

① Numerical methods in civil engineering are now used routinely in structural analysis to determine the member ~~of~~ forces & moments in structural system, prior to design, they are most useful in analysing civil engineering problems with complicated ~~in~~ geometrical analysis, material property and loading conditions where analytical methods are either very difficult or impossible to use.

- Numerical methods provide a way to solve problems quickly and easily compared to analytic solutions.
- Numerical methods can be used in problems of fluid surge! solving the balance of flow at a junction, for incompressible fluid flow in pipes.
- The problems of conservation of mass can be easily solved with the help of the numerical methods in various problems of air pollution or reactors in environmental & water waste engineering.

② Soln

Since the given number is in base 10, we will convert it to binary, octal and hexadecimal

To binary:

2	456	0
2	228	0
2	114	0
2	57	1
2	28	0
2	14	0
2	7	1
2	3	1
	1	

1.648	0.824	1
	*2	
1.296	0.648	1
	*2	
0.592	0.292	0
	*2	
1.184	0.892	1
	*2	
0.368	0.184	0
	*2	
0.736	0.368	0
	*2	

$\Rightarrow 456.824$ in binary is 111001000.110100

To octal: 111001000.110100
7 1 0 6 4

$\Rightarrow 456.824$ to octal will give us 710.64

To Hexa decimal:

456		
16	28	8
16	1	12
16	0	1

$\Rightarrow 1C8$

	0.824	
13.184	*16	13
3.944	0.184	2
	*16	
15.104	0.944	15
	*16	
1.664	0.104	1
	*16	
10.624	0.664	10
	*16	

$456.824 = \underline{\underline{1C8.02F1A}}$

$\Rightarrow 02F1A$

⑤ (a) $(2C.0B7)_{16}$

$$= 2 \times 16^1 + C \times 16^0 + 0 \times 16^{-1} + B \times 16^{-2} + 7 \times 16^{-3}$$

$$= 32 + 12 + 0.446773$$

$$= \underline{\underline{(44.0446773)_{10}}}$$

(b) $(48.D9)_{16}$

$$= 4 \times 16^1 + 8 \times 16^0 + D \times 16^{-1} + 9 \times 16^{-2}$$

$$= 64 + 8 + 8.8476562$$

$$= \underline{\underline{(72.8476562)_{10}}}$$

(c) $(74E.6A)_{16}$

$$= 7 \times 16^1 + 4 \times 16^0 + E \times 16^{-1} + 6 \times 16^{-2} + A \times 16^{-3}$$

$$= 112 + 64 + 14.4140625$$

$$= \underline{\underline{(1870.4140625)_{10}}}$$

⑨ req: Find % error in area of a rectangle.

soln: % error in length $= \frac{\Delta L}{L} \times 100 = \frac{1}{100} \times 100 = 1\%$

$$\% \text{ error in breadth} = \frac{\Delta b}{b} \times 100 = \frac{1}{100} \times 100 = 1\%$$

$$\% \text{ error in area} = 1\% + 1\% = \underline{\underline{2\%}}$$

⑪ ② req: % error in density

soln: $\rho = m/v$

$$\frac{d}{dx} (\log \rho = \log m - \log v)$$

$$\Rightarrow \frac{d\rho}{\rho} = \frac{dm}{m} + \frac{dv}{v}$$

$$= 5/500 + 2/130$$

$$= \frac{5 + 2 \times 4}{500} = \frac{13}{500}$$

$$= \underline{\underline{0.025}}$$

$$= 0.025 \times 100\%$$

$$= \underline{\underline{2.5\%}}$$

⑥ req: density of liquid

soln: $\rho = \rho' + \Delta\rho$

$$\Delta\rho = \frac{m' |\Delta v|}{v^2} + \frac{v' |\Delta m|}{v^2}$$

$$\Delta\rho = \frac{500(2)}{16900} + \frac{130(5)}{16900}$$

$$= 0.0615384 + 0.0384615$$

$$= 0.0999999$$

$$\rho' = \frac{m'}{v'} = \frac{500}{130} = 4$$

$$\rho = \rho' + \Delta\rho = 4 + 0.0999999$$

$$= \underline{\underline{4.0999999}}$$

(12) req: % error introduced in calculation of max bending stress
soln:

⇒ max bending stress developed in straight member

$$\sigma_{max} = \frac{M_c}{I} = \frac{50(10)^3(0.1)}{\frac{1}{12}(0.1)(0.2)^3} = \underline{\underline{75 \text{ Mpa}}}$$

⇒ stress developed on curved member.

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{0.1(0.2)}{0.10 \ln \frac{0.4}{0.2}} = \underline{\underline{0.288539 \text{ m}}}$$

$$\Rightarrow e = \bar{r} - R = \underline{\underline{0.011461 \text{ m}}}$$

* $M = 50 \text{ kNm}$, since it tends to decrease the curvature of the curved member

$$\sigma_B = \frac{M(R - r_B)}{Aer_B} = \frac{50(10)^3(0.288539 - 0.4)}{(0.1)(0.2)(0.011461)(0.4)} = 60.78 \text{ Mpa (compression)}$$

$$\sigma_A = \frac{M(R - r_A)}{Aer_A} = \frac{50(10)^3(0.288539 - 0.2)}{(0.1)(0.2)(0.011461)(0.2)} = 96.57 \text{ Mpa (tension)}$$

$$\% \text{ error} = \frac{96.57 - 75}{96.57} = 0.223$$

$$\Rightarrow 0.223 \times 100\% = \underline{\underline{22.3\%}}$$

(14) req: max error in the measurement of strain

Soln: Total error of strain

$$|\Delta \epsilon| = \left| \frac{\partial \epsilon}{\partial F} \Delta F \right| + \left| \frac{\partial \epsilon}{\partial A} \Delta A \right| + \left| \frac{\partial \epsilon}{\partial E} \Delta E \right|$$

$$\frac{\partial \epsilon}{\partial F} = 1/AE$$

$$\frac{\partial \epsilon}{\partial A} = -\frac{F}{A^2 E}$$

$$\frac{\partial \epsilon}{\partial E} = -F/AE^2$$

$$|\Delta \epsilon| = \left| \left(\frac{1}{(0.2)(210 \times 10^9)} \right) \times 0.5 \right| + \left| \frac{-50}{(0.2)^2 \times (210 \times 10^9)} \times 0.02 \right| + \left| \left(\frac{-50}{(0.2)(210 \times 10^9)^2} \right) \times 14.1 \right|$$

$$= 1.19 \times 10^{-11} + 1.19 \times 10^{-11} + 5.67 \times 10^{-12}$$

$$= \underline{\underline{2.95 \times 10^{-11}}}$$

17) a) req: Find 3rd Taylor polynomial

$$f(x) = 2x \cos(2x) - (x-2)^3$$

$$f'(x) = 2 \cos(2x) - 4x \sin(2x) - 2x + 4$$

$$f''(x) = -4 \sin(2x) - 8x \cos(2x) - 2$$

$$f'''(x) = (f''(x))' = -24 \cos(2x) + 16x \sin(2x)$$

$$\Rightarrow x_0 = 0$$

$$f(0) \Rightarrow -4$$

$$f'(0) \Rightarrow$$

$$f''(0) \Rightarrow -2$$

$$f'''(0) \Rightarrow -24$$

$$P_3(x) = \frac{f(0)}{0!} (x-0)^0 + \frac{f'(0)}{1!} (x-0)^1 + \frac{f''(0)}{2!} (x-0)^2 + \frac{f'''(0)}{3!} (x-0)^3$$

$$P_3(x) = -4 + 6x - x^2 + 4x^3$$

$$P_3(x) \text{ at } x=0.4$$

$$P_3(0.4) = -4 + 6(0.4) - (0.4)^2 + 4(0.4)^3$$
$$= -2.016$$

b) req: Find upper bound for error

$$\frac{f^{(4)}(c)}{4!} (0.4)^4 \leq \frac{76.8}{24} (0.4)^4 = \underline{\underline{0.08192}}$$

$$|f(0.4) - P_3(0.4)| = |-2.002634 + 2.016|$$

$$= 0.133653 \text{ approximated to } \underline{\underline{1.34 \times 10^{-3}}}$$