PROCESAREA SEMNALELOR - CURS 07

TRANSFORMATA FOURIER 2D, PROCESAREA IMAGINILOR

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CUPRINS

- **transformata Fourier 2D**

- **compresie JPEG**

DEFINIȚIE TRANSFORMATA FOURIER 2D

- pentru o funcție bi-dimensională $f(x, y)$ avem

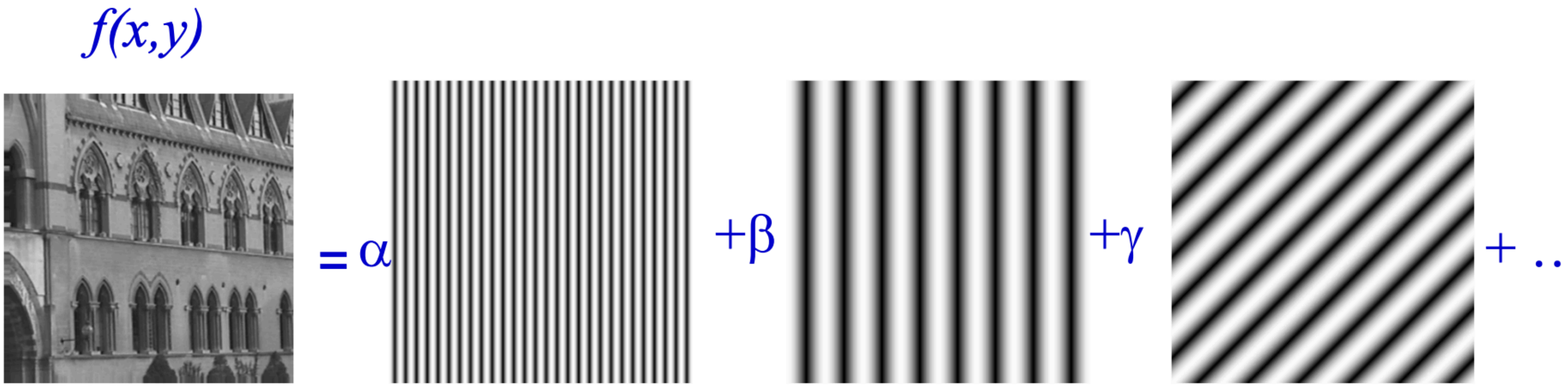
$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2j\pi(ux+vy)} dxdy$$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2j\pi(ux+vy)} dudv$$

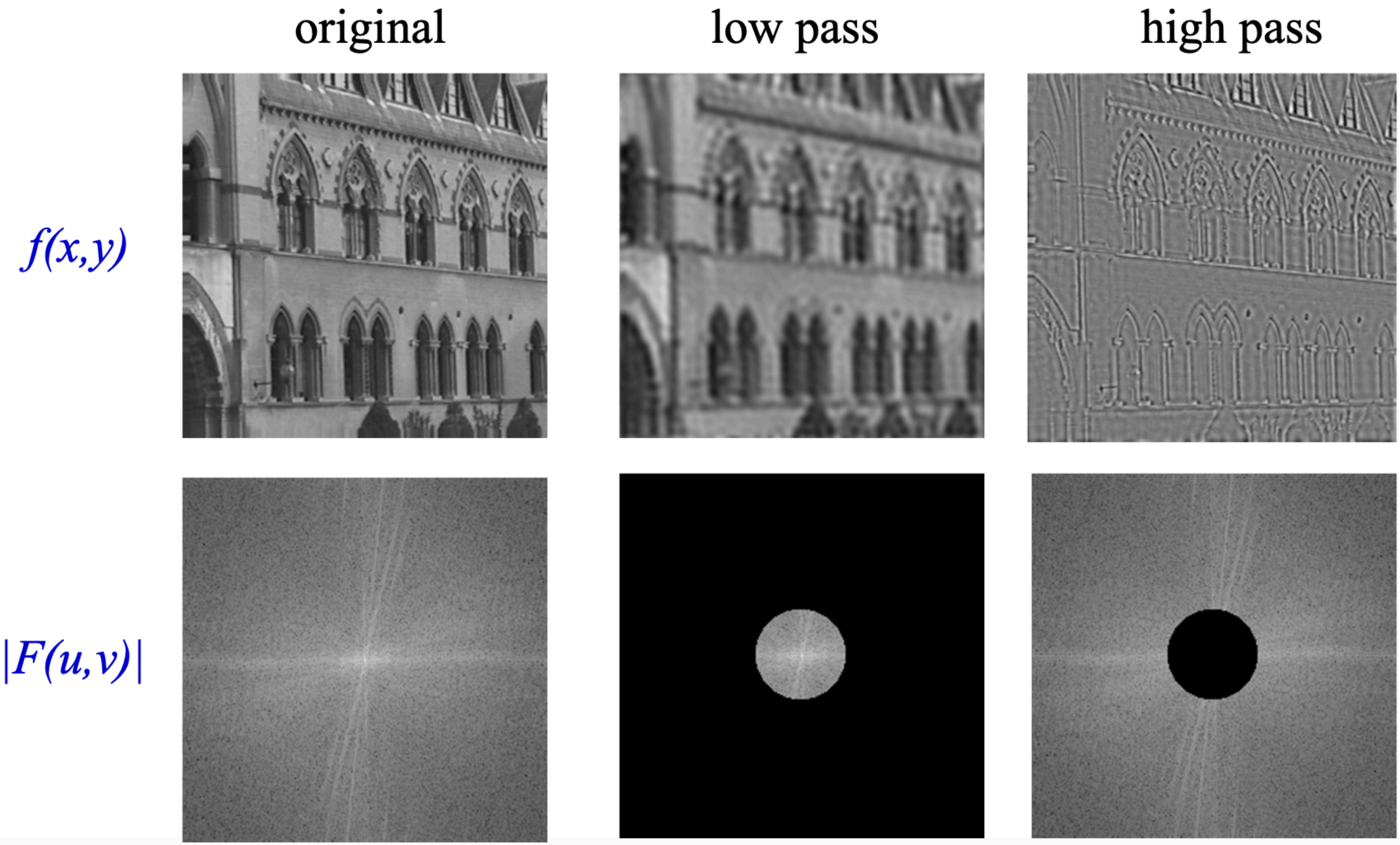
- în cazul 1D semnalele de bază erau sinusoidele
- cum arată semnalele de bază în cazul 2D?

TRANSFORMATA FOURIER 2D

- descompunerea funcției $f(x, y)$



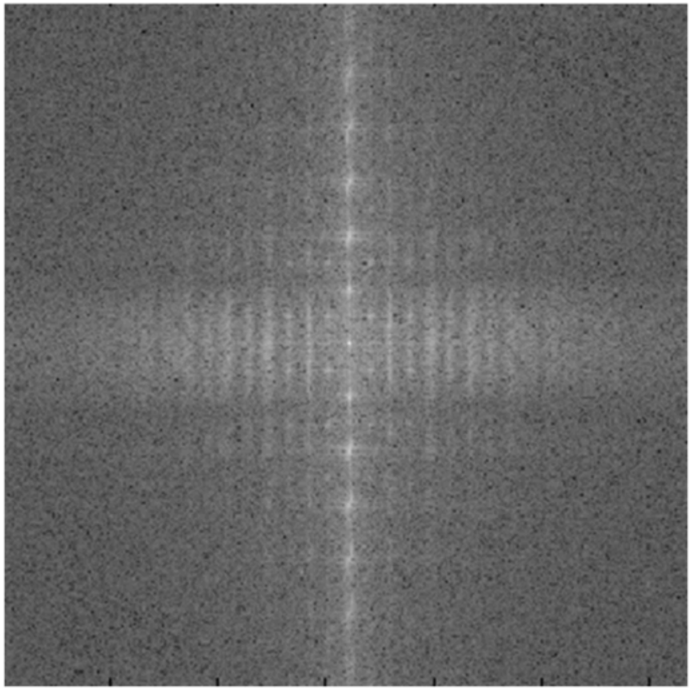
TRANSFORMATA FOURIER 2D



TRANSFORMATA FOURIER 2D

- structuri repetitive din imagini, se văd și în TF-2D

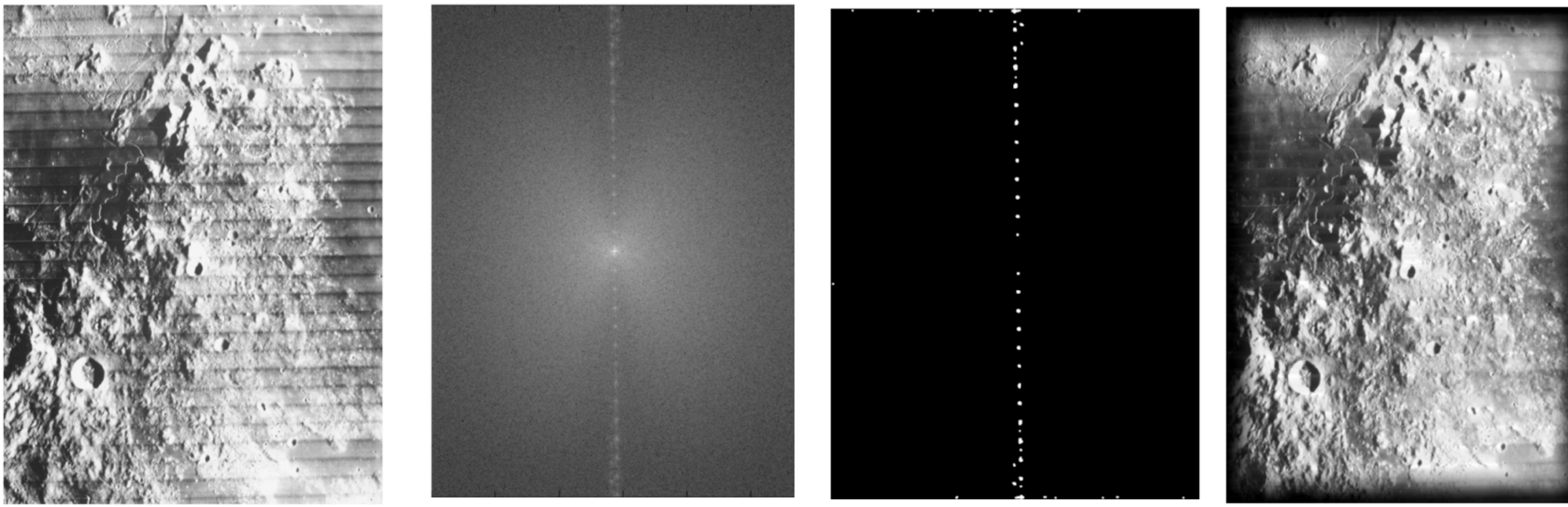
$f(x,y)$



$|F(u,v)|$

TRANSFORMATA FOURIER 2D

- lunar orbital image (1966)

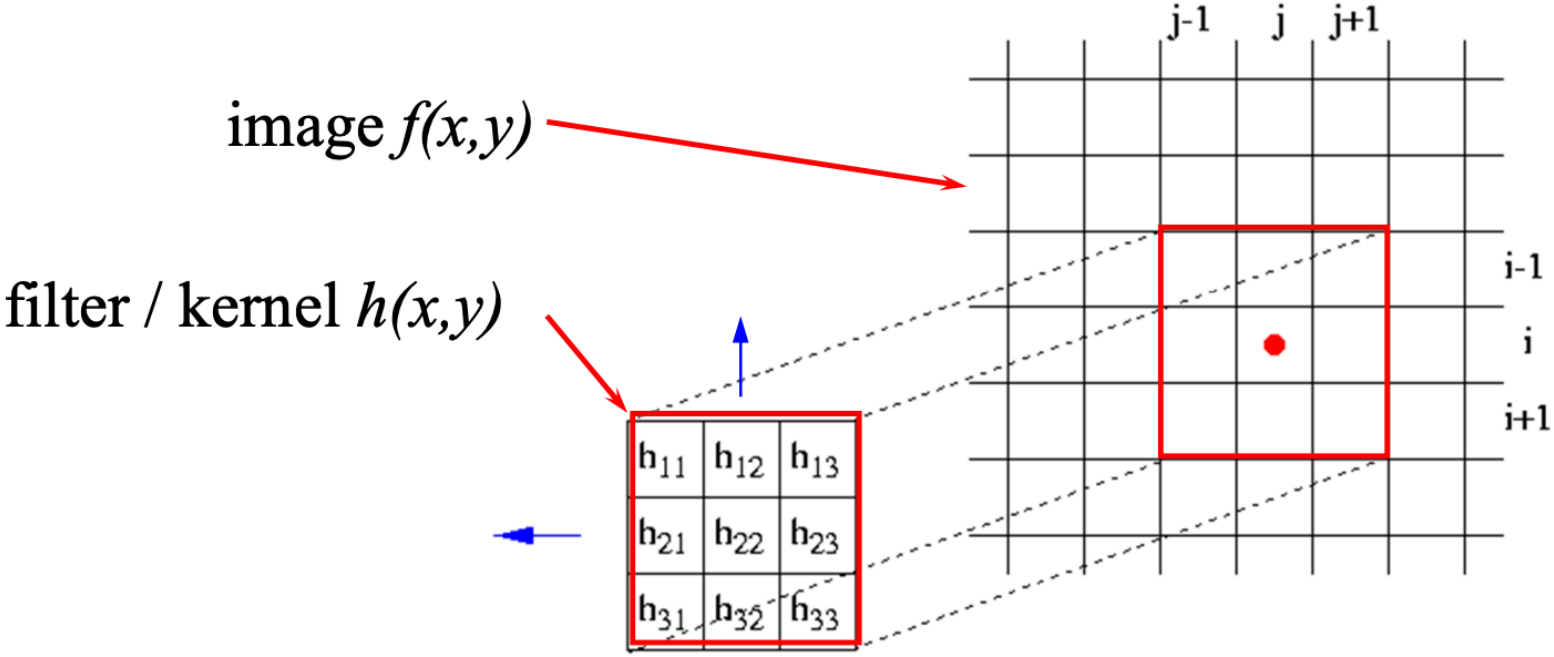


FILTRARE 2D

- convoluție:

$$g(x,y) = h(x,y) * f(x,y) = f(x,y) * h(x,y) = \int\int f(u,v)h(x-u,y-v)dudv$$

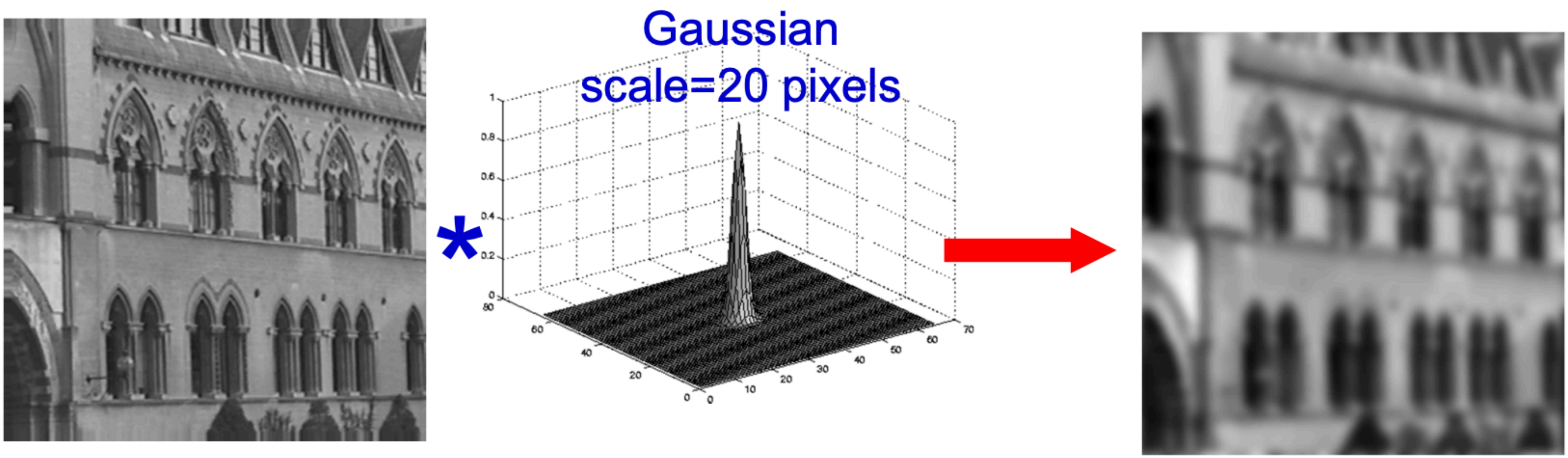
- filtrare:



$$g[x,y] = h_{11}f[i-1,j-1] + h_{12}f[i-1,j] + h_{13}f[i-1,j+1]$$
$$h_{21}f[i,j-1] + h_{22}f[i,j] + h_{23}f[i,j+1]$$
$$h_{31}f[i+1,j-1] + h_{32}f[i+1,j] + h_{33}f[i+1,j+1]$$

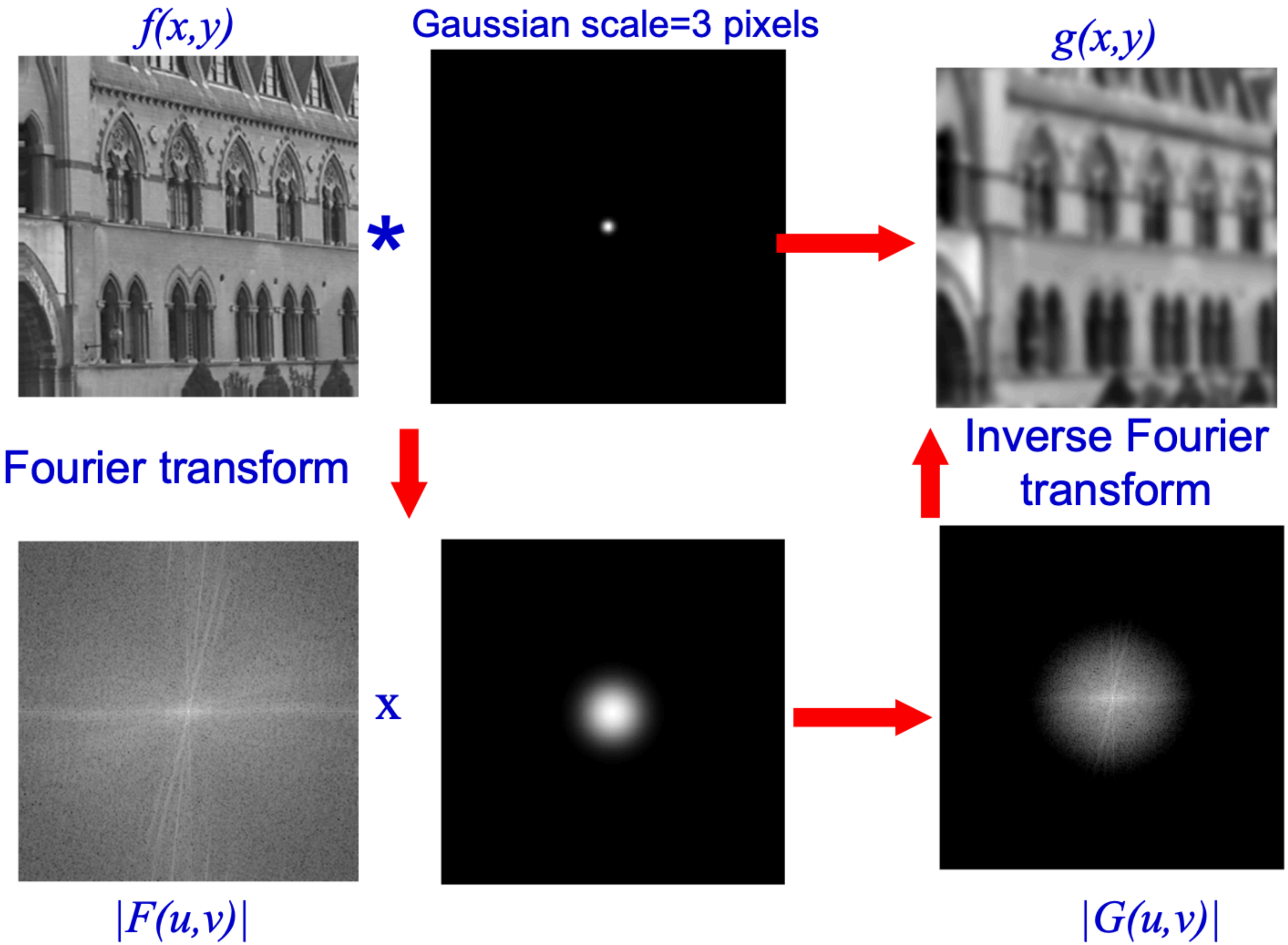
FILTRARE 2D

- exemplu



FILTRARE 2D

- exemplu



PROCESAREA IMAGINILOR

- pentru imagini folosim și transformări speciale (reale, nu complexe)

- 1D Discrete Cosine Transform (1D-DCT)

$$X[k] = \sum_{n=0}^{N-1} x[n] \cos\left(\frac{\pi}{N}\left(n + \frac{1}{2}\right)k\right)$$

- 2D Discrete Cosine Transform (2D-DCT)

$$X[k,m] = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f[x,y] \cos\left(\frac{(2x+1)k\pi}{2N}\right) \cos\left(\frac{(2y+1)m\pi}{2N}\right)$$

PROCESAREA IMAGINILOR

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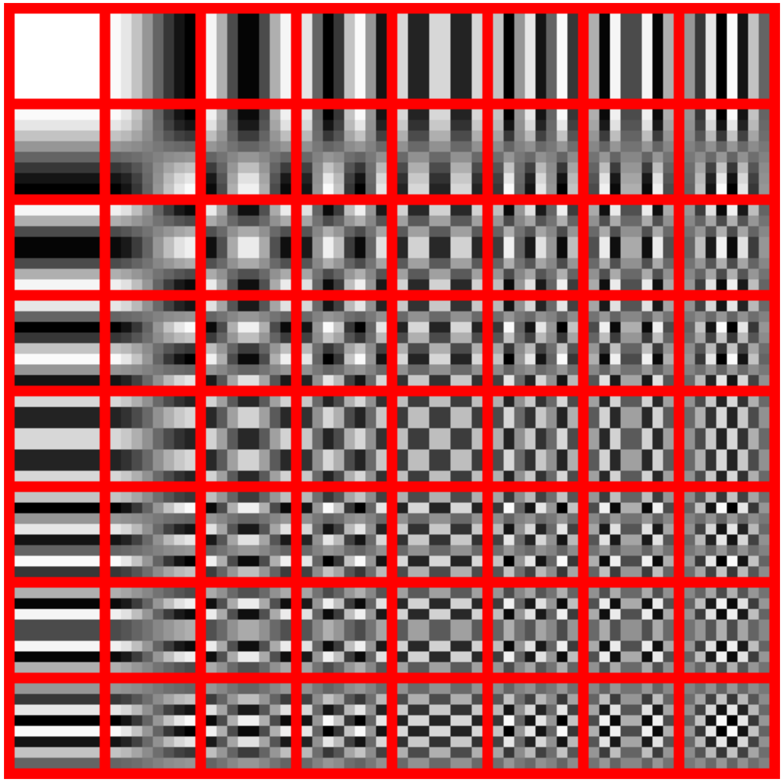


PROCESAREA IMAGINILOR

- un exemplu:

- pornim de la un patch de 8×8

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

- scoatem media din semnal

$$\xrightarrow{x}$$

$$\begin{bmatrix} -76 & -73 & -67 & -62 & -58 & -67 & -64 & -55 \\ -65 & -69 & -73 & -38 & -19 & -43 & -59 & -56 \\ -66 & -69 & -60 & -15 & 16 & -24 & -62 & -55 \\ -65 & -70 & -57 & -6 & 26 & -22 & -58 & -59 \\ -61 & -67 & -60 & -24 & -2 & -40 & -60 & -58 \\ -49 & -63 & -68 & -58 & -51 & -60 & -70 & -53 \\ -43 & -57 & -64 & -69 & -73 & -67 & -63 & -45 \\ -41 & -49 & -59 & -60 & -63 & -52 & -50 & -34 \end{bmatrix} \Big\downarrow y.$$

PROCESAREA IMAGINILOR

- aplicăm pe blocul predecent 2D-DCT

$$\begin{array}{cc} \xrightarrow{k} & \\ \begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.12 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.87 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix} & \Big\downarrow m \end{array}$$

- pentru a coda această matrice, trebuie să o cuantizăm cu matricea

$$Q = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

PROCESAREA IMAGINILOR

- matricea de date după cuantizare: $B = F/Q$

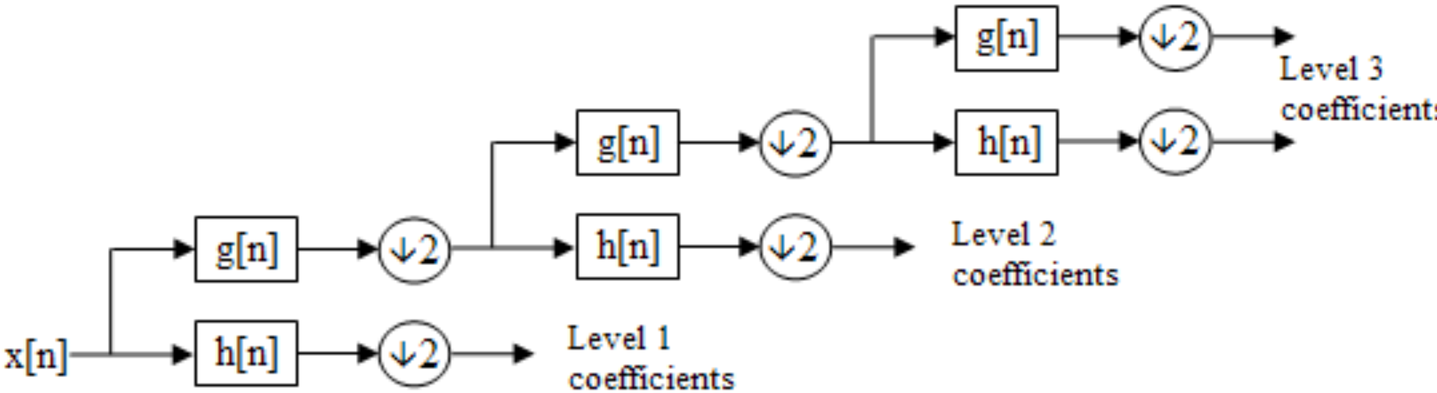
$$B = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

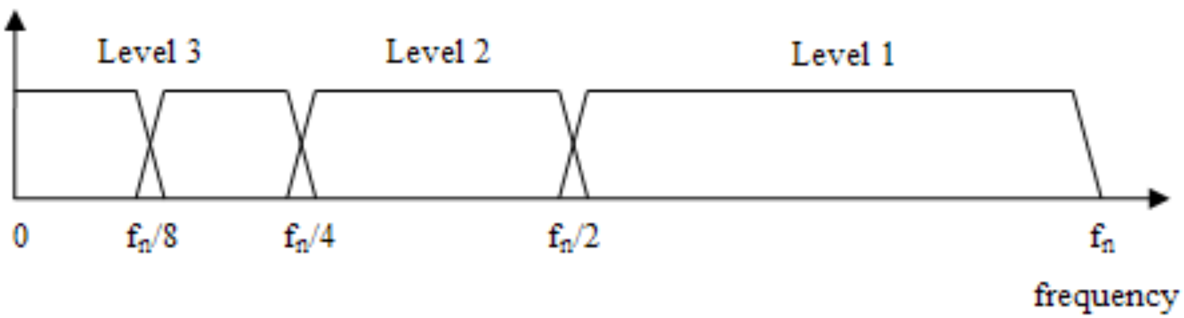
- matricea B este vectorizată (zig-zag-vec) și apoi codată (Huffman)

$$\begin{array}{rrrrrrrr} -26 \\ -3 & 0 \\ -3 & -2 & -6 \\ 2 & -4 & 1 & -3 \\ 1 & 1 & 5 & 1 & 2 \\ -1 & 1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 \\ 0 \end{array}$$

PROCESAREA IMAGINILOR: WAVELET

- vă reamintiți transformata wavelet

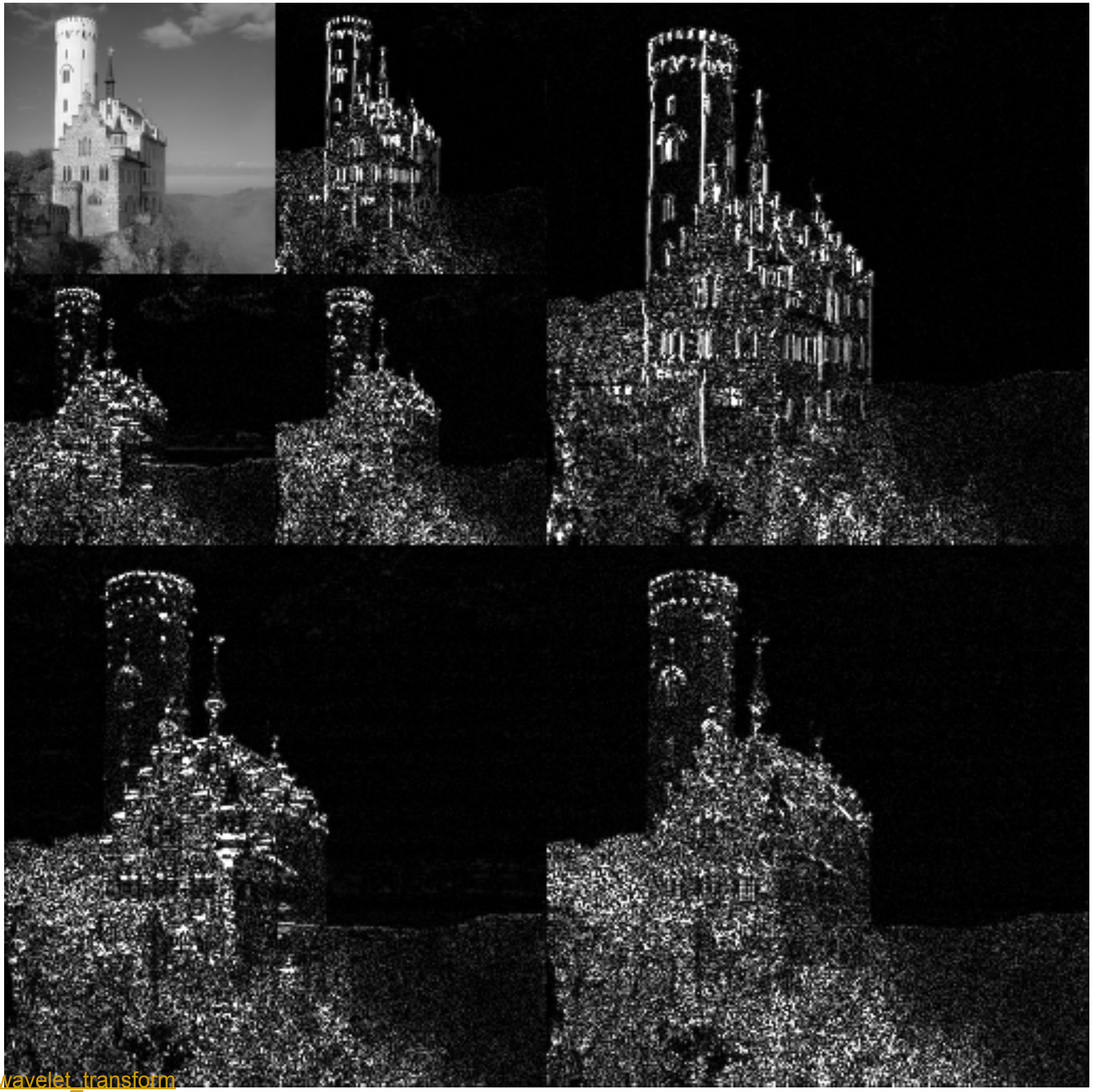




https://en.wikipedia.org/wiki/Discrete_wavelet_transform

PROCESAREA IMAGINILOR: WAVELET

- vă reamintiți transformata wavelet



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