STA 26W1

TEST NO. 1 **28 MARCH 2024** **Marks 50** **Time : 2hrs**

Question 1[20]

1.1 Consider the experiment of tossing a pair of fair dice.

1.1.1 Describe the sample space that might be appropriate for this experiment. (3)

1.1.2 Describe the event B that the total number of points rolled with the pair of dice is 7. (2)

1.1.3 Obtain the probability distribution for the total outcomes of the pairs of dice. (3)

1.2 An urn contains 12 articles, 8 of which are good and 4 of which are defective.

1.2.1 Suppose 3 articles are to be withdrawn from one after another without replacement, and the results of individual drawings are to be recorded.

1.2.2 Keeping the track of order of selection, how many ordered samples are possible? (1)

1.2.3 How many ordered samples are possible under the condition that the first two produce defective articles and the third yields a good piece? (2)

1.2.5 If the article is drawn at random, what is the probability of obtaining defective in the first two draws and good piece in the third draw? (2)

1.4 Use binomial theorem to expand $(4+3x)^5$. (3)

1.5 Write down the first four terms in the binomial series for this function $\sqrt[3]{(8-2x)}$. (4)

Question 1[30]

2.1 Let f(x) = x/15, x = 1, 2, 3, 4, 5, zero elsewhere, be the pmf of X. Find

2.1.1 P (X = 1 or 2), (3)

2.1.2 P (1/2 < X < 5/2). (3)

2.1.3 P (1 ≤ X ≤ 2). (3)

3.1 The density function for the random variable X is given by

$$f(x)=\begin{cases} kx & for \quad 0\le x\le 2 \\ 0 & otherwise \end{cases}$$

3.1.1 Find k and $P\left(\frac{1}{2}\le x\le 1\right)$ (4)

3.1.2 Find $F_X\left(x\right)$ (4)

4 Let X be a continuous random variable with the pdf $f\left(x\right)=2x$ which has support on the interval (0, 1). Suppose $Y=\dfrac{1}{1+X}$, Compute $E\left(Y\right)$. (3)

5. Find the moment generating function of the discrete random variable X which has probability distribution $f\left(x\right)=2\left(\dfrac{1}{3}\right)^x, \quad x=1, 2, 3, ...$
   and use it to determine μ'_1 *and* μ'_2. (7)

6. Find the moment generating function of the continuous random variable X whose probability density is given by $f(x)=\begin{cases}1 & for\, 0<x<1\\ 0 & elsewhere\end{cases}$ (3)