A linear formulation for the Synchronous Management of Energy Production and Consumption

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1 Introduction

The remainder of this paper is organized as follows. Section 2 formally introduces the *SMEPC*. Section 3 describes a MILP formulation of the problem. Section 4 presents theoretical properties of the linear relaxation as well as some new valid inequalities. Separation procedures are given in Section 5 and computational results derived from a Branch-And-Cut algorithm are proposed in Section 6. Finally, Section 7 concludes and outlines research perspectives.

2 Problem description, definitions and notations

2.1 *The model*

We define the *SMEPC* as follows. A vehicle powered by H^2 hydrogen fuel, has to visit a set of stations that need to be served, following a route which begins and ends at a special station: the *Depot*. Close to it, there is a Micro-Plant (MP) which is a charging station. The vehicle performs a tour starting from the *Depot* (labelled 0), passing through the sequence of stations $j = 1, \dots, M$ and returning to the *Depot* (labelled $M+1$). It leaves the depot at the date 0 and must return before the date T_{Max}. Travelling from one station j to the next one $j + 1$ incurs a driving time t_j that corresponds to the shortest path in time from j to $j + 1$ and an energy consumption e_j associated to this path. As the capacity of its tank is limited to C^{Veh}, the vehicle may need to periodically refuelling between two stations. The initial and final quantity of hydrogen in the vehicle's tank is denoted by E_0. Going from station j to the Micro-Plant takes a time (resp. an energy consumption) d_j (resp. ϵ_j), whereas the time (resp. the energy consumption) to return from MP to j is denoted by d^*_{j+1} (resp. ϵ^*_{j+1}), see Figure 1. We consider that the time values and energy ones are non null and satisfy the triangle inequality.

The micro-plant produces H^2 from water through a combination of photolysis and electrolysis. Resulting H^2 is stored inside a tank located directly beside the micro-plant, whose capacity (in energy units) is denoted by C^{MP}. We suppose that:

- The time space $\{0, 1, 2, \dots, T_{\text{Max}}\}$ is divided into periods $P_i = [p.i, p.(i + 1)[$, for $i \in \{0, \dots, N - 1\}$, all with a same length equal to p such that $T_{\text{Max}} = N.p$. For the sake of simplicity, we identify index i and period P_i.

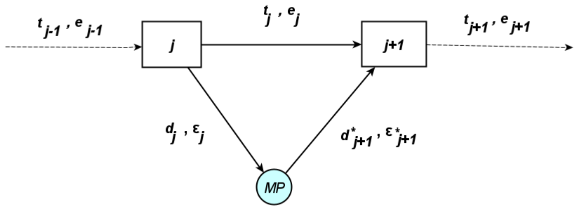

Figure 1: Time and Energy symbols between stations j, $j+1$ and the micro-plant

- If the micro-plant is active at some time during period i, then it is active during the whole period i, and produces R_i hydrogen fuel units, where production rate R_i depends on period i.
- At time 0, the current load of the micro-plant tank is equal to $H_0 \leq C^{\text{MP}}$ and the micro-plant is not active. We impose that the same situation holds at time T_{Max}.
- Because of safety concerns, the vehicle cannot refuel while the micro-plant is producing. Any vehicle refuelling transaction should start at the beginning of some period $i \in \{0, 1 \ldots, N-1\}$, and finish at the end of period i (which means that each refuelling operation lasts p units of time). Since vehicle refuelling and energy production are mutually exclusive, the vehicle may wait at the micro-plant before being allowed to refuel.

Producing H^2 fuel has a cost, which may be decomposed into *a fixed activation cost* denoted by $Cost^F$, charged every time the micro-plant is activated and *a time-dependent production cost* $Cost_i^V$ which corresponds to the power consumed during period i provided that the micro-plant is active during this period.

A solution of the SMEPC consists in a plant activity schedule that indicates the periods when the H^2 production happens, a vehicle timetable which specifies the arrival dates at each station and a refuelling policy which provides the energy quantities needed by the vehicle and the stations from which it will recharge.

2.2 *The objectif*

The Synchronized Management of Energy Production and Consumption problem ($SMEPC$) consists in scheduling both the vehicle and the micro-plant in such a way that:

- The vehicle starts from $Depot = 0$, visits all stations $j = 1, \ldots, M$ and comes back to $Depot = M + 1$ at some time $T_{M+1} \in [0, T_{\text{Max}}]$, refueling as many times as necessary at the micro-plant ;
- The micro-plant produces and stores in time the H^2 fuel needed by the vehicle;

- Both induced H^2 production cost $Cost$ and time T_{M+1} are the smallest possible. The two previous criteria are merged together into a unique one of the form: $Cost + \alpha.T_{M+1}$, where α is a conversion factor from time into economical cost.

3 A Mixed Integer Linear Programming formulation

We propose here a mathematical formulation by a linear program with mixed integer variables called $MILP_{SMEPC}$. For the sake of clarity, it will be defined on the basis of the following three elements:

- Hydrogen production: The Micro-Plant problem,
- Hydrogen consumption: The vehicle tour problem,
- Synchronization of consumption and production: The synchronization problem.

3.1 Input for $MILP_{SMEPC}$

1. **Vehicle related input**
   - M: number of stations ($Depot$ excluded)
   - $\Gamma = (Depot = 0, 1, \ldots, M, Depot = M + 1)$: vehicle tour (without refueling)
   - T_{Max}: maximal time for the vehicle to achieve its tour
   - C^{Veh}: vehicle tank capacity
   - E_0: initial vehicle hydrogen load
   - t_j: required time to go from station j to station $j + 1$
   - d_j: required time to go from station j to the micro-plant
   - d_j^*: required time to go from the micro-plant to station j
   - e_j: required energy to go from station j to station $j + 1$
   - ϵ_j: required energy to go from station j to the micro-plant
   - ϵ_j^*: required energy to go from the micro-plant to station j
2. **Micro-plant production related input**
   - N: number of production periods
   - p: duration (in time units) of one production period, $N.p = T_{\text{Max}}$
   - C^{MP}: micro-plant tank capacity
   - H_0: initial micro-plant hydrogen load
   - $Cost^F$: activation cost
   - $P_i = [p.i, p.(i + 1)[$: time interval related to production period i
   - R_i: production rate related to period i
   - $Cost_i^V$: production cost related to period i

The data above are supposed to be integer.

3.2 Variables and constraints

We first propose a Mathematical Programming oriented formulation, which allows to clearly identify main variables and constraints.

3.2.1 Variables

Vehicle: When the vehicle travels from a station $j \in \{0, \dots, M\}$ to the next one, we define

- a $0-1$ decision variable x_j, such that $x_j = 1$ when the vehicle refuels between j and $j+1$;
- the refuelling time $T_j^*, T_j^* \in \{0, p, 2p, \dots (N-1)p\}$, when the vehicle starts to refuel, if $x_j = 1$;
- the H^2 quantity $L_j, L_j \geq 0$ and integer , loaded by the vehicle, if $x_j = 1$.

We also consider, when the vehicle arrives at any station $j \in \{0, \dots, M+1\}$,

- its arrival time $T_j, T_j \in \{0, 1, 2, \dots T_{\text{Max}}\}$;
- the H^2 load $V_j^{\text{Veh}}, V_j^{\text{Veh}} \geq 0$ and integer , of its tank.

Production: For a period $i \in \{0, \dots N-1\}$,

- $y_i \in \{0, 1\}$ depicts whether the micro-plant is activated at the beginning of the period;
- $z_i \in \{0, 1\}$ takes the value one if the micro-plant is active along the period;
- $\delta_i \in \{0, 1\}$ indicates if the vehicle is refuelling during i;
- $L_i^*, L_i^* \geq 0$ and integer , is the quantity of H^2 loaded by the vehicle during the period in case $\delta_i = 1$.
- $V_i^{\text{MP}}, V_i^{\text{MP}} \geq 0$ and integer , is the H^2 load of the micro-plant tank at the beginning of the period (a fictitious period N is added).

Synchronization: The meeting point between the vehicle and the plant is the time when the refuelling takes place. Thus we consider

- the binary variable $U_{i,j}$ which is equal to one when the vehicle refuels during a period $i, i = 0, \dots N-1$, while travelling from a station $j, j = 0, \dots M$, to the next one;
- the quantity of fuel $m_{i,j}$ given to the vehicle while its travel from station j to $j+1$, at the period i, if $U_{i,j} = 1$.

3.2.2 Constraints

The previous variables are used in a mixed Integer Linear Program. We explain the way some of those constraints must be understood, specially if they result from a linearization of logical implications by the Big M technique.

Production: Let i be a period in $\{1, \dots, N-1\}$.

- The Micro-Plant is *activated* if the production begins whereas the micro-plant was shut down at the preceding period. The following constraints are obtained.

$$\begin{array}{cccl} y_i & -z_i & & \leq 0 \\ y_i & & +z_{i-1} & \leq 1 \\ -y_i & +z_i & -z_{i-1} & \leq 0. \end{array}$$

- Production and recharge cannot be carried out simultaneously during a period. This implies that

$$z_i \quad +\delta_i \quad \leq 1.$$

We also add the initial conditions $y_0 - z_0 = 0$ and $\delta_0 = 0$.

- The storage capacity induces

$$V_i^{\text{MP}} \leq C^{\text{MP}},$$

with the initial and final assumptions $V_0^{\text{MP}} = H_0$ and $V_N^{\text{MP}} \geq H_0$ (recall that N is a fictitious period).

- The production of the micro-plant generates flow type equation

$$V_{i+1}^{\text{MP}} = V_i^{\text{MP}} + R_i \cdot z_i - L_i^*.$$

- The energy L_i^* given by the plant to the vehicle must be non-negative and smaller than the capacity of the plant tank, that is $L_i^* \geq 0$ and $L_i^* \leq C^{\text{MP}} \cdot \delta_i$.

Vehicle: Assume that j is a station in $\{0 \dots M\}$.

- Taking into account storage capacity of the vehicle, we have

$$V_j^{\text{Veh}} \leq C^{\text{Veh}},$$

with the particular conditions at the Depot $V_0^{\text{Veh}} = E_0 \geq \epsilon_0$ and $V_{M+1}^{\text{Veh}} \geq E_0$ (the indices 0 and $M+1$ are assigned to the depot).

- The following set of constraints means that at any time, the vehicle must be able to go to the micro-plant and refuel, and relies on the Triangle Inequality for energy coefficients e_j and ϵ_j

$$V_j^{\text{Veh}} \geq \epsilon_j.$$

- The load of the vehicle verifies a flow type equation

$$V_{j+1}^{\text{Veh}} = V_j^{\text{Veh}} - e_j + (e_j - \epsilon_j - \epsilon_{j+1}^*) \cdot x_j + L_j.$$

- The quantity L_j loaded by the vehicle during its travel from j to $j+1$, if $x_j = 1$, is subject to:
  - $L_j \geq 0$,
  - $L_j \leq C^{\text{Veh}} \cdot x_j$,

– $L_j \leq C^{\text{Veh}} + \epsilon_j - V_j^{\text{Veh}}$. This last inequality expresses the residual capacity of the vehicle when it arrives at the plant.

- At any station, the time needed to reach the next one depends on the choice of the trajectory, a direct route or with a deviation through the plant. Thus, we have first

$$T_j^* \geq T_j + d_j.$$

Moreover, the arrival date T_{j+1} at station $j+1$ and the time T_j^* when a refuelling is eventually done are linked by the non linear inequality below.

$$T_{j+1} \geq (1 - x_j) \cdot (T_j + t_j) + x_j \cdot (T_j^* + p + d_{j+1}^*)$$

Either the vehicle takes the straight road to the next stage or the vehicle deviates through the plant. In that case, the next arrival time takes into account the waiting time at the tank before refuelling. This can be linearized according to the Big M technique:

$$T_{j+1} - T_j - t_j \geq -2T_{\text{Max}} \cdot x_j$$

and

$$T_{j+1} - (T_j^* + p + d_{j+1}^*) \geq -2T_{\text{Max}} \cdot (1 - x_j).$$

The boundary conditions are $T_0 = 0$ and $T_{M+1} \leq T_{\text{Max}}$.

Synchronization: In order to get a complete formulation, we must explain the way the activities of both the vehicle and the micro-plant are synchronized. We use the synchronization variable $U_{i,j}$ which will tell us, in case the vehicle decides to refuel between a station j and a station $j+1$, during which period i it will do it.

- For any station j, if the vehicle decides to refuel before going to the following station i.e. $x_j = 1$, exactly one period is concerned by this activity:

$$\sum_{i=0}^{N-1} U_{i,j} = x_j$$

- If any period i is corresponds to a refuelling, that is $\delta_i = 1$, then the vehicle comes from a unique station j:

$$\sum_{j=0}^{M} U_{i,j} = \delta_i$$

- The date to reach the micro-plant from a station j is at most the beginning of a period. This can be expressed by the next non linear inequality

$$\sum_{i=0}^{N-1} p.i.U_{i,j} \geq x_j \cdot (T_j + d_j).$$

This logical constraint can be linearized by:

$$\sum_{i=0}^{N-1} pi \cdot U_{i,j} \geq T_j + d_j - 2T_{\text{Max}} \cdot (1 - x_j)$$

- The date for refuelling between stations j and $j+1$ is at least the beginning of a period:

$$\sum_{i=0}^{N-1} pi \cdot U_{i,j} \leq T_j^*$$

- The amount of fuel distributed at a period corresponds to the quantity of fuel loaded during a travel, specifically at a period i that is matched with a station j:

$$L_i^* = \sum_{j=0}^{M} U_{i,j} \cdot L_j,$$

  And symmetrically,

$$L_j = \sum_{i=0}^{N-1} U_{i,j} \cdot L_i^*.$$

  To linearize this constraint we use variables $m_{i,j}$. Hence,

$$\begin{array}{lcl} L_i^* & = & \sum_{j=0}^{M} m_{i,j} \\ L_j & = & \sum_{i=0}^{N-1} m_{i,j} \\ m_{i,j} & \leq & C^{\text{Veh}} \cdot U_{i,j} \\ m_{i,j} & \geq & 0. \end{array}$$

3.3 The Mixed integer linear formulation $MILP_{SMEPC}$

$$\textbf{Minimize} \sum_{i=0}^{N-1} ((Cost^F.y_i) + (Cost_i^V.z_i)) + \alpha.T_{M+1} \tag{1}$$

1. **Production constraints:**

$$y_0 - z_0 = 0, \delta_0 = 0, \tag{2}$$
$$y_i - z_i \leq 0, \quad \text{For } i = 1, \dots N-1, \tag{3}$$
$$y_i + z_{i-1} \leq 1, \quad \text{For } i = 1, \dots N-1, \tag{4}$$
$$z_i - z_{i-1} - y_i \leq 0, \quad \text{For } i = 1, \dots N-1, \tag{5}$$
$$z_i + \delta_i \leq 1, \quad \text{For } i = 1, \dots N-1, \tag{6}$$
$$V_0^{\text{MP}} = H_0, \tag{7}$$
$$V_i^{\text{MP}} \leq C^{\text{MP}}, \quad \text{For } i = 0, \dots N-1, \tag{8}$$
$$V_N^{\text{MP}} \geq H_0, \tag{9}$$
$$V_{i+1}^{\text{MP}} = V_i^{\text{MP}} + R_i \cdot z_i - L_i^*, \quad \text{For } i = 0, \dots N-1, \tag{10}$$
$$V_i^{\text{MP}} \geq 0, L_i^* \geq 0, \quad \text{For } i = 0, \dots N-1, \tag{11}$$
$$L_i^* \leq C^{\text{MP}} \cdot \delta_i, \quad \text{For } i = 0, \dots N-1. \tag{12}$$

2. **Vehicle constraints:**

$$V_0^{\text{Veh}} = E_0, \tag{13}$$

$$V_j^{\text{Veh}} \leq C^{\text{Veh}}, \quad \text{For } j = 0, \dots M, \tag{14}$$

$$V_{M+1}^{\text{Veh}} \geq E_0, \tag{15}$$

$$V_j^{\text{Veh}} \geq \epsilon_j, \quad \text{For } j = 0, \dots M, \tag{16}$$

$$V_{j+1}^{\text{Veh}} = V_j^{\text{Veh}} - e_j + (e_j - \epsilon_j - \epsilon_{j+1}^*) \cdot x_j + L_j, \quad \text{For } j = 0, \dots M, \tag{17}$$

$$L_j \geq 0, \quad \text{For } j = 0, \dots M, \tag{18}$$

$$L_j \leq C^{\text{Veh}} \cdot x_j, \quad \text{For } j = 0, \dots M, \tag{19}$$

$$L_j \leq C^{\text{Veh}} + \epsilon_j - V_j^{\text{Veh}}, \quad \text{For } j = 0, \dots M, \tag{20}$$

$$T_0 = 0, \tag{21}$$

$$T_j \geq 0, \quad \text{For } j = 0, \dots M, \tag{22}$$

$$T_{j+1} - T_j - t_j \geq -2T_{\text{Max}} \cdot x_j, \quad \text{For } j = 0, \dots M, \tag{23}$$

$$T_{j+1} - (T_j^* + p + d_{j+1}^*) \geq -2T_{\text{Max}} \cdot (1 - x_j), \quad \text{For } j = 0, \dots M, \tag{24}$$

$$T_j^* \geq T_j + d_j, \quad \text{For } j = 0, \dots M, \tag{25}$$

$$T_{M+1} \leq T_{\text{Max}}. \tag{26}$$

3. **Synchronisation constraints:**

$$\sum_{i=0}^{N-1} U_{i,j} = x_j, \quad \text{For } j = 0, \dots M, \tag{27}$$

$$\sum_{j=0}^{M} U_{i,j} = \delta_i, \quad \text{For } i = 0, \dots N-1, \tag{28}$$

$$\sum_{i=0}^{N-1} pi \cdot U_{i,j} \leq T_j^*, \quad \text{For } j = 0, \dots M, \tag{29}$$

$$\sum_{i=0}^{N-1} pi \cdot U_{i,j} \geq T_j + d_j - 2T_{\text{Max}} \cdot (1 - x_j), \quad \text{For } j = 0, \dots M, \tag{30}$$

$$\sum_{j=0}^{M} m_{i,j} = L_i^*, \quad \text{For } i = 0, \dots N-1, \tag{31}$$

$$\sum_{i=0}^{N-1} m_{i,j} = L_j, \quad \text{For } j = 0, \dots M, \tag{32}$$

$$m_{i,j} \leq C^{\text{Veh}} U_{i,j}, \quad \text{For } i = 0, \dots, N-1, j = 0, \dots M, \tag{33}$$

$$m_{i,j} \geq 0, \quad \text{For } i = 0, \dots, N-1, j = 0, \dots M. \tag{34}$$

4. **Decision variable constraints:**

$$0 \leq y_i, 0 \leq z_i, 0 \leq \delta_i, \quad \text{For } i = 0, \dots N-1, \tag{35}$$

$$0 \leq x_j \leq 1, \quad \text{For } j = 0, \dots M, \tag{36}$$

and

$$y_i \in \{0,1\}, z_i \in \{0,1\}, \delta_i \in \{0,1\}, \quad \text{For } i = 0, \dots N-1, \tag{37}$$

$$x_j \in \{0,1\}, \quad \text{For } j = 0, \dots M. \tag{38}$$

Theorem 3.1. $MILP_{SMEPC}$ has a feasible solution if and only if $SMEPC$ has a solution with the same cost.

Proof. Consider an optimal solution $(\bar{z}, \overline{T}, (\bar{L}, \bar{x}))$ of $SMEPC$, which describes the plant activity schedule, the vehicle timetable and the refuelling policy respectively. We check that it can be turned into a feasible solution of the MILP with the same cost.

Let $\overline{J}$ be the set of indices $j \in \{0, \dots M\}$ for which x_j takes value one. By applying (13) and (17), the H^2 load $\overline{V}^{\text{Veh}}$ is iteratively calculated in the following way: if $j \in \overline{J}$, then $\overline{V}^{\text{Veh}}_{j+1} = \overline{V}^{\text{Veh}}_j - \epsilon_j + \overline{L}_j - \epsilon^*_{j+1}$, else $\overline{V}^{\text{Veh}}_{j+1} = \overline{V}^{\text{Veh}}_j - e_j$.

As the vehicle directly goes to the station after recharging which takes one period of time, $\overline{T}^*$ can be chosen such that

$$\overline{T}^*_j = \overline{T}_{j+1} - d^*_{j+1} - p \text{ and } \overline{T}^*_j \in \{0, p, \dots (N-1)p\}, \text{ for } j \in \overline{J}.$$

When j does not belong to $\overline{J}$, T^*_j can take any non negative value. The validity of the vehicle's route means that the vectors $\overline{T}$ and $\overline{T}^*$ verify (21)-(26).

Moreover, an index i_j can be assigned to each j in $\overline{J}$, by taking $i_j \cdot p = \bar{T}^*_j$. Let $\overline{I} = \{i_j : j \in \overline{J}\}$. One can assume that $\delta_i = 1, L^*_i = L_j$ for $i \in \overline{I}$, and $\delta_i = L^*_i = 0$ otherwise. Similarly, $U_{i,j} = 1$ if $i = i_j, j \in \overline{J}$, and $U_{i,j} = 0$ otherwise.

Let $\overline{I}^A = \{i \in \{0, N-1\} : \bar{z}_i = 1\}$. $\overline{I}^A$ can be partitionned into disjoint intervals of the form $[f_k, g_k], k = 1, \dots q$, on which the plant is active. Note than $\bar{z}_{f_k - 1} = 0 = \bar{z}_{g_k + 1}$, if $f_k > 0, g_k < N-1, 1 \le k \le N-1$. As the micro-plant cannot be active while the vehicle refuels, $\overline{I} \cap \overline{I}^A = \emptyset$; this implies that (6) is satisfied.

Let us define

$$\bar{y}_i = \begin{cases} 1 & \text{if } i = f_k, \text{ for some } k \le q, \\ 0 & \text{otherwise.} \end{cases}$$

(2)-(5) are therefore verified by $\bar{y}$ and $\bar{z}$. From (7), iteratively applying equation (10) enables us to determine the volume V^{MP}_i, for $i = 0, \dots N-1$. All other inequalities are capacity constraints that are necessary satisfied by a feasible solution of $SMEPC$.

Conversely, let us consider some optimal feasible solution

$$(\bar{y}, \overline{\delta}, \bar{x}, \bar{L}, \bar{z}, \overline{L}^*, \overline{T}, \overline{T}^*, \overline{V}^{\text{Veh}}, \overline{V}^{\text{MP}}, \overline{U})$$

of the above $MILP_{SMEPC}$.

By Constraints ((27)-(28)), the vector $\overline{U}$ defines a matching $j \to i_j$ between the sets $\bar{J} = \{j \in \{0, \dots, M\} : \bar{x}_j = 1\}$ and $\bar{I} = \{i \in \{0, \dots, N-1\} : \overline{\delta}_i = 1\}$. This matching is consistent with the standard linear ordering: if $j_1 < j_2$ then $i_{j_1} < i_{j_2}$. Indeed, inequalities (21)-(26) insures that $T_{j+1} > T_j, \forall j$ and $T_{j+1} > T^*_j > T_j, \forall j \in \bar{J}$. Hence $T^*_{j_2} > T_{j_2} \ge T_{j_1+1} > T^*_{j_1}$. From (29)-(30), we derive that $i_{j_2} p > T_{j_2} > T^*_{j_1} \ge i_{j_1} p$. Therefore, $i_{j_1} < i_{j_2}$.

By optimality of the solution, constraints (23), (24) and (29) are tight. Thus $\overline{T}$ defines a vehicle timetable. As $L^*_{i_j} = L_j, j \in \bar{J}$ from (31)-(34), the vehicle and the micro-plant tanks have feasible volumes due to flow type and capacity constraints.

We conclude that a solution to the $SMEPC$ can be extracted from the optimal feasible solution of the $MILP$, both of the same cost.

$\square$

4 Linear relaxation. Strengthening inequalities

Denote by $RMILP_{SMEPC}$ the linear relaxation of $MILP_{SMEPC}$, i.e. the linear program described by (1)-(36) and by $\bar{Z}_r$ its optimal value. Often, Big M techniques induce very weak linear relaxations. But here we have the following.

Lemma 4.1. If $RMILP_{SMEPC}$ is feasible, then $\bar{Z}_r > 0$.

Proof. Suppose on the contrary that there is an optimal solution

$$(\bar{y}, \overline{\delta}, \bar{x}, \bar{L}, \bar{z}, \overline{L}^*, \overline{T}, \overline{T}^*, \overline{V}^{\text{Veh}}, \overline{V}^{\text{MP}}, \overline{U})$$

of the $RMILP_{SMEPC}$ such that $\bar{Z}_r = 0$. Necessarily

$$\bar{y}_i = \bar{z}_i = 0, \quad \text{for } i = 0, \dots N-1.$$

Then from (10) we obtain that $V^{\text{MP}}_{i+1} = V^{\text{MP}}_i - L^*_i$, for $i = 0, \dots N-1$. By (7) and (9), $L^*_i = 0$, for $i = 0, \dots N-1$. (31) implies that $m_{i,j} = 0$, for $j = 0, \dots M, i = 0, \dots N-1$. By (32), $L_j = 0$, for $j = 0, \dots M$. Henceforth, (17) gives $V^{\text{Veh}}_{j+1} < V^{\text{Veh}}_j$, for $j = 0, \dots M$. But this contradicts the initial and final conditions (13) and (15). $\square$

The inequalities

$$T^*_{j+1} \geq T^*_j, \quad j = 0, \dots M, \tag{39}$$

$$T_{j+1} \geq T_j + t_j + x_j(d_j + d^*_{j+1} - t_j), \quad j = 0, \dots M, \tag{40}$$

$$L_j \geq x_j(\epsilon_j + \epsilon^*_{j+1} - e_j), \quad j = 0, \dots M. \tag{41}$$

are straightforwardly seen to be valid for $MILP_{SMEPC}$. (39) and (40) insure that the times form non decreasing sequences. By considering (41), the vehicle leaving any station will arrive at the next one with more hydrogen after a refuelling than if it followed the direct route between the two.

5 Separation procedures

6 Computational results

7 Conclusion