FEM: a end-user approach

Part III: h formulation

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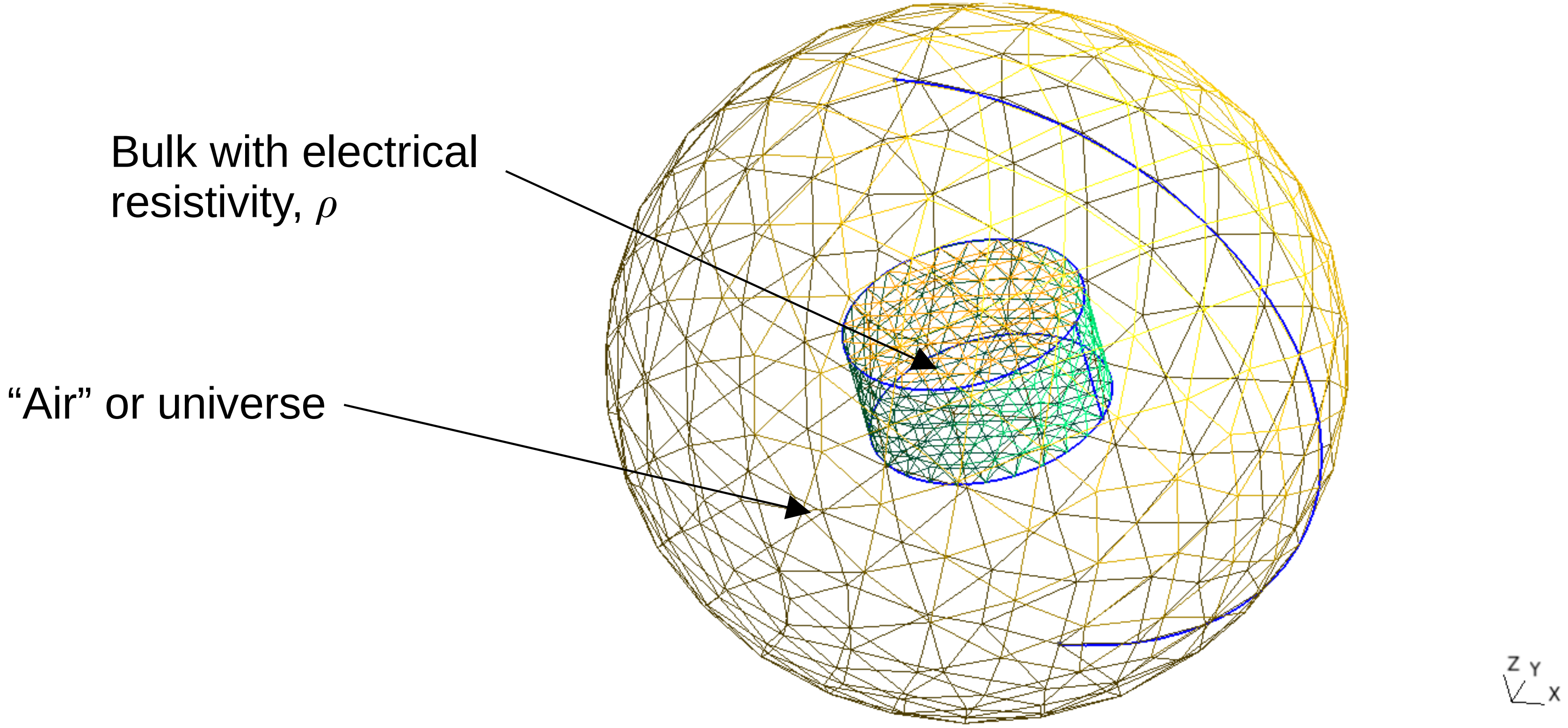
Case study

Induction of current in a metallic bulk under a varying applied magnetic field



Groups on meshes

<u>Volumes</u>

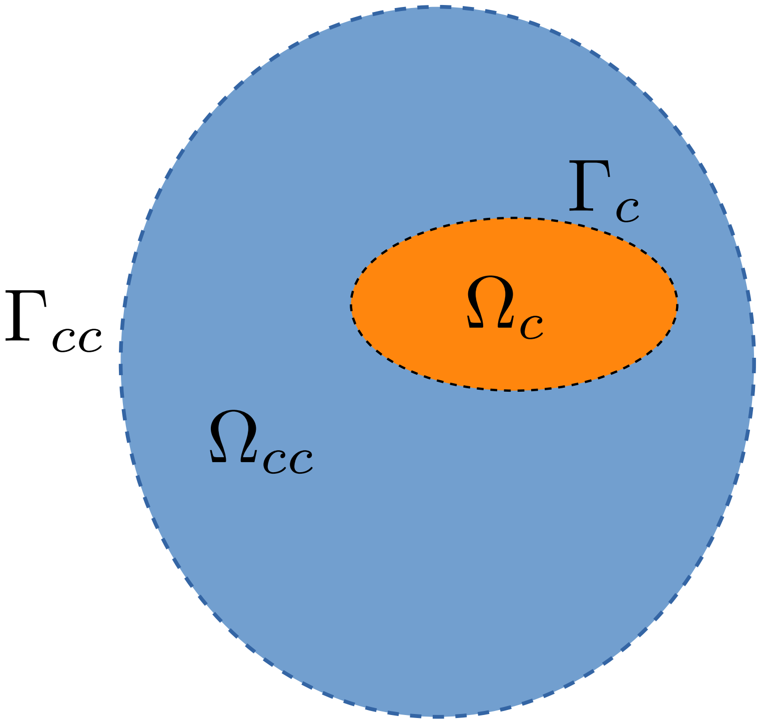
Material (bulk): Ω_c

Air: Ω_{cc}

<u>Boundaries</u>

Material: Γ_c

Air: Γ_{cc}



$$\Omega_{Bnd} = \Omega_c \cup \Omega_{cc} \cup \Gamma_{cc}$$
$$\Omega_{CBnd} = \Omega_c \cup \Gamma_c$$

Weak formulation

<u>Green's formulae:</u>

$$(\nabla \cdot \mathbf{u}\ v)_\Omega = -(\mathbf{u} \cdot \nabla v)_\Omega + \langle(\mathbf{u} \cdot \mathbf{n})\ v\rangle_\Gamma$$

u gradient-conform

$$u = \nabla \phi$$

$$(\nabla \times \mathbf{u} \cdot \mathbf{v})_\Omega = (\mathbf{u} \cdot \nabla \times \mathbf{v})_\Omega - \langle \mathbf{u} \times \mathbf{n} \cdot \mathbf{v}\rangle_\Gamma$$

u curl-conform

$$\mathbf{u} = \nabla \times \mathbf{w}$$

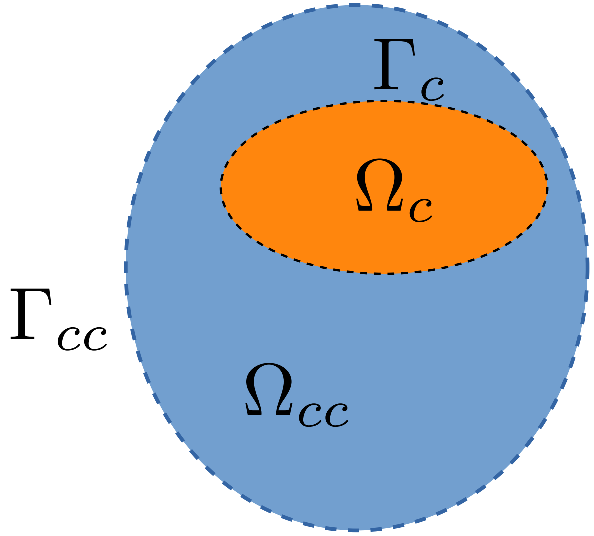
$$\boxed{\nabla \times \rho \nabla \times \mathbf{h} + \partial_t \mathbf{b} = 0}$$

$$\int_\Omega \rho \nabla \times \mathbf{h} \cdot \nabla \times \mathbf{h}^\star \mathrm{d}\Omega + \int_\Omega \partial_t (\mu \mathbf{h}) \cdot \mathbf{h}^\star \mathrm{d}\Omega - \int_{\Gamma_{cc}} \cancel{[\underbrace{(\rho \nabla \times \mathbf{h}) \times \mathbf{n}}_{\mathbf{e}}]} \cdot \mathbf{h}^\star \mathrm{d}\Gamma = 0$$

Time discretization: Euler

$$\partial_t(\mu\mathbf{h}) \simeq \frac{(\mu\mathbf{h})(t) - (\mu\mathbf{h})(t-\Delta t)}{\Delta t}$$

$$\mu = cst \quad \Rightarrow \quad \partial_t\mathbf{h} \simeq \frac{\mathbf{h}(t) - \mathbf{h}(t-\Delta t)}{\Delta t}$$



$$\int_\Omega \rho\nabla\times\mathbf{h}\cdot\nabla\times\mathbf{h}^\star d\Omega \boxed{+\int_\Omega \frac{\mu}{\Delta t}\mathbf{h}(t)\cdot\mathbf{h}^\star d\Omega - \int_\Omega \frac{\mu}{\Delta t}\mathbf{h}(t-\Delta t)\cdot\mathbf{h}^\star d\Omega = 0}$$

$$\mathbf{h}|_{\Gamma_{cc}} = \mathbf{h}_a(t)$$

Initial condition

Necessary initial condition: $\boxed{\mathbf{h}\,(t=0)=0}$ $\Rightarrow$ $\nabla\cdot\mathbf{b}=0$

<u>Demonstration (ensuring div(**b**) = 0)</u>

From Maxwell-Faraday: $\nabla\cdot(\nabla\times\rho\nabla\times\mathbf{h}+\partial_t\mathbf{b})=0$

as $\nabla\cdot(\nabla\times\rho\nabla\times\mathbf{h})=\rho\nabla\times\mathbf{h}\cdot(\nabla\times\nabla)=0$

then $\nabla\cdot\partial_t\mathbf{b}=0$ $\Rightarrow$ $\partial_t\,(\nabla\cdot\mathbf{b})=0$ $\Rightarrow$ $\underline{\nabla\cdot\mathbf{b}=cst_t}$

using the constitutive law, $\mathbf{b}=\mu\mathbf{h}$

and $\boxed{\mathbf{h}\,(t=0)=0}$

hence $\mathbf{b}\,(t=0)=0$ with $\underline{\nabla\cdot\mathbf{b}=cst_t}$

thus $\nabla\cdot\mathbf{b}=0$