Skull & Roses cards game

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1 Synonyms

S&R; *Skull*; *Skull & Roses*

2 Definition

The *Skull & Roses* (also known simply as *Skull*) is a multi-player cards game with small stochasticity, imperfect information, and partially observable outcomes. Consequently, like in the case of poker, it requires to mix bluffs, opponent modelling and coalitions as high-level strategic plays; achieving all this with a very simple set of rules and elements.

2.1 Notation

There are four main parameters of *Skull & Roses* to be considered the *number of players* P, the *initial number of skulls* $S = 1$, the *initial number of roses* $R = 3$, and the *number of wins* (successful challenges) required $W = 2$.

The original game has $3 \leqslant P \leqslant 6$, and the expansion – *Skull & Roses RED* – allows up to $3 \leqslant P \leqslant 12$.

3 History

It was created by Hervé Marly, illustrated by Rose Kipik, and edited by *lui-même* in 2010. After that, the game gains its popularity very quickly, being officially published in German, French and English. The game received the international price of as *d'or, jeu de l'année, Cannes 2011*.

This game is also known for the surprising fact that it took its designer, Hervé Marly,"15 years of perseverance" to properly balance it (Marly, 2010).

4 Game rules

A canonical game consists in a series of rounds composed by several turns, this decomposition is specified by the following rules (Marly, 2010):

Game sequence:

1. Play a round.
2. If last challenger is still playing, he takes the initiative.
3. If last challenger is not playing, next player, $(challenger + 1) \mod P$, takes the initiative.
4. Repeat until winning condition is reached.

Round sequence:

1. Do one *First turn*.
2. Until a challenge is set, every player in clock-wise order do either a *Card turn* or set a challenge,starting by the player that has the initiative.

3. Every player in order do either a *Bet turn* (aiming to become the challenger) or pass (definitively), until the challenger (player with the maximum final bet) is decided.

4. The challenger does a *Revelation turn*, until a skull is revealed or the number of roses revealed match his own bet.

5. Let w_p be the number of games that Challenger p has won so far. If the challenge is successful, increase w_p by one. Otherwise, do a *Discard turn*.

Turns:

First turn: Each player places simultaneously a card in his own pile from his hand.

Card turn: Place a card on the top your pile from your hand.

Bet turn: Increase the current bet.

Revelation turn: If your own pile is not empty, turn up the top card of your own pile. Otherwise, turn up the top card of any player's pile.

Discard turn: If the revealed skull is your card, choose a card to discard by yourself from your cards. Otherwise, let the owner of the revealed skull choose one of your cards to discard randomly. For both cases, the players other than you do not know which card was removed.

Conditions:

Starting Condition: Each player has 3 roses and 1 skulls and 0 wins. And Player 1 takes the initiative.

Winning condition: Either winning 2 challenges or being the last player standing.

Successful challenge: The challenge is successful, if the challenger revealed a number of roses equal to the bet without revealing a skull.

Unsuccessful challenge: The challenge is unsuccessful, if the challenger revealed a skull in the process of the challenge.

5 Game complexity

In order to measure the complexity of a game, there are two well-known complementary approaches: the computational complexity one, given by the complexity class; and the combinatorial game theory one, given by the state-space, game tree, average game length, and average branching factor.

5.1 Complexity class

Skull & Roses is a polynomially bounded game, where resources (hand cards) are used up, and is also a game of imperfect information (each player's hand and pile status).

According to standard book in the field by Hearn and Demaine, 2009 the natural extension (parametrization of the number of players, wins, skulls and roses) of games with this two key characteristics, the 2-player version of the game is in $PSPACE$, while the N-player version is in $NEXPTIME$. There are proofs to show that many games with those characteristics are in $PSPACE - complete$ and $NEXPTIME - complete$ respectively, but the proofs for *Skull & Roses* remain unknown.

5.2 State-space complexity

The state space complexity deals with the total number nodes in the game's state graph, this is given by the number of variables needed to fully represent a game-state and theirs domain sizes.

The upper bound below for the the natural extension of the game was proven in Gragera et al.,2013 by bounding the variables in each individual turn.

$$Size(\text{State-space}) \leqslant P^2(R+S)^2 \cdot \left[R+2+(R+1)^{P-1}\right] \times \left[4(S+1)^2(R+1)(W+1)\binom{S+R}{S}\right]^P$$

#Players	3	4	5	6	7	8	9	10	11	12
Upper bound	10^{13}	10^{16}	10^{20}	10^{24}	10^{27}	10^{31}	10^{34}	10^{38}	10^{42}	10^{45}

Table 1: The upper bound of state-space size for all possible players values in *Skull & Roses RED*

5.3 Game tree complexity

The game tree complexity considers the size of the game tree, a directed graph whose nodes are positions in a game and whose edges are the moves that were played between them. The size of this tree is measured by the number of leaf nodes in the game tree rooted at the game's initial position. This value is equivalent to the total number of possible different games that can be actually played.

An upper bound for the tree size of a single round was given in Gragera et al.,2013:

$$Size(\text{Round tree}) \leq \#placements \cdot \#bets \cdot \#revelations.$$

$\#Players$	3	4	5	6	7	8	9	10	11	12
Upper bound	10^{10}	10^{17}	10^{25}	10^{33}	10^{41}	10^{50}	10^{60}	10^{70}	10^{80}	10^{90}

Table 2: The upper bound of a single round tree size for all possible players values in *Skull & Roses RED*

And a very naive and loose upper bound was also obtained for the full game tree simply by using this bound (with the first round values) for all possible rounds (without considering the discard turn).

$$Size(\text{Game tree}) \leq (\#Placements \cdot \#Bets \cdot \#Revelations)^{\#Rounds}$$

5.4 Average game length and branching factor

The game length is the number of of position of each game, the depth of the tree. While the branching factor is the number of children at each node, the outdegree. Both higher branching factors and higher game lengths make search algorithms computationally more expensive and ineffective, due to the exponentially increasing number of nodes and the so called horizon effect.

#Players	3 players	4 players	5 players	6 players	7 players
Avg. game length	58	130	244	409	633
Avg. branching factor	5.262	9.175	14.445	21.868	32.897

#Players	8 players	9 players	10 players	11 players	12 players
Avg. game length	922	1282	1720	2242	2856
Avg. branching factor	50.119	78.320	126.218	209.773	358.761

Table 3: The average length of a game and the average branching factor for all possible players values in *Skull & Roses RED*

The previous averages were calculated based on the data collected by 10^7 games played by uniform random players (Gragera et. al, 2013).

6 Future directions

These conditions make it very attractive as a test-bed for game AIs in the big picture versus quick profit dilemma. As well as a good candidate to be included in General Game Playing test frameworks and competitions (Genesereth ed al. 2005).

Due the novelty of the game, still several important problems remain open, being the two most important to find a not naive game tree upper bound and to prove its exact complexity classes.

References

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