Welcome and Introduction

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https://github.com/philschulz/VITutorial

About us . . .

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- VI, Sampling methods, Machine Translation

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- VI, Machine Translation, Bayesian Models

Keywords

- supervised vs unsupervised learning
- density estimation
- probabilistic modelling
- deep learning
- maximum likelihood estimation
- gradient-based optimisation
- stochastic gradients
- latent variable models
- posterior inference
- variational inference

Problems

Supervised problems: "learn a distribution over observed data"

- sentences in natural language, images, videos, ...

Unsupervised problems: "learn a distribution over observed and unobserved data"

- sentences in natural language + parse trees, images + bounding boxes ...

Supervised problems

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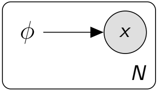
and proceed to estimate parameters that assign maximum likelihood to observations

Multiple problems, same language



(Conditional) Density estimation

	Side information (ϕ)	Observation (x)
Parsing	a sentence	its syntactic/semantic parse tree/graph
Translation	a sentence	its translation
Captioning	an image	caption in English
Entailment	a text and hypothesis	entailment relation

Where does deep learning kick in?

Let ϕ be all side information available
e.g. deterministic *inputs/features*

Have neural networks predict parameters of our probabilistic model

$$X|\phi \sim \mathrm{Cat}(\pi_\theta(\phi)) \quad \text{or} \quad X|\phi \sim \mathcal{N}(\mu_\theta(\phi), \sigma_\theta(\phi)^2)$$

and proceed to estimate parameters θ of the NNs

Task-driven feature extraction

Often our side information ϕ is itself some high dimensional data

- ϕ is a sentence and x a tree
- ϕ is the source sentence and x is the target
- ϕ is an image and x is a caption

and part of the job of the NNs that parametrise our models is to also deterministically encode that input in a low-dimensional space

NN as efficient parametrisation

From the statistical point of view NNs do not generate data

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Prediction is done by a decision rule outside the statistical model

- e.g. beam search

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MLE via gradient-based optimisation

If the log-likelihood is **differentiable** and **tractable** then backpropagation can give us the gradient

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and we can update θ in the direction

$$\gamma \nabla_\theta \mathcal{L}(\theta|x^{(1:N)})$$

to attain a local maximum of the likelihood function

Big Data

For large N, computing the gradient is inconvenient

$$\nabla_\theta \mathcal{L}(\theta|x^{(1:N)}) = \underbrace{\sum_{s=1}^{N} \nabla_\theta \log p(x^{(s)}|\theta)}_{\text{too many terms}}$$

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$$= \sum_{s=1}^{N} \mathcal{U}(s|{}^{1}/{}_{N}) N \nabla_\theta \log p(x^{(s)}|\theta)$$

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$$
\begin{aligned}
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&= \mathbb{E}_{S \sim \mathcal{U}({}^1\!/\!_N)} \left[N \nabla_\theta \log p(x^{(S)} | \theta) \right]
\end{aligned}
$$

S selects data points uniformly at random

Stochastic optimisation

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$$\overset{\text{MC}}{\approx} \frac{1}{M} \sum_{m=1}^{M} N \nabla_\theta \log p(x^{(s_i)} | \theta)$$

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and take a step in the direction

$$\gamma\frac{N}{M}\nabla_\theta \mathcal{L}(\theta|x^{(s_1:s_M)})$$

where $x^{(s_1:s_M)}$ is a random mini-batch of size M

DL in NLP recipe

Maximum likelihood estimation

- tells you which loss to optimise
  (i.e. negative log-likelihood)

Automatic differentiation (*backprop*)

- "give me a tractable forward pass and I will give you gradients"

Stochastic optimisation powered by backprop

- general purpose gradient-based optimisers

Tractability is central

Likelihood gives us a differentiable objective to optimise for

- but we need to stick with tractable likelihood functions

When do we have intractable likelihood?

Unsupervised problems contain unobserved random variables

$$p(x, z|\theta) = \overbrace{p(z)}^{\text{prior}} \quad \underbrace{p(x|z, \theta)}_{\text{observation model}}$$

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$$p(x, z|\theta) = \overbrace{p(z)}^{\text{prior}} \quad \underbrace{p(x|z, \theta)}_{\text{observation model}}$$

thus assessing the marginal likelihood requires marginalisation of latent variables

$$p(x|\theta) = \int p(x, z|\theta)\,\mathrm{d}z = \int p(z)p(x|z, \theta)\,\mathrm{d}z$$

Examples of latent variable models

Discrete latent variable, continuous observation

$$\underbrace{p(x|\theta) = \sum_{c=1}^{K} \mathrm{Cat}(c|\pi_1, \ldots, \pi_K) \underbrace{\mathcal{N}(x|\mu_\theta(c), \sigma_\theta(c)^2)}_{\text{forward pass}}}_{\textbf{too many forward passes}}$$

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Continuous latent variable, discrete observation

$$p(x|\theta) = \underbrace{\int \mathcal{N}(z|0, I) \underbrace{\mathrm{Cat}(x|\pi_\theta(z))}_{\text{forward pass}} \, \mathrm{d}z}_{\textbf{infinitely many forward passes}}$$

Gradient

Exact gradient is intractable

$$\nabla_\theta \log p(x|\theta)$$

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&= \mathbb{E}_{p(z|x,\theta)}\left[\nabla_\theta \log p(x,Z|\theta)\right]
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But why latent variable modelling?

Some reasons

- organise a massive collection of data
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- organise a massive collection of data
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- learn from unlabelled data
  e.g. semi-supervised learning
- induce discrete representations
  e.g. parse trees, dependency graphs, permutations, alignments
- uncertainty quantification
  e.g. Bayesian NNs

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  which is the reason why we are here today