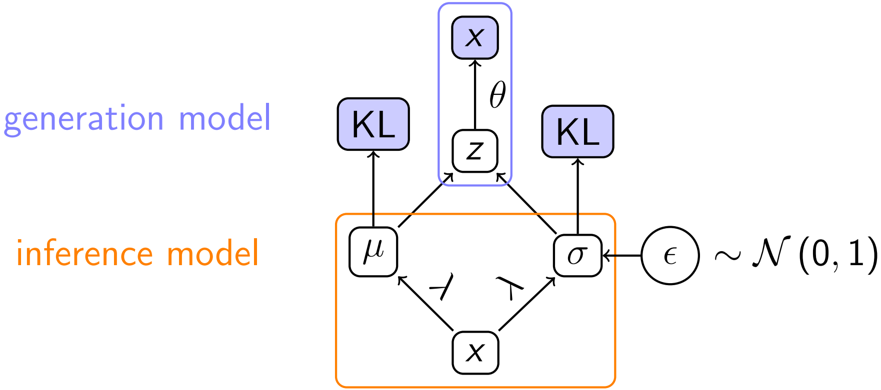
Reparametrisation Gradient



Pros and Cons

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- Cons
  - High Variance!

Baselines

We attempt to centre the gradient estimate. To do this we learn a quantity C that we subtract from the reconstruction loss.

$$\log q(z|\lambda)\left(\log p(x|z,\theta) - C\right)$$

We call C a baseline. It does not change the expected gradient (Williams, 1992).

Baselines

We can make baselines input-depdendent to make them more flexible.

$$\log q(z|\lambda)\left(\log p(x|z,\theta) - C(x)\right)$$

However, baselines may not depend on the random value z! Quantities that may depend on the random value ($C(z)$) are called **control variates**. See Blei et al. (2012); Ranganath et al. (2014); Gregor et al. (2014).

Baselines

Baselines are predicted by a regression model (e.g. a neural net). The model is trained using an L_2-loss.

$$\min \left(C(x) - \log p(x|z,\theta) \right)^2$$

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- Reparametrisation not available for discrete variables.
- Use score function estimator.
- High variance.
- Always use baselines for variance reduction!

Putting it all together

We now know how to handle continuous and discrete latent variables. Let us combine these two treat partially observed data.

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Transform an inflected form of a verb into another.

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Morphological Reinflection

Transform an inflected form of a verb into another.

- plays $\rightarrow$ played
- walking $\rightarrow$ walks

A Simple Model (Zhou and Neubig, 2017)

What do we need to correctly inflect a word?

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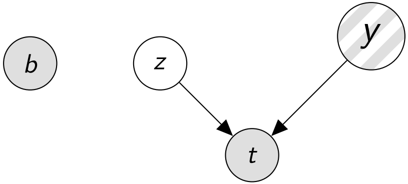
What do we need to correctly inflect a word?

- lemma (real vector)
- morphological information (discrete vector)

A Simple Model (Zhou and Neubig, 2017)



- z = lemma (continuous)
- y = morphological features (discrete)
- b = base form (inflected)
- t = target form (inflected)

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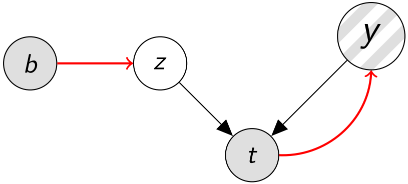

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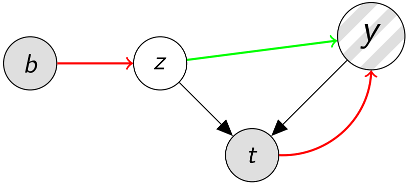

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Goodbye and thank you!

What we covered today

- Derivation of VI
- Continuous VAE
- Discrete VAE
- Semisupervised Models

Goodbye and thank you!

Next steps

- Do the coding tutorial (https://github.com/philschulz/VITutorial/blob/master/code/vae_notebook.ipynb)
- Come to our talk on Tuesday (10:30am, ROOM 203/204)
- Stay in touch
    - w.aziz@uva.nl
    - phschulz@amazon.com