Report on Saturn V Rocket

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I Introduction

In the 17th century, Isaac Newton made the discovery of the universal force of gravity. It was then noted that an object with mass has the property of exerting gravity. The greater the mass an object has, the greater the force of gravity it exerts on another object. Because of this, we are gravitationally bound to Earth. However, through the efforts of many scientists, mathematicians, and Newton, the work or energy needed to bring an object to point mass source is proportional to the mass of the point source and inversely proportional to the distance between them squared. Using the gravitational constant theoretically calculated by Newton and experimentally calculated by Henry Cavendish, the work needed to move an object through a gravitational field. The gravitational potential can be calculated by using the following equation:

$$\phi(r) = -\frac{GM}{r^2} \qquad (1)$$

Where G is the gravitational constant, M is the mass of point source, and r is the distance between the point mass source and the object being moved.

Keep in mind that the equation above assumes that the mass is a point source and the point source mass is significantly greater than the mass of the object that is being moved. However, the gravitational potential can be taken a step further by taking the derivative with respect to r or the negative gradient. Doing so would give the force of the pull of gravity that one mass exerts on another; this is called gravitational force.

$$F(r) = -\nabla\left(-\frac{GM}{r}\right) = -G\frac{m_1 m_2}{|r_{21}|^2}\widehat{r_{21}} \qquad (2)$$

G is the gravitational constant, m_1, m_2 are masses of the two objects and r is the distance between the two objects.

By knowing the gravitational potential and gravitational force, we are able to find out the energy needed to escape Earth and Moon gravity and understand the forces a rocket may experience in outer space; opening the potential of space travel or sending objects into orbit. To help understand the gravitational potential and gravitational force from the Earth-Moon system, we used Python NumPy to calculate and Matplotlib to plot the functions in various styles.

II The Gravitational Potential of the Earth-Moon System

Using the equation for gravitational potential, we are able to find the amount of energy per kilogram to move an object through the Earth-Moon system gravity. In addition, use Python Matplotlib and NumPy to calculate and plot the gravitational potential of the system.

To plot the gravitational potential of Earth, I first define the function to take 5 inputs (mass of point source and 2 coordinates) and input them into equation 1. In the function, a safety measure is placed to return no value if the distance between the point mass source and object is or very close to zero. The mass of Earth was taken to $5.9 \times 10^{24}\ kg$ and the Moon's mass is $7.3 \times 10^{22}\ kg$. The gravitational potential was calculated individually for the Earth and Moon. From there, the absolute values of the gravitational potential were added to get the total magnitude at a given distance.The gravitational potential is plotted from -1.5 to 1.5 times the distance between Earth and the Moon for both axes.

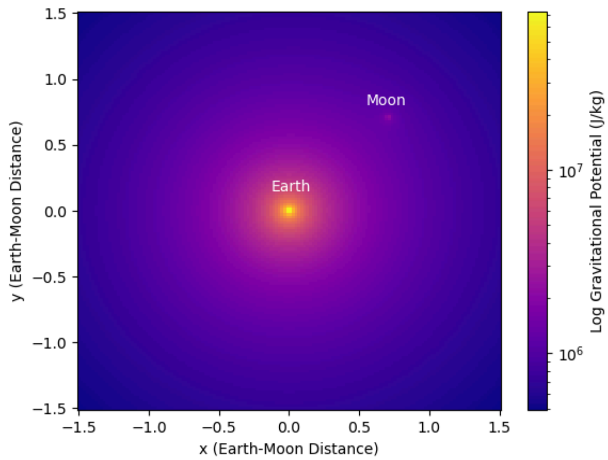

Figure 1: 2D colormesh plot of the gravitational potential of the Earth-Moon system. The Earth and the Moon were considered as point sources, where Earth has a x-y coordinate of$(0,\ 0)$ and the Moon is at $(\frac{d_{Em}}{\sqrt{2}}, \frac{d_{Em}}{\sqrt{2}})$.

According to the figure, the gravitational potential gets smaller the further it is from the mass source. Additionally, we can see that the gravitational potential is greater than the gravitational potential from the Moon, suggesting that it would be easier to move an object further away from Earth, and the Moon is much easier to transverse through its gravity.

III The Gravitational Force of the Earth-Moon System

Using the same procedure for plotting the gravitational potential of the Earth-Moon system, but changing the function to calculate the gravitational force and that it takes an additional mass input. Furthermore, the function returns an x and y component of the gravitational. Rather than plotting the same type of plot, a streamline plot was chosen to better reflect the gravitational field lines of the Earth-Moon system and the absolute value was not taken when adding the gravitational forces.

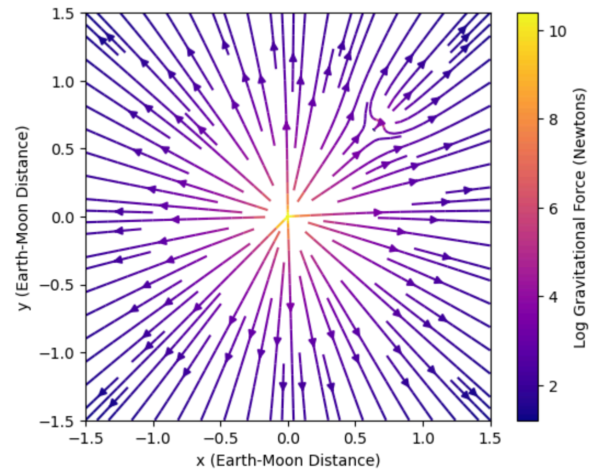

Figure 2: Streamline plot of the gravitational force vector lines that the Apollo 11 would experience from both Earth and the Moon. The Earth and the Moon have the same cartesian coordinates as the last figure.

Similar to the gravitational potential plot, the gravitational force decreases in magnitude the further it is from the point mass source. However, the gravitational force decreases much more rapidly as it is inversely proportional to the distance between the points squared. There is distortion in the gravitational force from the Moon, indicating that the Moon's gravitational force needs to be considered.

IV Projected Performance of the Saturn V Stage 1

In order for a rocket to enter space, it must first overcome the Earth's gravitational force. A rocket achieves this by using Newton's Third Law of Motion: for every force, there is an equal and opposite force. Using the conservation of momentum, the rocket expels mass by gas with great velocity from the bottom to create a thrust that pushes the rocket upward. However, rockets have many factors that must be considered, but thanks to the work of Konstantin Tsiolkovsky, the change in the rocket's velocity can be calculated. The Tsiolkovsky rocket equation is as follows:

$$\Delta v(t) = v_e ln(\frac{m_o}{m(t)}) - gt \quad (3)$$

Where v_e is the exhaust velocity of the rocket, m_o is the wet mass of the rocket, $m(t)$ is the amount of fuel at a given time, g is the gravitational acceleration of Earth, and t is time.

Using the Tsiolkovsky rocket equation and the known properties of the Saturn V rocket, we can calculate the change of velocity of the rocket. Although we can calculate the change in velocity of the rocket, the Tsiolkovsky rocket equation could be used to find the final altitude of the rocket by integrating it over some time.

$$h = \int_0^T \Delta v(t) dt \quad (4)$$

The T is the integral would be the burn time of the rocket or how long the rocket will last before it runs out of fuel. The burn time of a rocket is calculated as the following equation:

$$T = \frac{m_o - m_f}{\dot{m}} \quad (5)$$

Where $\dot{m}$ is the burn rate of the rocket, m_o is the wet mass of the rocket, and m_f is the dry mass of the rocket.

Taking the quantities of the Saturn V rocket, Saturn V rocket would have a theoretical burn time of 158 seconds and a theoretical final altitude of 74. 1 kilometers.

V Discussion and Future Work

During NASA's testing of the Saturn V rocket, they revealed that the rocket had a burn time of about 160 seconds and achieved an altitude of 70 km. When compared to our theoretical calculations, the burn time was an underestimation, and the final altitude was an overestimate. Although the equations used to calculate the burn time and altitude of the Saturn V were not exact, they provided a reasonable approximation of the Saturn V capabilities.

A possible reason for the underestimation of the burn time may be fuel preservation functions in the Saturn V to use fuel as efficiently as possible. First, the gravitational potential and force gets weaker the higher the rocket gets, so the rocket would need less force thus using less fuel. Second, the rocket loses mass as it burns its fuel, meaning the rocket needs to use less fuel the longer it goes. Moving on, the reason the final altitude calculated for the Saturn V being an overestimate is the assumptions made by the Tsiolkovsky rocket equation. The Tsiolkovsky rocket equation assumes that the rocket is traveling at constant velocity and burn time. It neglects drag force, air resistance, and other external forces that the rocket experiences as it takes off.

In conclusion, the Saturn V follows the properties of a standard rocket, as the Tsiolkovsky rocket equation and burn time equation provided reasonable approximations of the rocket performance. However, we can improve the calculations by considering external forces like drag force to increase the accuracy of the theoretical calculations for future rockets when calculating the final altitude. Opening the possibility of accurate computer simulations rather than live testing.