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An approximation-based approach for fuzzy multi-period production planning problem with credibility objective

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ABSTRACT

This paper develops a fuzzy multi-period production planning and sourcing problem with credibility objective, in which a manufacturer has a number of plants or subcontractors. According to the credibility service levels set by customers in advance, the manufacturer has to satisfy different product demands. In the proposed production problem, production cost, inventory cost and product demands are uncertain and characterized by fuzzy variables. The problem is to determine when and how many products are manufactured so as to maximize the credibility of the fuzzy costs not exceeding a given allowable invested capital, and this credibility can be regarded as the investment risk criteria in fuzzy decision systems. In the case when the fuzzy parameters are mutually independent gamma distributions, we can turn the service level constraints into their equivalent deterministic forms. However, in this situation the exact analytical expression for the credibility objective is unavailable, thus conventional optimization algorithms cannot be used to solve our production planning problems. To overcome this obstacle, we adopt an approximation scheme to compute the credibility objective, and deal with the convergence about the computational method. Furthermore, we develop two heuristic solution methods. The first is a combination of the approximation method and a particle swarm optimization (PSO) algorithm, and the second is a hybrid algorithm by integrating the approximation method, a neural network (NN), and the PSO algorithm. Finally, we consider one 6-product source, 6-period production planning problem, and compare the effectiveness of two algorithms via numerical experiments.

1. Introduction

Production planning is viewed as the plans and arrangements of the production mission and progress in production scheduled time. In recent years, uncertain production planning has received much attention in the field of production planning management, where uncertainty can be present as randomness and fuzziness in the production environments. This uncertainty will result in more realistic production planning models. However, the inclusion of uncertainty in the production system parameters is a more difficult task in terms of modeling and solving. Over the years, there has been much research and many applications with the aim of modeling the uncertainty in the production planning problems, including material requirements planning models [1,2], hierarchical production planning models [3,4], aggregate production planning model [5], supply chain models [6,7], and other well-known production planning models in the literature [8].

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To handle probabilistic uncertainty in production decision systems, some meaningful stochastic production planning models have been documented in the literature. Bitran [9] dealt with a stochastic production planning problem with a service level requirement and provided non-sequential and deterministic equivalent formulations of the model; Schmidt [10] presented a Markov decision process model that combines features of engineering design models and aggregate production planning models to obtain a hybrid model that links biological and engineering parameters to optimize operations performance; Yildrim et al. [11] studied a stochastic multi-period production planning and sourcing problem of a manufacturer with a number of plants or subcontractors and presented a methodology that a manufacturer can utilize to make its production and sourcing decisions, and Kelly et al. [12] extended the economic lot scheduling problem for the single-machine multi-product case with random demands, their objective was to find the optimal length of production cycles that minimizes the sum of set-up costs and inventory holding costs per unit of time and satisfies the demand of products at the required service levels.

On the other hand, with the development of fuzzy set and possibility theories [13–15], a number of researchers realized the importance to handle possibilistic uncertainty in decision systems, and applied the fuzzy theory to various production planning problems. Wang and Fang [16] presented a fuzzy linear programming model for solving the aggregate production planning problem with multiple objectives; Inuiguchi et al. [17] discussed the merits of possibilistic programming approach to production planning problems and applied a possibilistic programming based on possibility and necessity measures to solve the production planning problem. The reader who is interested in related issues in this field may refer to [18–20].

Since the credibility of fuzzy event and the expected value of fuzzy variable was defined in [21], an axiomatic approach called credibility theory has been developed in recent years (see [22,23]). Some interesting applications about credibility in production decision systems have been studied in the literature such as [24,25]. Maity et al. [24] proposed an optimal control approach to optimizing the production, recycling and disposal strategy so that the total expected profit is maximized, and Mandal et al. [25] developed an optimal production inventory model with fuzzy time period and fuzzy inventory costs for defective items and solved it under fuzzy space constraint. In the current development, we take credibility theory as the theoretical foundation of fuzzy optimization and formulate a novel class of multi-period production planning problems with credibility objective, in which product demands, production and inventory costs are uncertain and characterized by fuzzy variables. The objective of the problem is to maximize the credibility of the fuzzy costs not exceeding a given allowable invested capital, and this credibility can be regarded as the investment risk criteria under fuzzy environment. In general, the credibility functions in the service level constraints are difficult to compute, so we discuss the cases when demands are independent gamma fuzzy variables. In this situation, we can transform the credibility service level constraints to their equivalent deterministic forms. However, the analytical expression about the credibility objective is unavailable, and the equivalent production planning problem is neither linear nor convex, thus conventional optimization algorithms cannot be employed to solve it. Therefore, two heuristic solution methods are designed to solve the proposed production planning model. The first is the PSO algorithm (see [26,27]) combining with the approximation method [28], and the second is the hybrid PSO algorithm by integrating both the approximation method and an NN. One 6-product source, 6-period production planning problem is provided to compare the effectiveness of the designed algorithms.

The plan of this paper is as follows. Section 2 formulates a new class of fuzzy production planning models. In Section 3, we discuss the computation for the credibility objective of the production planning model, and deal with the convergence of computational method. In addition, due to the complexity of the proposed production planning problem, two heuristic solution methods are designed in this section. The first is the PSO algorithm based on the approximation method, and the second is a hybrid algorithm by integrating the approximation method, an NN and the PSO algorithm. Section 4 presents one 6-product source, 6-period production planning problem to compare the effectiveness of two algorithms. Section 5 concludes the paper.

2. Problem formulation

In this section, we will construct a new type of fuzzy programming models for multi-period production planning and sourcing problems with fuzzy parameters. The characteristic of this manufacturing system can be summarized as follows.

- There are N types of production sources (plants and subcontractors) in the system, and the decision of production levels to meet market demands must be taken for T periods. The demand in each period is not known with certainty and characterized by a fuzzy variable.
- The costs that are used in the objective function of the model consist of production cost and inventory carrying cost. The production and inventory cost coefficients are not known exactly and represented by fuzzy variables. In general, we assume fuzzy demands, production and inventory cost coefficients in different periods are mutually independent (see [29]).
- Constraints on the performance (related to backorders) of the system are imposed by requiring service levels which force the credibility of having no stock out to be greater than or equal to a predetermined service level requirement in each period.

The following indices and parameters are used to describe the manufacturing system.

Indices
i index of sources, $i = 1, 2, \ldots, N$
t index of periods, $t = 1, 2, \ldots, T$
Parameters
$c_{i,t}$ the fuzzy production cost at source i in period t
h_t the fuzzy unit cost of inventories in period t
I_t the inventory level at the end of period t
d_t the fuzzy demand for the specific product in period t
α_t the credibility service level requirement in period t
Decision variables
$x_{i,t}$ the production quantities at source i in period t
Objective function
The objective function includes the following costs
The total inventory cost during T periods

$$\sum_{t=1}^{T} h_t I_t^+,$$

where $I_t^+ = \max\{0, I_t\}$.

The total production cost from N sources during T periods:

$$\sum_{i=1}^{N} \sum_{t=1}^{T} c_{i,t} x_{i,t}.$$

As a consequence, the objective is to maximize the credibility that the total fuzzy cost is less than a preselected threshold f_0, i.e.,

$$\mathrm{Cr}\left\{\sum_{t=1}^{T}\left(h_t I_t^+ + \sum_{i=1}^{N} c_{i,t} x_{i,t}\right) \leqslant f_0\right\}.$$

Constraints:
I: Credibility constraint
The inventory balance equation for each period is

$$I_t = I_{t-1} + \sum_{i=1}^{N} x_{i,t} - d_t, \quad t = 1, \ldots, T,$$

or

$$I_t = I_0 + \sum_{i=1}^{N} \sum_{j=1}^{t} x_{i,j} - \sum_{j=1}^{t} d_j, \quad t = 1, \ldots, T,$$

which denotes the inventory level at the end of period t. As a consequence, the credibility service level constraint in each period is

$$\mathrm{Cr}\{I_t \geqslant 0\} \geqslant \alpha_t, \quad t = 1, \ldots, T.$$

The constraints impose the credibility of the fuzzy inventory levels at each period t are nonnegative.
II: System constraint:

$$x_{i,t} \geqslant 0, i = 1, \ldots, N, t = 1, \ldots, T.$$

The constraints state that the production quantities are nonnegative.

Using the notation above, we present a new approach to establishing a meaningful production planning problem with minimum risk criteria. We adopt the credibility criteria in the objective and build an N-product source, T-period production planning problem as the following mathematical programming model:

$$\begin{array}{ll} \max & \mathrm{Cr}\left\{\sum_{t=1}^{T}\left(h_t I_t^+ + \sum_{i=1}^{N} c_{i,t} x_{i,t}\right) \leqslant f_0\right\} \\ \text{subject to}: & \mathrm{Cr}\{I_t \geqslant 0\} \geqslant \alpha_t, \quad t = 1, \ldots, T, \\ & x_{i,t} \geqslant 0, \quad i = 1, \ldots, N, \; t = 1, \ldots, T, \\ & I_t^+ = \max\{0, I_t\}, \quad t = 1, \ldots, T, \end{array} \tag{1}$$

where $I_t = I_{t-1} + \sum_{i=1}^{N} x_{i,t} - d_t$ for $t = 1, \ldots, T$.

From the discussion above, the major differences between fuzzy production planning problem and stochastic production planning problem are summarized as follows:

- The uncertain data in fuzzy production planning problem are fuzzy variables with known possibility distributions, while the uncertain data in the stochastic production planning problem are random variables with known probability distributions.
- The objective function in fuzzy production planning problem is the computation for the credibility of fuzzy events, while in the stochastic problem, it is the computation for the probability of stochastic events. From the computation of credibility (see [21]), we can see it is quite different from that of probability.

These differences lead to the solution techniques developed for stochastic production planning problems can not be applied to fuzzy ones. In order to find optimal production decisions, the computational obstacles must be overcome, which will be addressed in the next section.

3. Solution methods

The implementation of conventional optimization methods demands easy access to function values and gradients, and even Hessians. Given the limitations of nonlinear fuzzy integrals, the credibility objective values in model (1) can very often be obtained numerically. In addition, it is difficult to check that the credibility function is differential, let alone the computation about its gradients and Hessians. Therefore, the conventional optimization methods cannot be applied to the solution of production planning problem (1), it is imperative to develop solution methods that do depend on approximation schemes. In this section, we develop two heuristic algorithms to solve the production planning problem (1). The first is the PSO algorithm based on the approximation method [28], and the second is a hybrid algorithm by integrating the approximation method, an NN and the PSO algorithm. Since the fuzzy production planning problem (1) is not generally a convex programming one, we can only obtain its approximate or near optimal solutions by the designed algorithms.

3.1. Handling the credibility constraints

In some case, we may transform the credibility service level constraints into their equivalent deterministic forms.

Theorem 1. *Let ξ be a gamma fuzzy variable with parameter λ such that its possibility distribution function is $\mu_\xi(x) = (x/(\lambda r))^r \exp(r - x/\lambda)$, where $x \geqslant 0, \lambda \in \Re$, and r is a fixed constant. Then, for any given credibility level $\alpha \in (0,1]$, we have:*

(i) *If $\alpha < 0.5$, then $\mathrm{Cr}\{\xi \leqslant t\} \geqslant \alpha$ is equivalent to $\ln 2\alpha + t/\lambda - r - r\exp(t/(\lambda r)) \leqslant 0$;*
(ii) *If $\alpha \geqslant 0.5$, then $\mathrm{Cr}\{\xi \leqslant t\} \geqslant \alpha$ is equivalent to $\ln 2(1-\alpha) + t/\lambda - r - r\exp(t/(\lambda r)) \geqslant 0$.*

Proof. By the possibility distribution of ξ and the computational method of $\mathrm{Cr}\{\xi \leqslant t\}$, we get

$$\mathrm{Cr}\{\xi \leqslant t\} = \begin{cases} 1 - \frac{1}{2}(t/(\lambda r))^r \exp(r - t/\lambda) & \text{if } t \geqslant \lambda r, \\ \frac{1}{2}(t/(\lambda r))^r \exp(r - t/\lambda) & \text{if } t < \lambda r. \end{cases}$$

If $\alpha < 0.5$, then we have

$$\frac{1}{2}(t/(\lambda r))^r \exp(r - t/\lambda) \geqslant \alpha,$$

i.e.,

$$(t/(\lambda r))^r \exp(r - t/\lambda) \geqslant 2\alpha.$$

Therefore, we can obtain

$$\exp(r - t/\lambda) \geqslant 2\alpha/(t/(\lambda r))^r,$$

which is equivalent to

$$\ln 2\alpha + t/\lambda - r - r\ln t - r\ln \lambda r \leqslant 0.$$

Similarly, if $\alpha \geqslant 0.5$, then we can obtain the equivalent deterministic form of $\mathrm{Cr}\{\xi \leqslant t\} \geqslant \alpha$ as follows

$$\ln 2(1-\alpha) + t/\lambda - r - r\ln t - r\ln \lambda r \geqslant 0. \qquad \square$$

According to Theorem 1, we have the following result:

Theorem 2. *Let*

$$g(x,\xi)=f_1(x)\xi_1+f_2(x)\xi_2+\cdots+f_n(x)\xi_n+f_0(x),$$

where gamma fuzzy variables $\xi_k,\ k=1,2,\ldots,n$, *are mutually independent with parameters* $\lambda_k,\ k=1,2,\ldots,n$, *and their possibility distributions are*

$$\mu_{\xi_k}(x)=(x/(\lambda_k r))^r\exp(r-x/\lambda_k),\ x\geqslant 0,\quad \lambda_k\in\Re,\ k=1,2,\ldots,n.$$

If we denote $f_k^+(x)=\max\{f_k(x),0\}$, *and* $f_k^-(x)=\max\{-f_k(x),0\}$ *for* $k=1,2,\ldots,n$, *then, for any given credibility level* $\alpha\in(0,1]$, *we have:*

(i) *If* $\alpha<0.5$, *then* $Cr\{g(x,\xi)\leqslant 0\}\geqslant\alpha$ *is equivalent to*

$$\ln 2\alpha - f_0(\mathbf{x})/\left(\sum_{k=1}^n\lambda_k f_k^+(x)+\sum_{k=1}^n\lambda_k f_k^-(x)\right)-r-r\ln(-f_0)-r\ln\left(r\left(\sum_{k=1}^n\lambda_k f_k^+(x)+\sum_{k=1}^n\lambda_k f_k^-(x)\right)\right)\leqslant 0;$$

(ii) If $\alpha\geqslant 0.5$, then $Cr\{g(x,\xi)\leqslant 0\}\geqslant\alpha$ is equivalent to

$$\ln 2(1-\alpha)-f_0(x)/\left(\sum_{k=1}^n\lambda_k f_k^+(x)+\sum_{k=1}^n\lambda_k f_k^-(x)\right)-r-r\ln(-f_0)-r\ln\left(r\left(\sum_{k=1}^n\lambda_k f_k^+(x)+\sum_{k=1}^n\lambda_k f_k^-(x)\right)\right)\geqslant 0.$$

Proof. By the negativity of $f_k^+(x)$ and $f_k^-(x)$, and $f_k(x)=f_k^+(x)-f_k^-(x)$, we have

$$g(\mathbf{x},\xi)=\sum_{k=1}^n f_k(x)\xi_k+f_0(x)=\sum_{k=1}^n[f_k^+(x)-f_k^-(x)]\xi_k+f_0(x)=\sum_{k=1}^n[f_k^+(x)\xi_k+f_k^-(x)\xi_k']+f_0(x),$$

where ξ_k is the gamma fuzzy variables with parameter $-\lambda_k,\ k=1,2,\ldots,n$. According to the independence of the fuzzy variables, $g(x,\xi)-f_0(x)$ is also a gamma fuzzy variable with parameter $\sum_{k=1}^n\left[f_k^+(x)\lambda_k-f_k^-(x)\lambda_k\right],\ k=1,2,\ldots,n$. It follows from Theorem 1 that assertions (i) and (ii) are valid. □

Now we consider the following credibility service level constraints

$$\mathrm{Cr}\{I_t\geqslant 0\}\geqslant\alpha_t,\quad t=1,\ldots,T,$$

which can be rewritten as

$$\mathrm{Cr}\left\{I_0+\sum_{i=1}^N\sum_{j=1}^t x_{i,j}-\sum_{j=1}^t d_j\geqslant 0\right\}\geqslant\alpha_t,\quad t=1,\ldots,T.$$

For simplicity, we assume that d_j are gamma fuzzy variables with parameters $\lambda_j,\ j=1,2,\ldots,T$. When fuzzy demands are mutually independent and $\alpha_t\geqslant 0.5,\ t=1,\ldots,T$, according to Theorem 2, the credibility service level constraints become the following equivalent deterministic forms.

$$\ln 2(1-\alpha_t)+\left(I_0+\sum_{i=1}^N\sum_{j=1}^t x_{i,j}\right)\Big/\left(\sum_{k=1}^n\lambda_j\right)-r-r\ln\left(I_0+\sum_{i=1}^N\sum_{j=1}^t x_{i,j}\right)-\left(r\ln\left(r\left(\sum_{k=1}^n\lambda_j\right)\right)\right)\geqslant 0.$$

As a consequence, production planning problem (1) can be turned into the following equivalent mathematical programming model

$$\begin{array}{ll}\max & \mathrm{Cr}\left\{\sum_{t=1}^T\left(h_tI_t^++\sum_{i=1}^N c_{i,t}x_{i,t}\right)\leqslant f_0\right\}\\ \text{subject to:} & \ln 2(1-\alpha_t)+\left(I_0+\sum_{i=1}^N\sum_{j=1}^t x_{i,j}\right)\Big/\left(\sum_{j=1}^n\lambda_j\right)-r-r\ln\left(I_0+\sum_{i=1}^N\sum_{j=1}^t x_{i,j}\right)\\ & \qquad -\left(r\ln\left(r\left(\sum_{k=1}^n\lambda_j\right)\right)\right)\geqslant 0,\quad t=1,\ldots,T,\\ & x_{i,t}\geqslant 0, i=1,\ldots,N,\quad t=1,\ldots,T,\\ & I_t^+=\max\{0,I_t\},\quad t=1,\ldots,T,\end{array}\tag{2}$$

where $I_t=I_{t-1}+\sum_{i=1}^N x_{i,t}-d_t,\ t=1,\ldots,T$. In the following, we focus our attention to the computation about the credibility objective of problem (2).

3.2. Computing credibility objective

Let

$$C(x,\xi(\gamma)) = \sum_{t=1}^{T} h_t I_t^+ + \sum_{i=1}^{N}\sum_{t=1}^{T} c_{i,t}x_{i,t}. \tag{3}$$

Then to solve the production planning problem (2), it is required to compute the credibility function

$$D : x \mapsto \mathrm{Cr}\{\gamma \in \Gamma | C(x,\xi(\gamma)) \leqslant f_0\}, \tag{4}$$

repeatedly, where $\xi(\gamma) = (h_1(\gamma), h_2(\gamma), \ldots, h_T(\gamma),\ c_{1,1}(\gamma), c_{1,2}(\gamma), \ldots, c_{N,T}(\gamma),\ d_1(\gamma), d_2(\gamma), \ldots, d_T(\gamma))$ is the fuzzy vector obtained by piecing together fuzzy inventory cost, fuzzy production cost and fuzzy demands in problem (2). For any given x, we can compute the value of credibility function (4) by the following method.

Suppose that $\xi = (\xi_1, \xi_2, \ldots, \xi_{NT+2T})$ is a continuous fuzzy vector with the following infinite support $\Xi = \prod_{j=1}^{NT+2T}[a_j, b_j]$, where $[a_j, b_j]$ is the support of ξ_j. In this case, we will try to use the approximation method [28] to approximate the possibility distribution of ξ by a sequence of possibility distributions of discrete fuzzy vectors $\{\zeta_s\}$. The process can be described as follows.

For each integer s, we define the discrete fuzzy vector $\zeta_s = (\zeta_{s,1}, \zeta_{s,2}, \ldots, \zeta_{s,NT+2T})$ by the following method:

For each $j \in \{1, 2, \ldots, NT+2T\}$, define fuzzy variables $\zeta_{s,j} = g_{s,j}(\xi_j)$ for $s = 1, 2, \ldots$, where the function $g_{s,j}$ is as follows

$$g_{s,j}(u_j) = \sup\left\{\frac{k_j}{s} | k_j \in Z \text{ such that } \frac{k_j}{s} \leqslant u_j\right\}, \quad u_j \in [a_j, b_j],$$

with Z being the set of integers.

Moreover, for each $j,\ 1 \leqslant j \leqslant NT+2T$, by the definition of $\zeta_{s,j}$, when ξ_j takes its values in $[a_j, b_j]$, the fuzzy vector $\zeta_{s,j}$ takes its values in the set $\{k_j/s | k_j = [sa_j], [sa_j]+1, \ldots, K_j\}$, where $[r]$ is the maximal integer such that $[r] \leqslant r$, and $K_j = sb_j - 1$ or $[sb_j]$ according as sb_j is an integer or not an integer. What's more, for each integer k_j, the fuzzy vector $\zeta_{s,j}$ takes the value k_j/s as ξ_j takes its values in the interval $[k_j/s, (k_j+1)/s)$. Therefore, the possibility distribution $\nu_{s,j}$ of the fuzzy variable $\zeta_{s,j}$ is

$$\nu_{s,j}\left(\frac{k_j}{s}\right) = \mathrm{Pos}\left\{\gamma | \frac{k_j}{s} \leqslant \xi_j(\gamma) < \frac{k_j+1}{s}\right\},$$

for $k_j = [sa_j], [sa_j]+1, \ldots, K_j$. By the definition of $\zeta_{s,j}$, for each $\gamma \in \Gamma$, we have

$$\xi_j(\gamma) - \frac{1}{s} < \zeta_{s,j}(\gamma) \leqslant \xi_j(\gamma),$$

for $j = 1, 2, \ldots, NT+2T$. Therefore, we have

$$|\xi_j(\gamma) - \zeta_{s,j}(\gamma)| < \frac{1}{s}.$$

Note that ξ and ζ_s are $NT+2T$-ary fuzzy vectors, and ξ_j and $\zeta_{s,j}$ are their jth components, respectively. As a consequence,

$$\|\zeta_s(\gamma) - \xi(\gamma)\| = \sqrt{\sum_{j=1}^{2T}(\zeta_{s,j}(\gamma) - \xi_j(\gamma))^2} < \frac{\sqrt{NT+2T}}{s}, \quad \gamma \in \Gamma,$$

which implies that the sequence $\{\zeta_s\}$ of fuzzy vectors converges to fuzzy vector ξ uniformly.

We now provide an example to illustrate the approximation method described above.

Example 1. Suppose ξ_1 and ξ_2 are mutually independent gamma fuzzy variables, their possibility distributions are $x\exp(1-x),\ x \in [0,5]$, and $(x/2)\exp(1-x/2),\ x \in [0,10]$, respectively. In this case, we denote $\xi = (\xi_1, \xi_2)$. Determine the possibility distributions of discrete fuzzy vectors $\zeta_s = (\zeta_{s,1}, \zeta_{s,2}),\ s = 1, 2, \ldots$, where the fuzzy variables $\zeta_{s,i} = g_{s,i}(\xi_i),\ i = 1, 2$, with

$$g_{s,1}(u_1) = \sup\left\{\frac{k_1}{s} | k_1 \in Z \text{ such that } \frac{k_1}{s} \leqslant u_1\right\}, \quad u_1 \in [0,5],$$

and

$$g_{s,2}(u_2) = \sup\left\{\frac{k_2}{s} | k_2 \in Z \text{ such that } \frac{k_2}{s} \leqslant u_2\right\}, \quad u_2 \in [0,10].$$

We first derive the possibility distributions of fuzzy variables $\zeta_{s,1},\ s = 1, 2, \ldots$.

Let $s=1$. Then fuzzy variable $\zeta_{1,1}$ takes the value 0 as ξ_1 takes its value in $[0,2.5)$, and takes the value 1 as ξ_1 takes its value in $[2.5,5]$. Therefore, we have

$$\nu_{1,1}(0)=\mathrm{Pos}\{0\leqslant\xi_1<2.5\}=1,\quad \nu_{1,1}(1)=\mathrm{Pos}\{2.5\leqslant\xi_1\leqslant 5\}=1,$$

i.e., the fuzzy variable $\zeta_{1,1}$ takes on values 0 and 1 with possibility 1 each.

Let $s=2$. Then fuzzy variable $\zeta_{2,1}$ takes the values 0, 1.25, 2.5 and 3.75 as the fuzzy variable ξ_1 takes its values in the intervals $[0,1.25)$, $[1.25,2.5)$, $[2.5,3.75)$ and $[3.75,5]$, respectively. Therefore, we have

$$\begin{aligned}\nu_{2,1}(0)&=\mathrm{Pos}\{0\leqslant\xi_1<1.25\}=1,\quad \nu_{2,1}(1.25)=\mathrm{Pos}\{1.25\leqslant\xi_1<2.5\}=(5/4)e^{-1/4},\quad \nu_{2,1}(2.5)\\&=\mathrm{Pos}\{2.5\leqslant\xi_1<3.75\}=(5/2)e^{-3/2},\quad \nu_{2,1}(3.75)=\mathrm{Pos}\{3.75\leqslant\xi_1\leqslant 5\}=(15/4)e^{-11/4}.\end{aligned}$$

That is, the fuzzy variable $\zeta_{2,1}$ takes on values 0, 1.25, 2.5, and 3.75 with possibility 1, $(5/4)e^{-1/4}$, $(5/2)e^{-3/2}$ and $(15/4)e^{-11/4}$, respectively.

Generally, the fuzzy variable $\zeta_{s,1}$ takes on values $k_1/s,\ k_1=0,1,\ldots,5s$, and the possibility that $\zeta_{s,1}$ takes the value k_1/s is

$$\nu_{s,1}\left(\frac{k_1}{s}\right)=\begin{cases}\frac{k_1+1}{s}\exp\left(1-\frac{k_1+1}{s}\right), & \text{if } 0\leqslant k_1<s,\\ \frac{k_1}{s}\exp\left(1-\frac{k_1}{s}\right), & \text{if } s\leqslant k_1\leqslant 5s,\\ 0, & \text{otherwise.}\end{cases}\tag{5}$$

Also, by the definition of $\zeta_{s,1}$, one has

$$\xi_1-\frac{1}{s}<\zeta_{s,1}<\xi_1,\quad s=1,2,\ldots.\tag{6}$$

Using the similar method, we can derive the possibility distribution of fuzzy variable $\zeta_{s,2}$ as

$$\nu_{s,2}\left(\frac{k_2}{s}\right)=\begin{cases}\frac{k_2+1}{2s}\exp\left(1-\frac{k_2+1}{2s}\right), & \text{if } 0\leqslant k_2<s,\\ \frac{k_2}{2s}\exp\left(1-\frac{k_2}{2s}\right), & \text{if } s\leqslant k_2\leqslant 10s,\\ 0, & \text{otherwise,}\end{cases}\tag{7}$$

and the following link between $\zeta_{s,2}$ and ξ_2,

$$\xi_2-\frac{1}{s}<\zeta_{s,2}<\xi_2,\quad s=1,2,\ldots.\tag{8}$$

By (5) and (7), the possibility distribution μ_s of fuzzy vector $\boldsymbol{\zeta}_s=(\zeta_{s,1},\zeta_{s,2})$ is as follows

$$\mu_s\left(\frac{k_1}{s},\frac{k_2}{s}\right)=\begin{cases}\min\left\{\frac{k_1+1}{s}\exp\left(1-\frac{k_1+1}{s}\right),\frac{k_2+1}{2s}\exp\left(1-\frac{k_2+1}{2s}\right)\right\}, & \text{if } 0\leqslant k_1<s,\ 0\leqslant k_2<s,\\ \min\left\{\frac{k_1+1}{s}\exp\left(1-\frac{k_1+1}{s}\right),\frac{k_2}{2s}\exp\left(1-\frac{k_2}{2s}\right)\right\}, & \text{if } 0\leqslant k_1<s,\ s\leqslant k_2\leqslant 10s,\\ \min\left\{\frac{k_1}{s}\exp\left(1-\frac{k_1}{s}\right),\frac{k_2+1}{2s}\exp\left(1-\frac{k_2+1}{2s}\right)\right\}, & \text{if } s\leqslant k_1\leqslant 5s,\ 0\leqslant k_2<s,\\ \min\left\{\frac{k_1}{s}\exp\left(1-\frac{k_1}{s}\right),\frac{k_2}{2s}\exp\left(1-\frac{k_2}{2s}\right)\right\}, & \text{if } s\leqslant k_1\leqslant 5s,\ s\leqslant k_2\leqslant 10s,\\ 0, & \text{otherwise.}\end{cases}$$

In addition, it follows from (6) and (8) that

$$\|\boldsymbol{\zeta}_s-\boldsymbol{\xi}\|=\sqrt{(\zeta_{s,1}-\xi_1)^2+(\zeta_{s,2}-\xi_2)^2}<\frac{\sqrt{2}}{s},$$

which implies that the sequence $\{\boldsymbol{\zeta}_s\}$ of discrete fuzzy vectors converges uniformly to the continuous fuzzy vector $\boldsymbol{\xi}$.

We now compute $\mathrm{Cr}\{\gamma\in\Gamma|C(\mathbf{x},\boldsymbol{\xi}(\gamma))\leqslant f_0\}$ according to the method proposed above. Let $\{\boldsymbol{\zeta}_s\}$ be the discretization of the fuzzy vector $\boldsymbol{\xi}$. For each fixed s, the vector $\boldsymbol{\zeta}_s$ takes on K values $\hat{\boldsymbol{\zeta}}_s^k=\left(\hat{\zeta}_{s,1}^k,\hat{\zeta}_{s,2}^k,\ldots,\hat{\zeta}_{s,2T}^k\right),\ k=1,2,\ldots,K$, with $K=K_1K_2\ldots K_{2T}$, where K_i is the number of discrete points of ξ_i, $i=1,2,\ldots,2T$. We now replace the possibility distribution of $\boldsymbol{\xi}$ by that of $\{\boldsymbol{\zeta}_s\}$, and approximate $\mathrm{Cr}\{\gamma\in\Gamma|C(x,\boldsymbol{\xi}(\gamma))\leqslant f_0\}$ by $\mathrm{Cr}\{\gamma\in\Gamma|C(x,\boldsymbol{\zeta}_s(\gamma))\leqslant f_0\}$. Then the credibility $D_s(x)=\mathrm{Cr}\{\gamma\in\Gamma|C(x,\boldsymbol{\zeta}_s(\gamma))\leqslant f_0\}$ can be computed by the formula

$$D=\frac{1}{2}\left(1+\max\left\{\nu_k|C\left(x,\hat{\boldsymbol{\zeta}}_s^k(\gamma)\right)\leqslant f_0\right\}-\max\left\{\nu_k|C\left(x,\hat{\boldsymbol{\zeta}}_s^k(\gamma)\right)>f_0\right\}\right).\tag{9}$$

The process to compute the credibility function is summarized as

3.3. Approximation method to credibility objective

Step 1. Generate K points $\hat{\zeta}_s^k = \left(\hat{\zeta}_{s,1}^k, \hat{\zeta}_{s,2}^k, \ldots, \hat{\zeta}_{s,NT+2T}^k\right)$ uniformly from the support Ξ of ξ for $k = 1, 2, \ldots, K$;
Step 2. Calculate $C\left(x, \hat{\zeta}_s^k(\gamma)\right)$ according to formula (3);
Step 3. Calculate $v_{s,i}\left(\hat{\zeta}_{s,i}^k\right)$ via formulas (5) and (7) for $i = 1, 2, \ldots, NT + 2T$; $k = 1, \ldots, K$;
Step 4. Set $v_k = v_{s,1}\left(\hat{\zeta}_{s,1}^k\right) \wedge v_{s,2}\left(\hat{\zeta}_{s,2}^k\right) \wedge \cdots \wedge v_{s,NT+2T}\left(\hat{\zeta}_{s,NT+2T}^k\right)$ for $k = 1, 2, \ldots, K$;
Step 5. Return the credibility D via the estimation formula (9).

In what follows, we refer to the sequence $\{\zeta_s\}$ of discrete fuzzy vector as the discretization of the fuzzy vector ξ.

The convergence of the approximation method is ensured by the following theorem. As a consequence, the original credibility function $\mathrm{Cr}\{\gamma \in \Gamma | C(x, \xi(\gamma)) \leqslant f_0\}$ can be estimated by the approximating credibility function $\mathrm{Cr}\{\gamma \in \Gamma | C(x, \zeta_s(\gamma)) \leqslant f_0\}$ provided that s is sufficiently large.

Theorem 3. *Consider fuzzy production planning problem (2). If ξ is a continuous and bounded fuzzy vector and the sequence $\{\zeta_s\}$ of fuzzy vectors is the discretization of ξ, then for each feasible decision x, we have*

$$\lim_{s\to\infty} \mathrm{Cr}\{\gamma \in \Gamma | C(x, \zeta_s(\gamma)) \leqslant f_0\} = \mathrm{Cr}\{\gamma \in \Gamma | C(x, \xi(\gamma)) \leqslant f_0\},$$

provided that the objective function $Cr\{C(x, \xi) \leqslant f\}$ is continuous at $f = f_0$.

Proof. For any feasible solution x, $C(x, \xi)$ is continuous on the support Ξ of ξ, which together with the supposition on the credibility function $\mathrm{Cr}\{C(x, \xi) \leqslant f\}$, satisfy the conditions of [28, Theorem 1]. As a consequence, the theorem is valid. □

We now provide an example to help our understanding the result of Theorem 3.

Example 2. Suppose $T = 2$, $h_1 = 1$, $h_2 = 0$, $c_{1,1} = c_{1,2} = 1$, $x_{1,1} = x_{2,1} = 10$, $I_0 = 0$, $\xi = (\xi_1, \xi_2)$ is the fuzzy vector defined in Example 1, and $\{\zeta_s\}$ is the discretization of ξ. In this case, the original credibility objective becomes $\mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\}$ and the approximative credibility objective is $\mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\}$. We will show that

$$\lim_{s\to\infty} \mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\} = \mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\}. \tag{10}$$

By the possibility distribution of ξ_1, we have

$$\mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\} = \begin{cases} 0, & \text{if } f_0 < 5, \\ (10 - f_0)\exp(f_0 - 9), & \text{if } 5 \leqslant f_0 < 9, \\ 1, & \text{otherwise}. \end{cases}$$

On the other hand, according to the possibility distribution (5) of the fuzzy variable $\zeta_{s,1}$, we have

$$\mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\} = \begin{cases} 0, & \text{if } f_0 < 5, \\ \left(10 - \frac{k_1}{s}\right)\exp\left(\frac{k_1}{s} - 9\right), & \text{if } \frac{k_1}{s} < f_0 \leqslant \frac{k_1+1}{s},\ 5s \leqslant k_1 < 9s, \\ \left(10 - \frac{k_1+1}{s}\right)\exp\left(\frac{k_1+1}{s} - 9\right), & \text{if } \frac{k_1}{s} < f_0 \leqslant \frac{k_1+1}{s},\ 9s \leqslant k_1 \leqslant 10s, \\ 1, & \text{otherwise}. \end{cases}$$

We now show that (10) holds for every $f_0 \in (-\infty, 5) \cup (5, +\infty)$.

If $f_0 < 5$ or $f_0 \geqslant 10$, then by the computational results above, we have

$$\mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\} = \mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\},$$

which implies that (10) holds.

If $f_0 \in (5, 9)$, then there exists an integer $k_1, 5s \leqslant k_1 < 9s$ such that $k_1/s < f_0 \leqslant (k_1 + 1)/s$, which is equivalent to

$$\left(10 - \frac{k_1+1}{s}\right)\exp\left(\frac{k_1+1}{s} - 9\right) \leqslant (10 - f_0)\exp(f_0 - 9) < \left(10 - \frac{k_1}{s}\right)\exp\left(\frac{k_1}{s} - 9\right).$$

Noting that

$$\mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\} = (10 - f_0)\exp(f_0 - 9),$$

and

$$\mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\} = \left(10 - \frac{k_1}{s}\right)\exp\left(\frac{k_1}{s} - 9\right),$$

then one has

$$\mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\} - m \leqslant \mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\} < \mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\},$$

where $m = \left(10 - \frac{k_1}{s}\right) \exp\left(\frac{k_1}{s} - 9\right) - \left(10 - \frac{k_1+1}{s}\right) \exp\left(\frac{k_1+1}{s} - 9\right)$.

Therefore, $0 \leqslant \mathrm{Cr}\{\gamma \in \Gamma | 10 - \zeta_{s,1}(\gamma) \leqslant f_0\} - \mathrm{Cr}\{\gamma \in \Gamma | 10 - \xi_1(\gamma) \leqslant f_0\} < m$, which implies that (10) holds.

In the case when $f_0 \in [9, 10]$, it is similar to show (10) holds too.

3.4. Approximation-based PSO algorithm

PSO algorithm, originally developed by Kennedy and Eberhart [26], is a method for optimization on metaphor of social behavior of flocks of birds and/or schools of fish. Compared to other evolutionary algorithms, PSO has a faster convergence rate and much less parameters to adjust, which makes it particularly easy to implement. Recently the PSO algorithm has attracted much attention and been successfully applied in the fields of evolutionary computing, unconstrained continuous optimization problems and many others [27]. As for constrained optimization problems, Dong et al. [30] proposed a PSO algorithm embedded with constraint fitness priority-based ranking method.

PSO is based on a collection of pop_size n-dimensional particles, each of which indicates a possible solution of the problem space. Each particle has its own best position (pbest) representing the personal biggest objective value so far at time t. The global best particle (gbest) represents the best particle found so far at time t in the colony.

As a consequence, the new velocity of the ith particle is updated by the following formula

$$V_i(t+1) = \omega V_i(t) + c_1 r_1 (P_i(t) - X_i(t)) + c_2 r_2 (P_g(t) - X_i(t)), \tag{11}$$

while the new position of the ith particle is renewed by

$$X_i(t+1) = X_i(t) + V_i(t+1), \tag{12}$$

where $i = 1, 2, \ldots, pop\_size; \omega$ is called the inertia coefficient; c_1 and c_2 are learning rates and usually $c_1 = c_2 = 2,\ r_1$ and r_2 are two independent random numbers generated randomly in the unit interval [0,1].

Based on the discussion above, the process of the approximation-based PSO algorithm is summarized as follows.

Algorithm 1. PSO algorithm).

Step 1. Initialize pop_size particles with random positions and velocities, and compute their corresponding objective values by the approximation method.
Step 2. Set pbest of each particle and its objective value equal to its current position and objective value, and set gbest and its objective value equal to the position and objective value of the best initial particle;
Step 3. Renew the velocity and position of each particle according to formulas (11) and (12), respectively.
Step 4. Calculate the objective values for all particles by the approximation method.
Step 5. For each particle, compare the current objective value with that of its pbest. If the current objective value is bigger than that of pbest, then renew pbest and its objective value with the current position and objective value.
Step 6. Find the best particle of the current swarm with the biggest objective value. If the objective value is bigger than that of gbest, then renew gbest and its objective value with the position and objective value of the current best particle.
Step 7. Repeat the third to six steps for a given number of cycles.
Step 8. Return the gbest and its objective value as the (near) optimal solution and the (approximate) optimal value.

3.5. NN-based PSO algorithm

So far we have discussed the computation for the credibility function $\mathrm{Cr}\{\gamma \in \Gamma | C(\mathbf{x}, \xi(\gamma)) \leqslant f_0\}$ by the approximation method. During the solution of Algorithm 1, the objective values of all particles are computed by the approximation method. Therefore, Algorithm 1 is a time-consuming process provide that the number of sample points is large enough. To speed up the solution process, we desire to replace the credibility function by an NN since a trained NN has the ability to approximate functions as well as high speed of operations. In this paper, we employ the fast BP algorithm to train a feedforward NN to approximate the credibility function. Usually, an NN with two hidden layers is better in generation than the NN with one hidden layer. But in most applications, an NN with one hidden layer is enough to be a universal approximator for any integrable functions. Thus, in this paper, we only consider the NN with input layer, one hidden layer and output layer connected in a feedforward way, in which there are n_1 input neurons in input layer representing the input values of decision variables, p neurons in hidden layer and 1 neuron in output layer representing the output value of the objective function. Let $\{(x_i, y_i)|\ i = 1, 2 \ldots, n\}$ be a set of input–output data generated by the approximation method. The training process is to find the best weight vector $\boldsymbol{w}_i$ that minimizes the following error function

$$Err(w_i) = \frac{1}{2} \sum_{i=1}^{n} |F(x_i, w_i) - y_i|.$$

In this section, we incorporate the approximation method [28], an NN and the PSO algorithm to produce a hybrid PSO algorithm for solving fuzzy production planning problem (2). In this hybrid algorithm, the approximation method is used to generate a set of input–output data for the credibility objective of model (2). Using the training data, an NN is trained to approximate the credibility objective. After the NN is well-trained, it is embedded into a PSO algorithm to search for the (near) optimal solution to model (2). This solution process includes the following several steps.

Algorithm 2. A hybrid PSO algorithm

Step 1. Generate a set of input–output data for the credibility objective

$$D : x \to \mathrm{Cr}\{\gamma \in \Gamma | C(x, \xi(\gamma)) \leqslant f_0\},$$

by the approximation method;

Step 2. Train an NN to approximate the credibility objective $\mathrm{Cr}\{\gamma \in \Gamma | C(x, \xi(\gamma)) \leqslant f_0\}$ by the generated data;

Step 3. Initialize pop_size particles with random positions and velocities, and compute the objective values for all particles by the trained NN;

Step 4. Set pbest of each particle and its objective value equal to its current position and objective value, and set gbest and its objective value equal to the position and objective value of the best initial particle;

Step 5. Renew the velocity and position of each particle according to formulas (10) and (11), respectively;

Step 6. Calculate the objective values for all particles by the trained NN;

Step 7. For each particle, compare the current objective value with that of its pbest. If the current objective value is bigger than that of pbest, then renew pbest and its objective value with the current position and objective value;

Step 8. Find the best particle of the current swarm with the biggest objective value. If the objective value is bigger than that of gbest, then renew gbest and its objective value with the position and objective value of the current best particle;

Step 9. Repeat the fifth to eighth steps for a given number of cycles;

Step 10. Return the gbest and its objective value as the (near) optimal solution and the (approximate) optimal value.

Remark 1. Algorithm 2 is different from Algorithm 1. During the solution of Algorithm 2, the objective values of all particles are computed by the trained NN. In the next section, we provide one 6-product source, 6-period production planning problem to compare the effectiveness of Algorithms 1 and 2 via numerical experiments.

4. Numerical experiments

In this section, we provide a production planning problem, and solve it by Algorithms 1 and 2. The numerical experiments are implemented on a personal computer (Lenovo 14002 with Intel T2370 1.73 GHz processor and 1024 memory). The problem statement and the solution processes are reported in the following subsections, respectively.

4.1. Problem statement

Consider the following 6-product source, 6-period production planning problem with initial inventory $I_0 = 0$.

$$\begin{array}{ll} \max & \mathrm{Cr}\left\{\sum_{t=1}^{6}\left(h_t I_t^+ + \sum_{i=1}^{6} c_{i,t}x_{i,t}\right) \leqslant f_0\right\} \\ \text{subject to}: & \mathrm{Cr}\{I_t \geqslant 0\} \geqslant \alpha_t, \quad t = 1,\ldots,6, \\ & x_{i,t} \geqslant 0, \quad t = 1,\ldots,6, \\ & I_t^+ = \max\{0, I_t\}, \quad t = 1,\ldots,6, \end{array} \tag{13}$$

where $I_t = I_{t-1} + \sum_{i=1}^{6} x_{i,t} - d_t,\ t = 1,\ldots,6$, the required data set for this manufacturing system is collected in Table 1. The possibility distributions of fuzzy costs in this production planning problem are also provided in Table 1. The demands $d_t,\ t = 1,\ldots,6$ are gamma fuzzy variables with the following possibility distributions

$$\begin{aligned} &\mu_{d_1} = (x/8)\exp(1 - x/8),\ x \in [0,40], \quad \mu_{d_2} = (x/10)\exp(1 - x/10),\ x \in [0,50], \\ &\mu_{d_3} = (x/15)\exp(1 - x/15),\ x \in [0,75], \quad \mu_{d_4} = (x/7)\exp(1 - x/7),\ x \in [0,35], \\ &\mu_{d_5} = (x/12)\exp(1 - x/12),\ x \in [0,60], \quad \mu_{d_6} = (x/8)\exp(1 - x/8),\ x \in [0,40]. \end{aligned}$$

In addition, the fuzzy variables involved in this problem are mutually independent.

According to Theorem 2, model (13) can be turned into the following equivalent mathematical programming problem with credibility objective

$$\begin{aligned}
\max \quad & \mathrm{Cr}\left\{\sum_{t=1}^{6}\left(h_t I_t^+ + \sum_{i=1}^{6} c_{i,t}x_{i,t}\right) \leqslant 11850\right\}\\
\text{subject to}: \quad & \ln 0.16 + \sum_{i=1}^{6} x_{i,1}/2 - 1 - \ln\left(\sum_{i=1}^{6} x_{i,1}/2\right) - \ln 2 \geqslant 0,\\
& \ln 0.14 + \sum_{i=1}^{6}\sum_{j=1}^{2} x_{i,j}/5 - 1 - \ln\left(\sum_{i=1}^{6}\sum_{j=1}^{2} x_{i,j}/5\right) - \ln 5 \geqslant 0,\\
& \ln 0.18 + \sum_{i=1}^{6}\sum_{j=1}^{3} x_{i,j}/6 - 1 - \ln\left(\sum_{i=1}^{6}\sum_{j=1}^{3} x_{i,j}/6\right) - ln6 \geqslant 0,\\
& \ln 0.12 + \sum_{i=1}^{6}\sum_{j=1}^{4} x_{i,j}/10 - 1 - \ln\left(\sum_{i=1}^{6}\sum_{j=1}^{4} x_{i,j}/10\right) - \ln 10 \geqslant 0,\\
& \ln 0.20 + \sum_{i=1}^{6}\sum_{j=1}^{5} x_{i,j}/15 - 1 - \ln\left(\sum_{i=1}^{6}\sum_{j=1}^{5} x_{i,j}/15\right) - \ln 15 \geqslant 0,\\
& d\ln 0.16 + \sum_{i=1}^{6}\sum_{j=1}^{6} x_{i,j}/21 - 1 - \ln\left(\sum_{i=1}^{6}\sum_{j=1}^{6} x_{i,j}/21\right) - \ln 21 \geqslant 0,\\
& x_{i,t} \geqslant 0, \quad i = 1,\ldots,6,\ t = 1,\ldots,6,\\
& I_t^+ = \max\{0, I_t\}, \quad t = 1,\ldots,6,
\end{aligned} \tag{14}$$

where $I_t = I_{t-1} + \sum_{i=1}^{6} x_{i,t} - d_t,\ t = 1,\ldots,6.$

We now solve the equivalent production planning problem (14) by Algorithms 1 and 2, respectively, and compare their effectiveness via numerical experiments.

4.2. Algorithm 1

We first employ Algorithm 1 to solve production planning problem (14). During the solution process, for each particle x, we generate 3000 sample points via the approximation method to compute the credibility objective value of problem (14)

$$D : x \mapsto \mathrm{Cr}\left\{\gamma \in \Gamma \mid \sum_{t=1}^{6}\left(h_t I_t^+ + \sum_{i=1}^{6} c_{i,t}x_{i,t}\right) \leqslant 11850\right\}. \tag{15}$$

To identify the parameters' influence on the solution quality, a numerical study is implemented to compare the solutions obtained by running the approximation-based PSO algorithm with careful variation of parameters. The computational results are reported in Table 2, where parameter 'Objective 1' is the optimal credibility objective value obtained by Algorithm 1; parameter 'Time 1' in the fifth column is the computational time consumed by Algorithm 1 to get the (near) optimal solutions listed in the third column, and parameter 'relative error' in the last column is defined as

$$\frac{\text{optimal value-objective 1}}{\text{optimal value}} \times 100\%,$$

Table 1
The data set for production planning problem (13).

Product source	Periods					
	1	2	3	4	5	6
Production profit matrix (c_{it})						
1	(4,7,10)	(5,9,11)	(5,7,9)	(6,8,10)	(3,7,10)	(4,7,11)
2	(7,9,11)	(5,7,9)	(8,10,12)	(4,7,10)	(3,6,9)	(5,7,10)
3	(3,7,11)	(4,6,8)	(5,7,9)	(2,4,6)	(8,10,12)	(7,8,12)
4	(8,10,12)	(7,9,11)	(5,8,9)	(6,9,12)	(3,5,8)	(5,7,12)
5	(5,8,13)	(5,7,10)	(3,6,9)	(3,5,8)	(5,10,12)	(5,7,12)
6	(2,6,9)	(5,8,10)	(4,8,12)	(3,5,10)	(2,7,12)	(5,7,12)
Inventory cost matrix (h_t)						
	(2,3,4)	(1,2,3)	(0.5,1.5,2.5)	(1,5,6)	(2,4,5)	(3,5,6)
Service level constraints (α_t)						
	0.90	0.92	0.91	0.93	0.90	0.92

Table 2
Comparison solution of Algorithm 1.

Pop_size	Gen	Optimal solution	Objective 1	Time 1 (min)	Error %
40	400	(28.0000, 12.2956, 5.0000, 28.0000, 5.0000, 5.4055 8.1900, 5.0128, 27.9999, 8.2944, 21.3578, 5.0000 15.9473, 5.0000, 25.6392, 6.2072, 28.0000, 10.6613 11.2182, 5.0000, 5.0000, 5.0923, 27.9889, 20.3500 6.1639, 28.0000, 5.0000, 14.8491, 28.0000, 20.6332 27.4760, 5.0000, 17.5636, 5.0785, 5.5893, 28.0000)	0.891667	53.90	0.74
30	300	(27.4760, 5.0000, 17.5636, 5.0785, 5.5893, 28.0000 26.3001, 26.8129, 5.0000, 5.0000, 16.7520, 13.6086 16.4088, 8.9973, 19.0387, 26.3904, 8.3526, 13.5564 9.8378, 17.1292, 18.0985, 28.0000, 20.0000, 26.7808 28.0000, 25.6061, 5.0000, 5.0093, 5.2438, 5.0000 28.0000, 21.8517, 21.4822, 5.5142, 22.9570, 12.6315)	0.896667	43.70	0.19
30	400	(19.1955, 23.4216, 5.0000, 13.8206, 6.4482, 10.5238 6.2691, 25.8273, 25.1071, 5.5463, 5.0000, 5.0000 9.0758, 6.4306, 26.4941, 7.7461, 26.0943, 23.1459 20.3433, 5.0000, 27.5043, 5.0000, 6.2673, 6.1816 16.3915, 28.0000, 27.6562, 16.6037, 5.3805, 8.0179 25.1267, 19.3427, 8.8141, 13.1312, 5.0000, 9.9090)	0.898333	50.40	0.00
40	500	(25.1898, 15.7797, 9.8448, 5.0096, 28.0000, 15.9414) 7.0730, 10.5435, 5.7415, 6.3855, 27.2635, 28.0000 13.5710, 27.4698, 21.6474, 5.0000, 28.0000, 8.4685 15.8106, 16.5618, 5.0000, 23.9197, 5.4824, 8.9140 25.8936, 26.1702, 28.0000, 5.0256, 22.1088, 5.0000 27.9319, 18.6195, 11.4926, 24.2975, 26.4092, 16.9767)	0.886667	56.90	1.30
50	300	(5.0000, 20.5743, 5.0000, 5.0207, 11.7402, 9.4520 23.0157, 21.5344, 5.0000, 20.3418, 24.4115, 16.2782 13.3157, 12.8578, 5.0000, 18.1090, 5.0000, 13.0000 20.6983, 6.0371, 5.4369, 8.5395, 28.0000, 28.0000 6.6237, 6.1256, 8.8302, 12.2463, 5.0000, 28.0000 24.3121, 9.5440, 20.5732, 20.0000, 28.0000, 3.7855)	0.898333	62.30	0.00
50	400	(5.0000, 10.6130, 12.3530, 22.0143, 14.3064, 5.0000 10.1802, 7.8489, 5.0000, 22.8113, 7.1726, 12.028675 5.0680, 28.0000, 12.1228, 25.0011, 22.760850, 10.5168 5.2060, 20.0343, 16.4241, 10.9631, 5.5063, 26.0008 20.3560, 10.0318, 5.0000, 17.5801, 20.4830, 16.6156 10.4385, 17.1843, 13.0000, 9.5631, 14.5878, 19.5479)	0.895000	65.00	0.37

with the optimal value being the biggest one of the six 'Objective 1' values in the fourth column. It can be seen from Table 2 that the relative errors do not exceed 1.30% when various parameters of PSO algorithm are selected, which implies the approximation-based PSO algorithm is robust to parameters settings. However, during the solution process of Algorithm 1, we are required to employ the approximation method to compute the objective values of all particles, which results in the solution process is time-consuming. This can be seen from the values of the parameter 'Time 1' in the fifth column.

4.3. *Algorithm 2*

In this subsection, we apply Algorithm 2 to the solution of production planning problem (14). Firstly, according to (15), we employ approximation method to produce a set $\{(x_i, D(x_i))|\ i = 1, \ldots, 3000\}$ of input–output data for credibility objective $D(x)$. That is, for each $x_i, D(x_i)$ is computed by the approximation method described in the above (with 3000 sample points in this example). Secondly, we train an NN by the generated input–output data to approximate the credibility objective. For problem (14), the NN contains 36 input neurons representing the value of decision $x = \{x_{i,t}|\ i = 1, \ldots, 6;\ t = 1, \ldots, 6\}$, 15 hidden neurons and 1 output neurons representing the output objective value of the trained NN. After the NN is well-trained, it is embedded into a PSO algorithm to search for the (near) optimal solutions to problem (14). During the solution process of Algorithm 2, the credibility objective $D(x)$ for all particles are computed by the trained NN instead of using the approximation method, thus much time can be saved. In order to further test the effectiveness of Algorithm 2, we repeat more experiments with different parameters in the PSO algorithm. The computational results of Algorithm 2 are provided in Table 3, in which parameter 'Objective 2' is the optimal credibility objective value obtained by Algorithm 2; parameter 'Time 2' in the fifth column is the computational time consumed by Algorithm 2 to get the (near) optimal solutions listed in the third column, and parameter 'Error' is defined as $(optimal\ value\text{-}objective2)/optimal\ value \times 100\%$ with the optimal value being the biggest one of the six 'Objective 2' values in the fourth column. It can be seen from Table 3 that the relative errors do not exceed 2.19% when various parameters of the PSO algorithm are selected, which demonstrate that Algorithm 2 is also robust to parameters settings.

Before concluding this section, we compare the computational results about Algorithms 1 and 2. In addition to the parameters 'Objective 1' and 'Objective 2' defined in the above, we also adopt the following two parameters, 'deviation' and 'time

Table 3
Comparison solution of Algorithm 2.

Pop_size	Gen	Optimal solution	Objective 2	Time 2 (min)	Error %
40	400	(5.0000, 28.0000, 28.0000, 5.0000, 28.0000, 5.0000 5.0000, 26.7157, 28.0000, 28.0000, 28.0000, 21.5742 28.0000, 28.0000, 28.0000, 28.0000, 26.1551, 28.0000 5.0000, 28.0000, 27.8774, 24.8343, 11.0824, 5.0000 5.1933, 10.6609, 28.0000, 27.7310, 28.0000, 13.3109 5.0000, 7.3097, 28.0000, 5.0000, 7.8143, 10.8022)	0.901262	28.50	0.00
30	300	(5.0000, 28.0000, 28.0000, 5.0000, 28.0000, 27.31021 5.0000, 28.0000, 28.0000, 28.0000, 5.0000, 28.0000 28.0000, 5.0000, 28.0000, 28.0000, 28.0000, 28.0000 5.0000, 28.0000, 28.0000, 28.0000, 6.5471, 5.3012 28.0000, 5.0000, 28.0000, 20.3850, 28.0000, 5.0000 27.5440, 5.0000, 10.7127, 28.0000, 28.0000, 30.4400)	0.900847	20.30	0.05
30	400	(5.0000, 28.0000, 28.0000, 5.0000, 28.0000, 27.2677 5.0000, 28.0000, 28.0000, 28.0000, 5.0000, 28.0000 28.0000, 5.0000, 28.0000, 28.0000, 28.0000, 28.0000 5.0000, 28.0000, 27.9815, 28.0000, 6.5789, 5.2751 28.0000, 5.0000, 28.0000, 20.2840, 28.0000, 5.0000 27.7744, 5.0000, 10.6031, 28.0000, 28.0000, 21.8992)	0.900926	25.10	0.04
40	500	(9.9358, 28.0000, 28.0000, 5.0000, 17.2570, 28.0000) 5.0367, 28.0000, 28.0000, 28.0000, 27.9365, 17.0780 28.0000, 28.0000, 27.9899, 28.0000, 27.9811, 28.0000 5.6347, 28.0000, 28.0000, 27.6065, 28.0000, 5.0000 5.0000, 16.3444, 28.0000, 28.0000, 28.0000, 12.8598 5.0000, 5.0950, 12.2723, 26.7716, 28.0000, 34.5674)	0.901260	32.40	0.00
50	300	(5.0000, 28.0000, 7.0854, 5.0000, 28.0000, 5.0000 5.0000, 26.4578, 28.0000, 28.0000, 25.7458, 21.5742 28.0000, 28.0000, 28.0000, 28.0000, 26.1551, 28.0000 5.0000, 28.0000, 27.8202, 22.8542, 15.0774, 5.0000 28.0000, 10.6456, 28.0000, 25.7478, 28.0000, 28.0000 5.0000, 10.3498, 28.0000, 5.0000, 7.1450, 28.0000)	0.881536	38.60	2.19
50	400	(5.0000, 28.0000, 8.0775, 5.0000, 28.0000, 5.0000 5.0000, 26.7784, 28.0000, 28.0000, 24.5424, 21.3242 28.0000, 28.0000, 28.0000, 28.0000, 26.4585, 28.0000 5.0000, 28.0000, 28.0000, 22.2548, 15.0784, 5.0000 28.0000, 10.5426, 28.0000, 24.7478, 28.0000, 28.0000 5.0000, 12.3424, 28.0000, 5.0000, 7.1743, 28.0000)	0.884252	40.20	1.89

Table 4
Comparison of Algorithms 1 and 2.

Pop_size	Gen	Objective 1 (Algorithm 1)	Objective 2 (Algorithm 2)	Deviation %	Time difference (min)
40	400	0.891667	0.901262	1.06	25.40
30	300	0.896667	0.900847	0.46	23.40
30	400	0.898333	0.900926	0.29	25.30
40	500	0.886667	0.901260	1.62	24.50
50	300	0.898333	0.881536	1.91	23.70
50	400	0.895000	0.884252	1.22	24.80

difference'. The 'deviation' is defined as $|objective\ 2 - objective\ 1|/objective\ 2 \times 100\%$, while the 'time difference' is defined as *Time 2-Time 1*. The comparison of two algorithms is summarized in Table 4, from which we can see the relative objective 'deviation' does not exceed 1.91%. Thus the optimal objective values obtained by Algorithms 1 and 2 have no obvious difference. However the 'time difference' parameter demonstrates that Algorithm 2 can save much time compared with Algorithm 1, which depend on problem size, the number of sample points, pop_size and generation. Therefore, from the computational viewpoint, we can conclude that Algorithm 2 is more effective than Algorithm 1 when we employ them to solve fuzzy production planning problem (14).

5. Conclusions

When optimal production decisions must be reached in an environment beset with possibilistic uncertainty, not only the formulation of the decision model requires a deeper probing of the aspirations criteria in order to give to the optimization problem its appropriate form, but often significant computational obstacles must be overcome to find optimal production decisions. In these two aspects, the major new contributions of the current development are as follows.

(i) Based on credibility criteria in the model formulation, we developed a new class of fuzzy production planning problems, in which the production cost, inventory cost and product demands were characterized by fuzzy variables. In addition, a manufacturer has a number of plants and subcontractors and has to meet the product demands according to various credibility service levels prescribed by its customers.

(ii) When product demands are mutually independent gamma fuzzy variables, we have turned the credibility level constraints into their equivalent deterministic forms (see Theorems 1 and 2). However, in this situation, the analytical expression about the credibility objective of problem (2) is still unavailable. To overcome this obstacle, the approximation method is adopted to compute the credibility objective. The reasonableness about the method has also been discussed (see Theorem 3).

(iii) Since production planning problem (14) is neither linear nor convex, conventional optimization algorithms cannot be employed to solve it. In this paper, two heuristic solution methods were designed to solve problem (14). Algorithm 1 is a combination of the approximation method and the PSO algorithm, while Algorithm 2 integrates the the approximation method, an NN and the PSO algorithm. The numerical experiments about problem (14) have been made to compare the effectiveness of two algorithms. The computational results demonstrated that the optimal objective values obtained by Algorithms 1 and 2 have no obvious deviations, but Algorithm 2 can save much time compared with Algorithm 1. From the computational viewpoint, we concluded that Algorithm 2 is more effective than Algorithm 1 when we employed them to solve production planning problem (14).

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