### Linear Programming in Healthcare Optimization

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#### Question 1: Consider 3 possible objectives for a hospital: minimize costs, maximize profits, and maximize net worth. In general, these objectives do not lead to the same optimal solution. Give an example where minimizing cost does not maximize profit.

Hospitals have commonly ended up in a situation where they are confronted with various inconsistent goals in opting to optimize their operations. There are three objectives: reduction of costs, maximisation of profits and maximisation of net worth. Cost minimization aims at making the organization as efficient as it can be by cutting down on costs such as staffing, supplies and utilities (Wang & Demeulemeester, 2023). Profit maximisation is meant to maximise the difference between the revenue and costs obtained by the services and is usually achieved by maximising the number of patients or procedures whose profit remains high. An increase in the net worth is aimed at increasing the long-term financial health, including by investing in an asset or reducing debts, which may require reinvesting instead of distributing profits.

These goals are not usually associated with the same best solution, as they give preference to different trade-offs. An example is the reduction of costs that may include either the reduction of staff or the utilization of inexpensive sourcing, which may negatively affect the service quality and patient satisfaction, eventually leading to the decline of profits and net worth. Profit maximisation may drive aggressive pricing or growth in volume to boost short-term revenue (Wang & Demeulemeester, 2023). However, it runs the risk of inflating costs or incurring regulatory fines to the detriment of net worth. The maximization of net worth may imply conservative measures, such as cash reserves, which may restrict the current profit potential or raise the operational expenses by not entirely investing in efficiency.

A good example is the allocation of a bed in the hospital during the peak season of flu, when the primary aim is to reduce cost but not to increase profit. Assume a hospital has 100 beds and it is in demand. The administration could reduce costs, such as restricting overtime among the nurses and reusing any single-use supplies where they can, to ensure that the operating cost on a daily basis is $5,000.00/bed. It enables the occupation of all 100 beds with the low reimbursement rate of 6,000 per patient day and earns a low profit of 100,000 (1000 profit per bed). To maximise profits, the hospital may invest more in advertising and high-quality staffing (costing $5,500 per bed) to bring in higher-paying elective surgery cases that will reimburse per day at $8,000. Placing 80 beds with such patients will result in a profit of 200,000 dollars in one month (2,500 dollars per bed), which is way above the cost-minimization approach. In this case, the cost-cutting approach foregoes the opportunities for higher profits to save on short-term results. It is in line with the lecture that focuses on presenting issues in the form of quantifiable objectives, such that objectives need to be balanced to prevent suboptimal methods.  
 **Question 2:** In the HU supply/truck example in the online lecture, instead of shipping medical supplies, imagine that we were shipping fruit (obviously, food is part of any healthcare system) and that the variables represented the number of tons of fruit needed to be shipped from the east coast to the west coast. Now imagine that at each transshipment depot, the fruit must be offloaded and reloaded, resulting in 5% of the fruit being ruined. How would you modify the linear program to account for this loss and still meet the fruit requirements on the west coast?

Based on the HU Logistics case presented in the lecture, which represents a traditional linear programming (LP) network flow model, optimising the transportation cost of truckloads of medical supplies between East Coast depots (NYC and PHL) and transhipment points (HAR and RMD) into destinations on the West Coast (ROK, MAR, PIT). NYC and PHL have 200 truckloads of supply and 300 of demand, while ROK, MAR and PIT have 100, 150 and 200 needs, respectively. Costs depend on a per-mile basis (e.g. $68 NYC-HAR), and restrictions make sure that there is a limited supply, a specific demand quantity, and that there is conservation of flow when at transshipments.  
 Scaling this to shipping of fruit makes it perishable: at transhipment depots (HAR and RMD) 5% ruinage per offloading/reloading. The measure of fruit is in tons, thus variables are tons shipped at each link (e.g. xNYC−HAR tons shipped between NYC and HAR). The objective is to minimise total cost; however, we should consider losses in order to provide precise demand tons (e.g. 100 tons to ROK) following spoilage. In order to adjust the LP, we took into consideration the 5% loss through inflows inflation at transshipments. Incoming tons (loading), which are reloaded to be shipped out, are also only 95 per cent (0.95 x incoming). Therefore, to produce y tonnes in a depot, one will have to feed y/0.95 tonnes. It produces a non-conservation of mass; the input = output, but output = 0.95 x input.

Key changes:

1. Decision Variables: Retain the 10 variables for tons shipped per link (non-negative reals), e.g., xNYC−HAR,xNYC−RMD,xNYC−PIT, xPHL−HAR ,xPHL−RMD, xHAR−ROK ,xHAR−MAR, xHAR−PIT, xRMD−MAR, xRMD−PIT.   
    Note: Direct xNYC−PIT, bypasses transhipment, so no loss there.
2. Objective Function: Unchanged minimise total cost: 68xNYC−HAR +264xNYC−PIT +129xNYC−RMD + 109xPHL−HAR +118xPHL−RMD + 212xHAR−ROK +214xHAR−MAR + 203xHAR−PIT + 162xRMD−MAR +135xRMD−PIT.
3. Supply Constraints: Same limited resource bounds: xNYC−HAR+ xNYC−RMD +xNYC−PIT ≤ 200 (NYC supply) Xphl−har + xPHL−RMD ≤300 (PHL supply)
4. Demand Constraints: Adjusted for surviving tons only. For ROK (only from HAR): 0.95×xHAR−ROK ≥100 → xHAR−ROK ≥100/0.95= 105.26 tons shipped from HAR.  
    For MAR: 0.95×(xHAR−MAR +xRMD−MAR)≥150 → xHAR−MAR xRMD−MAR ≥150/0.95≈157.89   
   For PIT: xNYC−PIT +0.95×(xHAR−PIT +xRMD−PIT)≥200 (direct from NYC has no loss).
5. Transhipment Constraints: Modified for loss. At HAR: Incoming = outgoing / 0.95 xNYC−HAR +xPHL−HAR =(xHAR−ROK +xHAR−MAR +xHAR−PIT) /0.95.   
   At RMD: xNYC−RMD+xPHL−RMD =(xRMD−MAR +xRMD−PIT) /0.95 (assuming no RMD-ROK link per diagram). It ensures enough is shipped in to cover 95% survival, equaling outgoing needs.
6. Non-negativity: All x ≥ 0.

This change ensures that the model remains linear (it is permissible to multiply by some constant 0.95). It overships to transshipments (e.g., 5.26 additional tons to the ROK path), such that costs begin to rise slightly, but demands are satisfied precisely after loss. In the case of compounded losses (compounded by several stops), we would chain (e.g., 0.95 2 ) of transshipments; however, in this example, that of depots. The model is reflective of actual healthcare logistics of perishable goods, such as vaccines, and provides integrity of delivery.   
  
**Question 3:** What are the main components of a linear program? (List them all)

The main components of a linear program (LP), as outlined in the lecture, form a structured mathematical model for optimisation. They are:

1. Decision Variables: Measurable quantities we control, e.g., number of trucks or tons shipped along each route. Represented as x1,x2,…, they must be non-negative ( xi ≥0) for standard LPs.
2. Objective Function: A linear expression to maximise or minimise, e.g., minimise cost c1x1+c2x2+⋯+cnxn, where ci are coefficients (costs/profits).
3. Constraints: Linear inequalities or equalities limiting variables, e.g., supply ∑xi ≤S, demand ∑xi ≥D, or conservation ∑inxi =∑outxi. They define a feasible region.
4. Non-negativity Constraints: Implicit or explicit xi ≥0, ensuring realistic (non-negative) decisions like shipment amounts.

These components ensure the model is solvable via methods like simplex, as in the HU example.

#### Extra Credit: Worth 2 points, but assignment point total will not go over 50. In healthcare informatics, can you think of any decision variable that could be (or is) negative? Why do you state this/explain?

Decision variables in healthcare informatics are often non-negative, and some of the quantities, such as doses given or beds occupied, are reflected in these decision variables. Nonetheless, one of the variables that might be negative is the change in stock levels of pharmaceutical inventories. As an example, in an LP model of drug distribution between hospitals, x should be a net change in inventory that is on hand at a central warehouse (Pardede, 2019). When x is negative, it will be discarding either excess or expired drugs (negative adjustment to lower the waste costs). This will be needed since it will be possible not to have to pay penalties such as spoilage charges of overstocks, as well as restrictions to ensure that the total supply matches the demand. In practice, systems such as those in supply chain informatics (like Epic or Cerner modules) model such variables as unrestricted in sign to treat returns or write-offs, which can be optimised more realistically without incurring artificial positivity that may inflate costs.

References

Pardede, A. M. H. (2019). Limited resources optimization of health care services with a linear integer programming approach.

Wang, L., & Demeulemeester, E. (2023). Simulation optimization in healthcare resource planning: A literature review. *Iise Transactions*, *55*(10), 985-1007.