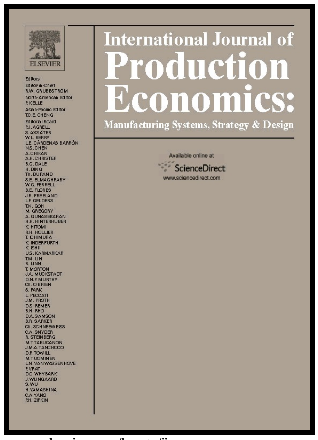

www.elsevier.com/locate/ijpe

Quantitative Insights into the Integrated Supply Vehicle Routing and Production Planning Problem

Fanny. Hein, Christian Almeder

PII: S0925-5273(16)30038-X
DOI: http://dx.doi.org/10.1016/j.ijpe.2016.04.014
Reference: PROECO6390

To appear in: *Intern. Journal of Production Economics*

Received date: 3 August 2015
Revised date: 13 March 2016
Accepted date: 18 April 2016

Quantitative Insights into the Integrated Supply Vehicle Routing and Production Planning Problem

Fanny Hein*, Christian Almeder [1]

European University Viadrina Frankfurt (Oder), Chair for Supply Chain Management Große Scharrnstraße 59, 15230 Frankfurt (Oder), Germany

Hein@europa-uni.de

Almeder@europa-uni.de

*Corresponding author. +49 (0335) 5534 2965

Abstract

In this work we assess the benefits of an integrated planning approach for the supply of raw material and the subsequent production process. The supply part is concerned with the collection of raw materials from geographically dispersed suppliers, while the production planning part addresses the conversion of those raw materials into final products to satisfy customer demand. The proposed model is an extension of the model introduced by Kuhn and Liske (2011) considering dynamic demand and general structures of the bill of materials. We investigate two scenarios: one including raw material inventories at the production site, and the other supposing just-in-time (JIT) supply. Numerical experiments show that substantial cost savings are possible with an integrated planning approach compared to a classical sequential approach. The JIT scenario and situations with a rather low utilization in the production system benefit most from the integration. The proposed supply vehicle routing and production planning problem has a kind of reverse structure compared to the well-studied production-distribution systems. Surprisingly, a sensitivity analysis on the dependency of the cost savings on different parameters show a quite similar behavior for both types of planning problems.

Keywords: integrated planning; supply-production planning; vehicle routing; just-in-time; mixed-integer programming

[1] +49 (0335) 5534 2254

1 Introduction

There is a growing awareness among companies to enhance their supply chain performance in order to enhance their overall performance. The integration and coordination across supply chain functions is regarded as the next source of competitive advantage, and thus has become of great interest (e.g. Chandra and Fisher, 1994; Thomas and Griffin, 1996; Sarmiento and Nagi, 1999; Fumero and Vercellis, 1999; Lei et al., 2006; Bard and Nananukul, 2010; Adulyasak et al., 2015; Díaz-Madroñero et al., 2015). While researchers mainly concentrate on coordinating production and distribution, studying the benefits from coordinating supply and production is a relatively new approach. In fact, findings on production-routing problems (PRP) cannot be directly transferred to supply-production problems (SPP) due to the different position of the production plant in the supply chain. PRPs involve a classical inventory routing problem (IRP) of type one-to-many, while SPPs embed a reverse IRP of type many-to-one. Another aspect is that, unlike PRPs, in SPPs the production plan subjects to the availability of raw materials, and thus, to the underlying routing problem. In order to save transportation costs, there is a tendency to purchase larger amounts of raw materials provoking earlier production.

Regardless of the perspective, whether PRP or SPP, conflicting interests between supply chain partners typically entail inefficiencies. Suppliers normally prefer large shipping quantities and stable volumes. Conversely, buyers prefer small batches and frequent deliveries due to changing demands and their unwillingness to hold larger amounts of inventories. In order to achieve optimal operational performance on supply chain level, planning decisions should be made jointly. Since decisions on production and logistics are known for their complexity, most companies manage these two functions independently. The sequential approach is assumed to be an appropriate method if inventories are involved decoupling supply (or distribution) and production operations from each other (Chandra and Fisher, 1994; Díaz-Madroñero et al., 2015). If storage of input materials (at the plant) is possible, procurement and consumption of those materials can differ in terms of time and quantity facilitating to consolidate shipments and to save transportation costs. However, inventory costs, the trend towards just-in-time (JIT) operations and supply chain agility are creating pressure to cut inventories. Low inventory levels, in turn, affect the suitability of decoupled planning practices such that a closer collaboration among supply chain partners becomes essential.

The importance of stock-keeping in the context of supply chain coordination has been regarded as elementary, and therefore, potential savings are evaluated for two scenarios. In the first scenario, we suppose an unlimited storage of input materials at the plant, whereas in the second scenario, we suppose that storage of input materials is not allowed at the plant intending to model JIT supply.

The objective of this paper is to investigate the advantage of simultaneous versus sequential supply-production planning for the two described problem settings. First, we will prove whether coordination is still beneficial even if inventories are involved (Scenario I). Second, we will investigate how potential savings change under JIT supply (Scenario II) where the value of coordination is expected to be much greater.

In doing so, we developed an integrated supply-production model minimizing the aggregate of setup costs, system holding costs and transportation costs over a finite planning horizon subject to deterministic, dynamic demand that must be fully met. The resulting problem comprises three critical decisions: how many end items to produce in each period, when to replenish the raw materials and in which quantities, and which routes to use when visiting material suppliers.

Main contributions of this work are:

- studying the benefits of integrated planning for a complex supply-production network embedded in a dynamic, discrete-type system
- identifying the role of raw material inventories for supply-production decisions
- establishing managerial implications for supply-production systems and comparison to similar production-distribution problems

The remainder of this paper is organized as follows. Section 2 overviews relevant literature where emphasis is put on papers tackling lot sizing and vehicle routing as a combined problem. Section 3 contains the problem statement and suggests a mixed-integer program to model the integrated supply-production problem distinguishing between two scenarios. After that, a sequential solution approach is presented in Section 4 used as benchmark solution to estimate relative cost savings. Section 5 reports on the computational experiments and elaborates under which conditions integrated supply-production planning has the potential to provide a competitive advantage. Lastly, Section 6 summarizes main findings and identifies topics for further research.

2 Literature Review

The integrated Capacitated Lot Sizing and Supply Side Vehicle Routing Problem (CLSVRP) is based on a combination of two well-known problems: the Capacitated Lot Sizing Problem (CLSP) and the Vehicle Routing Problem (VRP).

The CLSP is a single-level lot-sizing model used for problems with capacity restrictions and dynamic demand. The basic idea is to find production amounts for each period such that the sum of holding and setup cost are minimized. It stems for a well-known model introduced by Wagner and Whitin (1958). Quadt and Kuhn (2008) as well as Buschkühl et al. (2010) provide reviews on the CLSP and its extensions. In addition to dynamic demand models, there are also simplified models assuming constant demand and an infinite planning horizon. The Economic Lot Scheduling Problem (ELSP) is one of them (cf. Elmaghraby, 1978) and has been applied by Kuhn and Liske (2011) as basis for simultaneous supply-production planning.

The collection of raw materials is basically a VRP. It consists of designing optimal delivery routes from one or multiple depots to a number of geographically dispersed customers or cities, subject to side constraints (Laporte, 1992; Toth and Vigo, 2002). A variant of the VRP suitable to model the collection of raw materials is the Inventory Routing Problem (IRP). The IRP tackles vehicle routing and inventory management problems in an integrated manner of which many variants emerged within the past 30 years. A literature review is provided by Coelho et al. (2014). Usually, the IRP deals with delivering goods from one depot to several customer locations while limited stocks at the customer sites are allowed. The work by Lee et al. (2003) is among the rare papers tackling an in-bound logistic network of the type many-to-one in the context of an automotive parts supply chain. However, the IRP approach in general neglects the production process (cf. Campbell et al., 1998; Sarmiento and Nagi, 1999; Kuhn and Liske, 2011).

The integration of production and transport planning has been studied intensively in recent decades whereas the majority of articles focus on the strategic or tactical aspects of the integration (cf. Vidal and Goetschalckx, 1997; Sarmiento and Nagi, 1999; Erengüc et al., 1999; Fahimnia et al., 2013). The group of PRPs combining lot sizing and distribution routing aspects at a more detailed level is of particular interest for our research. A comprehensive summary of existing PRP formulations and solution techniques is given by Adulyasak et al. (2015) and Díaz-Madroñero et al. (2015). One of the first researchers proposing an integrated production scheduling and vehicle routing model are Chandra and Fisher (1994) considering one manufacturing plant with a finished goods warehouse attached and multiple retailers. Through computational experiments they assess the value of coordinating production and distribution. Fumero and Vercellis (1999) apply a Lagrangian decomposition method to solve a problem similar to that studied by Chandra and Fisher (1994). Lei et al. (2006) study the integrated Production, Inventory and Distribution Routing Problem (PDRP) allowing heterogeneous transporters and propose a two-phase solution approach. Adulyasak et al. (2013) present two branch-and-cut approaches to solve the PRP. Absi et al. (2014) propose an iterative, MIP-based solution procedure for the PRP with uncapacitated production resources. Amorim et al. (2013) overviewed production-distribution models for perishable goods in make-to-order production systems concentrating more on the operational aspects.

Research on the combination of supply and production planning on the operational level is considerably less common. In fact, to the best of our knowledge, Kuhn and Liske (2011, 2014) are the only publications dedicated to the combination of a supply side routing problem with a production planning problem which is of special interest for assembly-oriented manufacturers. Kuhn and Liske (2011) combine an ELSP representing the production system and a VRP representing the collection of raw materials from a set of suppliers to the so-called Economic Lot and Supply Scheduling Problem (ELSSP). The continuous-like system faces constant demand and production rates, and uses a common cycle policy for determining the optimal production cycle time. To gain new insights into the ELSSP, sensitivity analysis was conducted to test the impact of the inventory holding cost ratio, the distance ratio as well as setup cost. Results underline that the simultaneous solution approach can achieve total cost reductions up to 90%. This seems rather too optimistic and might be due to the fact that a very simplified representation of the production system is used. Additional assumptions such that each input material is dedicated to exactly one end item and that consolidation of shipments is only possible among input materials which are needed for production of the same product might be too restrictive as well. In a subsequent publication dedicated to the ELSSP, Kuhn and Liske (2014) identified the benefits of the power-of-two policy compared to a common cycle approach.

The work of Kuhn and Liske (2011) is the starting point of the present work where we extend the approach by a more complex production system and loosening some of the assumptions. Hence, we are able to assess, in a more realistic way, the benefits of integrated supply and production planning, in particular how a JIT system compared with a traditional approach including a raw material inventory is affected by the integrated planning.

3 An integrated supply routing and production model

3.1 Problem Description

We base our model formulation on the supply and production planning problem of a classical assembly-oriented manufacturer who produces a set of end products for which predefined amounts of raw materials are needed. The problem is similar to that introduced by Kuhn and Liske (2011).

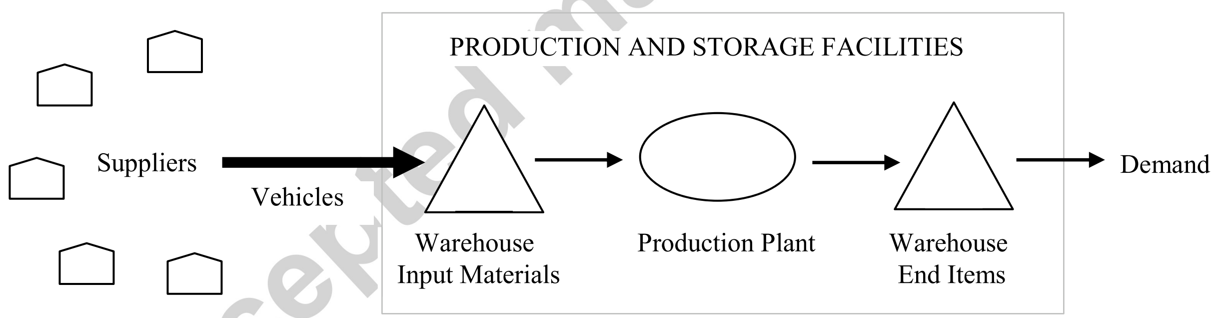

Figure 1: The supply chain of the CLSVRP (cf. Kuhn and Liske, 2011)

Figure 1 depicts the underlying routing, production and inventory system consisting of a single production plant, a warehouse for input materials, a warehouse for end items, a transportation fleet managing the transfer of materials and a set of suppliers each of them providing a certain set of materials. The manufacturer has to solve a standard lot sizing problem in order to meet deterministic but time-dependent demand. The production quantities per period are restricted by a finite capacity. For the sake of simplicity we omit intermediate production steps and assume single-stage manufacturing process. Input materials are withdrawn from the input material inventories and transformed into final goods. There is a general relationship between input materials and final products, i.e., the same input material might be used for several end products and an end product is usually made out of several different input materials. In order to replenish input material inventories the manufacturer has to collect those materials using a homogeneous fleet of vehicles with limited capacities at different suppliers which are geographically dispersed in close proximity to the production location. There is a many-to-one relationship of input materials and suppliers, i.e. each input material is only available from one supplier, whereas one supplier can provide multiple input materials. Besides, we assume an unlimited supply at the suppliers. Every tour of a vehicle starts and ends in the same period, so that materials collected in the current period are available for production in the following period. Holding costs arising during the supply lead time of one period formally represent a constant factor over the entire planning horizon and, therefore, they can be omitted in the objective function.

Transportation costs are distance-based whereby effects of truckload on, e.g., speed or fuel consumption are neglected. Supposing that travel costs are symmetric, it is natural to define the supply-routing problem on an undirected graph $\mathcal{G} = (\mathcal{V} \cup \{0\}, \mathcal{E})$ where every pair of vertex set $\mathcal{V} \cup \{0\} = \{0,1,\dots,S\}$ is connected via a unique edge of set $\mathcal{E} = \{(s,s'): s,s' \in \mathcal{V} \cup \{0\}, s < s'\}$. Vertices of $\mathcal{V}$ denote the suppliers and vertex 0 denotes the manufacturing facility. Symmetry means here that the direction of how edges are traversed has no influence on travel costs. This edge-based formulation allows to decrease the number of binary variables improving computational efficiency (cf. Archetti et al., 2007; Coelho and Laporte, 2013). A mathematical formulation for the asymmetric counterpart is provided in Appendix A.1. Transportation costs expressed by matrix $tc_{ss'}$ consist of variable and fixed cost components. We incorporated variable transportation costs proportional to the travel distance by multiplying the travel cost per distance unit, denoted as tc^{var}, with the distance between two locations. Besides, we considered vehicle fixed costs, denoted as tc^{fix}, arising every time a vehicle leaves the depot, e.g. for driver wages, maintenance, preparation, insurance or rental. Fixed cost are included in total transportation cost $tc_{ss'}$ such that all edges incident to the depot at vertex 0 are additionally associated with half of the vehicle fixed cost. Clearly, since every vehicle k operated in period t must leave and return to the depot once, exactly two edges linked to the depot must be part of each feasible round trip. We suppose that each vehicle can perform maximum one trip per period so that terms "vehicle" and "trip" can be used interchangeably. Note that it is theoretically possible to visit the same supplier multiple times per period by different vehicles (during different trips). Vehicles being idle stay in the depot without causing decision relevant cost. Fleet size is fixed by parameter K. Except for very short-term planning, limitation on fleet size is not close to reality. This fleet restriction can be easily loosened by choosing a high value for K like, for instance, the number of trucks needed to collect all materials at once meaning that any supply plan can be executed.

The overall objective of the described planning problem is to minimize the sum of setup cost, holding cost for end items, holding cost for input materials, and transportation cost.

3.2 Mathematical formulation

Before introducing the model formulation, the common notation used throughout this paper is provided below.

Indices and Sets:

$i \in \mathcal{M}$	(set of) input materials
$j \in \mathcal{N}$	(set of) end items
$s, s' \in \mathcal{V} \cup \{0\}$	(set of) suppliers with $\mathcal{V} = (1, \ldots, S)$ and $\{0\}$ corresponding to the depot
$t \in \mathcal{T}$	(set of) periods
$k \in \mathcal{K}$	(set of) homogenous vehicles

Parameters:

a_{ij}	amount of input material i needed to produce one unit of end item j (gozinto factor)
b_{is}	equals 1, if input material i is delivered by supplier s, 0 otherwise
C_t	production capacity in period t
d_{jt}	demand of end item j in period t
h_i^r	unit holding cost of input (raw) material i
h_j^f	unit holding cost of end (finished) item j
I_{i0}^r	initial inventory of input material i
pt_j	production time per unit j
Q	vehicle capacity
sc_j	setup cost incurred, if end item j is produced in period t

$tc_{ss'}$ total costs for traveling from vertex s to s'

Decision Variables:

I^r_{it} inventory of input material i at the end of period t

I^f_{jt} inventory of end item j at the end of period t

p_{jt} production quantity (lot size) of end item j in period t

q^k_{it} amount of input material i supplied in period t by vehicle k

z_{jt} equals 1, if end item j is setup in period t, 0 otherwise

$x^k_{ss't}$ equals 1, if the direct connection between two suppliers $s, s' \in \mathcal{V}$ with $s < s'$ is part of vehicle k's tour in period t, 0 otherwise

$x^k_{0s't}$ equals 1, if supplier $s' \in \mathcal{V}$ is scheduled first or last on vehicle k's tour in period t, equals 2, if supplier $s' \in \mathcal{V}$ is visited on a direct tour by vehicle k in period t, 0 otherwise

v^k_{st} equals 1, if supplier (depot) $s \in \mathcal{V} \cup \{0\}$ is part of vehicle k's tour in period t, 0 otherwise

The CLSVRP combines lot sizing and supply scheduling in one model formulation. The production part corresponds to the "inventory and lot size" formulation proposed for the Multi-Level Capacitated Lot Sizing Problem (ML-CLSP) (in Tempelmeier and Derstroff, 1996; Buschkühl et al., 2010). The second block of equations refers to the supply-routing aspect of the problem which is in essence an IRP exhibiting a network structure of type many-to-one studied by Lee et al. (2003).

The CLSVRP for coordinating the supply of multiple input materials from multiple suppliers and production of multiple end items at a single plant can be formulated as follows:

Model CLSVRP:

$$\min \sum_{j\in\mathcal{N}} \sum_{t\in\mathcal{T}} sc_j z_{jt} + \sum_{j\in\mathcal{N}} \sum_{t\in\mathcal{T}} h^f_j I^f_{jt} + \sum_{i\in\mathcal{M}} \sum_{t\in\mathcal{T}} h^r_i I^r_{it} + \sum_{\substack{s,s'\in\mathcal{V}\cup\{0\} \\ s<s'}} \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} tc_{ss'} x^k_{ss't} \quad (1)$$

subject to

$$I^r_{it} = I^r_{i,t-1} + \sum_{k\in\mathcal{K}} q^k_{i,t-1} - \sum_{j\in\mathcal{N}} a_{ij} p_{jt} \qquad \forall\, i, t \in \mathcal{T} \setminus \{1\} \quad (2)$$

$$I^r_{i1} = I^r_{i0} - \sum_{j\in\mathcal{N}} a_{ij} p_{j1} \qquad \forall\, i \quad (3)$$

$$I_{jt}^{f} = I_{j,t-1}^{f} + p_{jt} - d_{jt} \qquad \forall\, j,t \qquad (4)$$

$$p_{jt} \leq \sum_{\tau=t}^{T} d_{j\tau} z_{jt} \qquad \forall\, j,t \qquad (5)$$

$$\sum_{j\in\mathcal{N}} pt_j p_{jt} \leq C_t \qquad \forall\, t \qquad (6)$$

$$\sum_{i\in\mathcal{M}} q_{it}^{k} \leq Q\, v_{0t}^{k} \qquad \forall\, t,k \qquad (7)$$

$$\sum_{i\in\mathcal{M}} b_{is}\, q_{it}^{k} \leq Q\, v_{st}^{k} \qquad \forall\, t,k,s \qquad (8)$$

$$\sum_{\substack{s'\in\mathcal{V}\cup\{0\} \\ s'>s}} x_{ss't}^{k} + \sum_{\substack{s'\in\mathcal{V}\cup\{0\} \\ s'<s}} x_{s'st}^{k} = 2\, v_{st}^{k} \qquad \forall\, t,k,s \qquad (9)$$

$$\sum_{\substack{s,s'\in\mathcal{L} \\ s<s'}} x_{ss't}^{k} \leq \sum_{s\in\mathcal{L}} v_{st}^{k} - v_{gt}^{k} \qquad \forall\, t,k,\mathcal{L}\subseteq\mathcal{V}, g\in\mathcal{L} \qquad (10)$$

$$z_{jt}, v_{st}^{k}, x_{ss't}^{k} \in \{0,1\}\ ,\ x_{0s't}^{k} \in \{0,1,2\} \qquad \forall\, j,t,k,s,s' \qquad (11)$$

$$p_{jt},\ q_{it}^{k},\ I_{it}^{r},\ I_{jt}^{f} \geq 0 \qquad \forall\, i,j,t,k \qquad (12)$$

The objective function (1) minimizes the sum of production setup costs, holding costs of end items, holding costs of input materials as well as transportation costs over the entire planning horizon. Variable production costs, which are assumed to be proportional to the production volume can be omitted. Constraints (2) - (4) reflect typical inventory balance constraints ensuring that external demands and dependent demands are met in every period. Initial inventories of input materials I_{i0}^{r} and end items I_{j0}^{f} entering balance equations (3) and (4) are directly available for consumption in the first period. In contrast to the replenishment lead time of one period, the production lead time is assumed to be zero. This leads to the fact that items produced in $t = 1$ can be immediately used to satisfy customer demand of that period given that enough input materials are on hand. In order to prevent shortages in $t = 1$, and thus prevent infeasible problem settings, initial inventories of materials I_{i0}^{r} must be sufficient to produce the demand of $t = 1$ not covered by initial inventories of end items. Thus, for all materials i it must hold: $I_{i0}^{r} \geq \sum_{j\in\mathcal{N}} a_{ij}(d_{j1} - I_{j0}^{f})$. As known from the basic CLSP, constraints (5) ensure that a production run can only start if the resource is in the correct setup state. Constraints (6) and (7) guarantee that both production and vehicle capacity limits are respected. Constraints (8) linking supply quantities q_{it}^{k} to routing variables state that q_{it}^{k} is only positive if supplier s providing material i is part of vehicle k's route in period t. The assignment matrix b_{is} indicates the relationship between input materials and suppliers. Since we suppose a many-to-one relationship between input materials and suppliers, for each input material i holds: $\sum_{s\in\mathcal{V}} b_{is} = 1$.[2] Though vehicle capacity Q could be replaced by any arbitrarily large number, it is reasonable to select Q to tighten the constraints stating that vehicle k is able to pick up maximum Q items in total at supplier site s. Degree constraints given in (9) guarantee route continuity while subtour-breaking constraints proposed for symmetric IRPs are given in (10) (cf. Fischetti et al., 1998; Archetti et al., 2007; Coelho and Laporte, 2013). Finally, integrality and non-negativity of decision variables are enforced by (11) and (12).

3.3 Just-in-time supply

The problem tackled in Section 3.1 and 3.2 supposes that there is no limitation on input material storage (hereinafter referred to as Scenario I). In this case, the sequential planning approach might work sufficiently well since inventories serving as buffer allow to reduce supply frequency and to better utilize vehicle capacities. However, this assumption does not hold for a variety of industry branches. Companies processing goods which, e.g. have a short lifespan or are highly perishable, typically aim at rapid inventory turnovers resulting in intermediate inventories kept to a minimum. Common examples are the food processing industry, the chemical or pharmaceutical industry where raw materials deteriorate quickly such that carrying larger amounts of stock is not an acceptable option. End items being preserved during the production process are considered as non-perishable. Besides, also in assembly-oriented processes, the JIT concept can be employed for strategic reasons.

Due to the fact that inventories allow to reduce overall logistics cost by exploiting economies of scale in freight transportation and help to overcome inefficiencies in managing the logistics system (e.g., Ghiani et al., 2004), we believe that the value of coordination is highly dependent on the opportunity to carry inventories. Thus, we additionally study the scenario where storage of input materials is not possible which is consistent with the JIT principle (hereinafter referred to as Scenario II).

Given JIT supply, the system of the CLSVRP is downsized by the input material warehouse as depicted in Figure 2.

[2] A general relationship between input materials and suppliers can be easily expressed by defining the corresponding assignment matrix b_{is} where for each i holds: $\sum_{s\in\mathcal{V}} b_{is} \geq 1$.

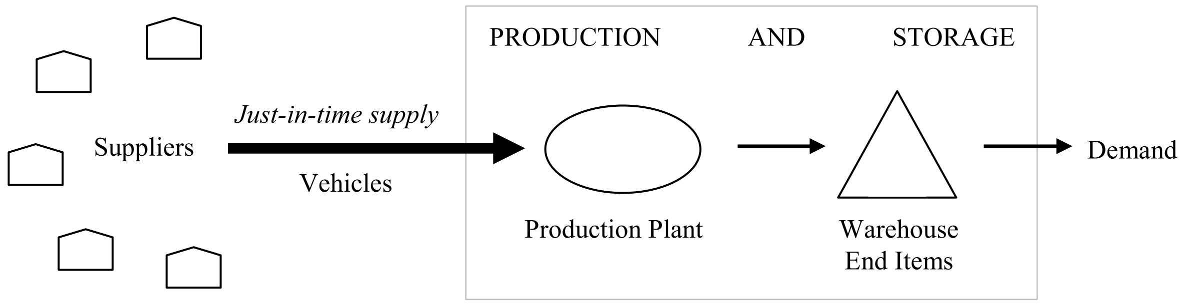

Figure 2: The supply chain of the CLSVRP under JIT supply

In order to formulate the JIT counterpart of the supply and production planning problem, denoted as CLSVRP-JIT, an additional constraint (13) is imposed to the CLSVRP (presented in Section 3.2) stating that the inventory level of input materials at the end of each period must be zero:

$$I_{it}^{r} = 0 \qquad \forall\, i, t \qquad (13)$$

Hence, the objective function now reduces to the problem of minimizing the sum of setup costs, holding costs of end items, and transportation costs over the entire planning horizon. Assuming that $I_{it}^{r} = 0$, the inventory balance constraints of input materials (see (2) - (3)) indicate that materials procured in period $t-1$ must exactly meet total production requirements in period t. The initial inventory of input materials I_{i0}^{r} is understood as the quantity ordered in $t = 0$ and arriving in $t = 1$ which is consistent with the concept of rolling horizons. Obviously, the supply process in Scenario II represents a strict form of JIT supply forcing input materials to immediately go into production without being temporarily stocked.

3.4 Illustrative Example

An illustrative example is introduced to facilitate the understanding of the CLSVRP model and to prove its proper functioning. The selected case represents one example instance of the small data set A being investigated during our computational study (see Section 5). Let us consider a network of four contractors who supply one manufacturer with eight different input materials needed for production of three different products over a planning horizon of five periods. Every supplier provides two different input materials. Figure 3 depicts the bill-of-materials as well as the links between input material and suppliers. Demand data, cost parameters and capacity assumptions are summarized in Table 1. The total transportation cost consisting of variable and fix cost components is given in Table 2. Note that the first row of the cost matrix includes half of the vehicle fixed costs.

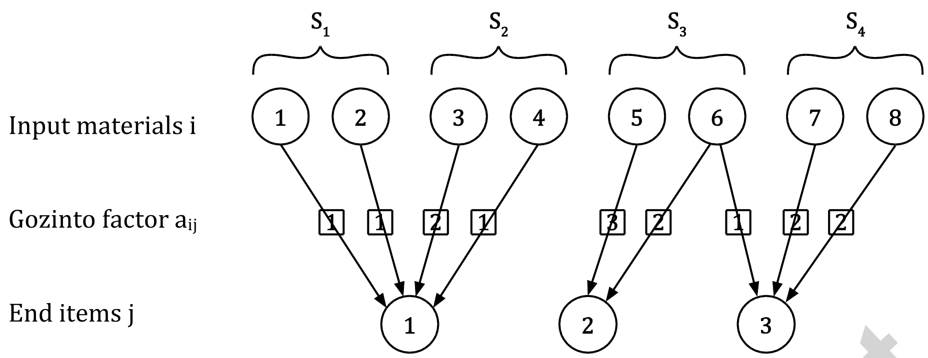

Figure 3: Bill of materials of the illustrative example

Table 1: Parameters of the illustrative example

Parameter		*Value*				
d_{1t}	$t=1,\dots,5$	{ 77	82	90	100	107 }
d_{2t}	$t=1,\dots,5$	{ 96	107	84	105	91 }
d_{3t}	$t=1,\dots,5$	{106	124	102	100	96 }
sc_j	$j=1,\dots,3$	250				
pt_j	$j=1,\dots,3$	1				
h^r_i	$i=1,\dots,8$	1				
h^f_j	$j=1,\dots,3$	5				
C_t	$t=1,\dots,5$	425 (70% mean utilization)				
tc^{var}		10				
tc^{fix}		500				
Q		1485 (factor 1)				
K		3				

Table 2: Total transportation cost matrix of the illustrative example

	Depot	*S₁*	*S₂*	*S₃*	*S₄*
Depot	0	720	570	790	780
S_1			740	900	160
S_2				640	830
S_3		not used			870
S_4					

The small test instance was solved in 1 second. The best integer solution found yield total costs of 13507 (in monetary units) whereof 1750 account for setup activities, 707 and 3010 for holding inventories of input materials and end items, respectively, and 8040 for transportation (including 2500 fixed transportation cost). According to the optimal supply schedule given in Table 3 which is graphically supported by Figure 4, supply is concentrated on the first three periods where maximum two vehicles, denoted as v_1 and v_2, are operated at the same time. Whenever a supplier is visited, input materials provided by the same supplier are preferably consolidated into a single shipment to save transportation costs. In period $t = 1$ and period $t = 2$, vehicles achieve nearly full truckloads. Capacity is particularly underutilized in $t = 3$ with vehicles v_1 and v_2 showing utilization rates of 80% and 40%. Due to relatively low setup cost, it is not surprising that production is split in several small batches with frequent setup operations.[3] When comparing production and supply schedule, the delay of one period is immediately recognizable and can be explained by the proposed replenishment lead time of one period. Since $t = 2$ starts with zero inventories of end items, each end item must be setup for production in $t = 2$ to satisfy the demand of that period.

Table 3: The optimal production and supply schedule of the illustrative example

$\boldsymbol{p_{jt}}$												
	$j \setminus t$	1		2		3		4		5		$\sum$
	1	-		82		90		207		-		379
	2	-		117		270		-		-		387
	3	-		226		-		196		-		422
	$\sum$	-		425		360		403		-		1,188
$\boldsymbol{q_{it}^k}$												
	$i \setminus k$	v_1	v_2	v_1	v_2	v_1	v_2	v_1	v_2	v_1	v_2	$\sum$
S_1	1	172	-	-	-	207	-	-	-	-	-	379
	2	172	-	-	-	207	-	-	-	-	-	379
S_2	3	-	344	-	-	-	414	-	-	-	-	758
	4	-	172	-	-	-	207	-	-	-	-	379

[3] The test set used for the computational experiments (see Section 5) also contains instances with setup costs larger than 250.

S_3	5	-	412	627	-	-	-	-	-	-	-	1,161
	6	-	460	858	-	-	-	-	-	-	-	1,196
S_4	7	452	-	-	-	392	-	-	-	-	-	844
	8	452	-	-	-	392	-	-	-	-	-	844
	∑	1,248	1,388	1,485	-	1,198	621	-	-	-	-	5,940

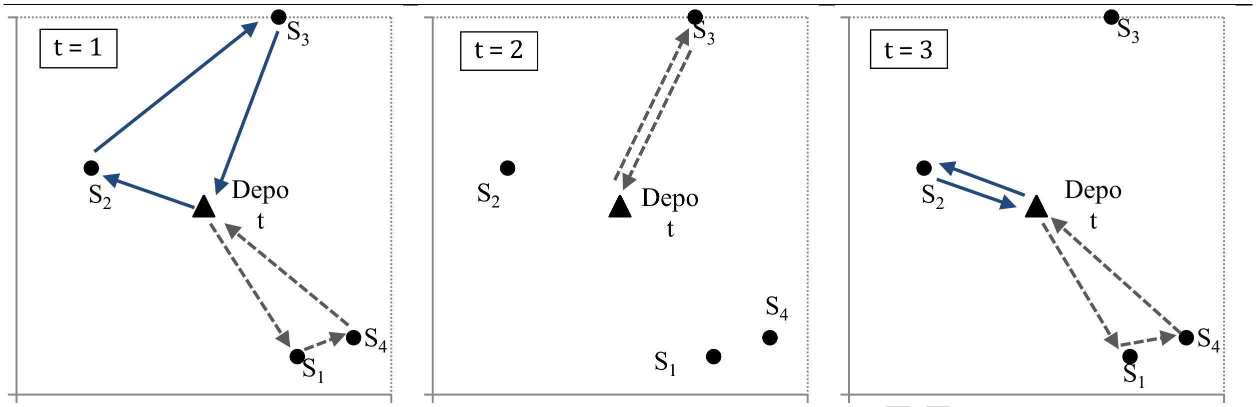

Figure 4: Locations and optimal routing during supply periods of the illustrative example

4 A sequential modeling approach

In practice and in theory, sequential solution procedures are commonly applied to the production and supply (and distribution) routing problem where priority is usually set on the production side (Chandra and Fisher, 1994; Kuhn and Liske, 2011). Decoupling lot sizing from supply decisions technically means that after having generated the optimal production plan in the first step, the supply schedule complying with the optimal production plan is determined subsequently.

From modeling perspective, the lot sizing step corresponds to a standard CLSP with the objective to minimize the sum of setup cost and holding cost for end products as stated in (14). While the first planning step is identical in both scenarios, the objective function of the second planning step does depend on the underlying system. Scenario I is similar to an IRP of type many-to-one where delivery quantities subject to inventory balance constraints for input materials (see (16) - (19)). Scenario II assuming JIT supply, by contrast, reduces to a standard VRP where delivery quantities are directly linked to the material requirements determined in the lot sizing step (see (20) - (23)). Consequently, the JIT principle entails a less flexible supply process as delivery dates and quantities are fixed.

Step 1: *Lot sizing*

$$\min \sum_{j\epsilon\mathcal{N}} \sum_{t\epsilon\mathcal{T}} sc_j z_{jt} + \sum_{j\epsilon\mathcal{N}} \sum_{t\epsilon\mathcal{T}} h_j^f I_{jt}^f \tag{14}$$

subject to(4) - (6)

$$p_{jt},\ I_{jt}^f \geq 0 \qquad \forall\, j, t \tag{15}$$

Step 2: p_{jt}^* denote the optimal production quantities obtained in Step 1

Inventory routing (Scenario I)

$$\min \sum_{i\epsilon\mathcal{M}} \sum_{t\epsilon\mathcal{T}} h_i^r I_{it}^r + \sum_{\substack{s,s\prime\epsilon\mathcal{V}\cup\{0\}\\ s<s\prime}} \sum_{t\epsilon\mathcal{T}} \sum_{k\epsilon\mathcal{K}} tc_{ss\prime}\, x_{ss\prime t}^k \tag{16}$$

subject to (7) - (11)

$$I_{it}^r = I_{i,t-1}^r + \sum_{k\epsilon\mathcal{K}} q_{i,t-1}^k - \sum_{j\epsilon\mathcal{N}} a_{ij}\, p_{jt}^* \qquad \forall\, i, t \in \mathcal{T} \setminus \{1\} \tag{17}$$

$$I_{i1}^r = I_{i0}^r - \sum_{j\epsilon\mathcal{N}} a_{ij}\, p_{j1}^* \qquad \forall\, i \tag{18}$$

$$q_{it}^k,\ I_{it}^r \geq 0 \qquad \forall\, i, t, k \tag{19}$$

Vehicle routing (Scenario II)

$$\min \sum_{\substack{s,s\prime\epsilon\mathcal{V}\cup\{0\}\\ s<s\prime}} \sum_{t\epsilon\mathcal{T}} \sum_{k\epsilon\mathcal{K}} tc_{ss\prime}\, x_{ss\prime t}^k \tag{20}$$

subject to (7) - (11)

$$\sum_{k\epsilon\mathcal{K}} q_{i,t-1}^k = \sum_{j\epsilon\mathcal{N}} a_{ij}\, p_{jt}^* \qquad \forall\, i, t \in \mathcal{T} \setminus \{1\} \tag{21}$$

$$I_{i0}^r = \sum_{j\epsilon\mathcal{N}} a_{ij}\, p_{j1}^* \qquad \forall\, i \tag{22}$$

$$q_{it}^k \geq 0 \qquad \forall\, i, t, k \tag{23}$$

5 Computational Study

This section reports on the computational experiments that have been conducted to evaluate the cost advantage of integrated over sequential supply and production planning subject to the underlying scenario. In doing so, solutions of 3,888 test cases organized into data sets A and B were investigated with respect to different parameter combinations. As appropriate problem instances for the integrated supply and production planning are not yet readily available from the literature, analysis relies on self-generated test instances incorporating randomness. However, the instance generation process and the dimension of parameter values are oriented on related papers (cf. Chandra and Fisher, 1994; Tempelmeier and Derstroff, 1996; Kuhn and Liske, 2011).

5.1 Instance Generation

Data sets A and B only differing in problem size are described by the number of input materials (M), end items (N) , suppliers (S) and the length of the planning horizon (T) with A being the smaller set. While test instances of set A consider 4 suppliers, 8 input materials, 3 end items, and 5 periods planning horizon, problem size of set B increases to 6 suppliers, 12 input materials, 4 end items and 10 periods. Each test instance of A and B is further characterized by key parameters, namely production capacity (C_t), coefficient of variation (CV) of the demand, vehicle capacity (Q), setup costs (sc_j), unit inventory costs of input materials (h_i^r) and end items (h_j^f), transportation costs per distance unit traveled (tc^{var}), and fixed transportation costs per vehicle (tc^{fix}).

The demand per period and end item is assumed to be normally distributed with a constant mean but different CV. Mean demands are standardized to 100 units per period and the CV is set to either 10% or 50%. Each input material requires the same amount of vehicle capacity, and each end item requires the same processing time per unit (pt_j) which is uniformly set to 1. The available production capacity is fixed at three levels meaning that the average utilization rate over all periods yields either 50%, 70% or 90%. Initial inventories of input materials are set to zero while initial inventories of end items are chosen such that they are just sufficient to meet the first period's demand. Note that due to the assumption of zero initial inventories and a supply lead time of one period, the actual planning horizon reduces to 4 and 9 periods for data set A and B, respectively.

Setup costs, unit holding costs, unit transportation cost as well as fixed vehicle costs do not vary by product or period. Four different demand scenarios are generated (two scenarios per data set as described above). For each demand scenario, the demand per item and period is taken from the same distribution.

Travel distances are derived from the Euclidean distance between two locations. All supplier locations are arbitrarily positioned on a (0, 0) to (100, 100) grid. The manufacturing facility and nearby warehouses (= depot) are positioned in the middle of the grid at point (50, 50). Fleet size is chosen sufficiently large intending not to constraint the optimal solution. Vehicle capacity is expressed by a factor indicating the average coverage of raw materials if only one vehicle is operated. For instance, a factor of 3 means that one vehicle manages to collect the average dependent demand of 3 periods. Parameter values used in the numerical experiments are provided together with the results in Table 5.

5.2 Computational Results

In order to adequately measure the advantage of the simultaneous planning approach, the sequential approach serves as benchmark. In doing so, (nearly) optimal solutions resulting from the integrated model, denoted as TC_{int}, are compared with solutions produced by the sequential one, denoted as TC_{seq}. With respect to the underlying scenario, relative cost savings ΔTC and ΔTC^{JIT} reflecting the discrepancy in operational efficiency are defined as follows:

$$\Delta TC = \frac{TC_{seq} - TC_{int}}{TC_{seq}} \quad , \qquad \Delta TC^{JIT} = \frac{TC_{seq}^{JIT} - TC_{int}^{JIT}}{TC_{seq}^{JIT}} \tag{24}$$

Table 4 reports on the number of problems that need to be solved for each test instance in order to estimate potential cost savings for the two proposed scenarios.

Table 4: Problems involved to assess the savings potential of an integrated planning approach

	Scenario I (incl. input material storage)	Scenario II (JIT-supply)
Integrated	CLSVRP	CLSVRP^{JIT}
Sequential	CLSP + IRP	CLSP + VRP
Cost Savings	**ΔTC**	**ΔTC^{JIT}**

Table 5: Cost savings ΔTC and ΔTC^{JIT} (in %) from an integrated approach compared to a sequential one subject to different parameter combinations (50%, 70%, 90% refer to the average utilization rate of the production plant)

			Avg. ΔTC in % (Scenario I)								Avg. ΔTC-JIT in % (Scenario II)								
			Set A				Set B					Set A				Set B			
		#	50%	70%	90%	All	50%	70%	90%	All	#	50%	70%	90%	All	50%	70%	90%	All
Total		**648**	**5.5**	**2.2**	**0.4**	**2.7**	**7.4**	**5.0**	**2.2**	**4.9**	**324**	**15.2**	**7.0**	**1.8**	**8.0**	**15.1**	**10.8**	**5.3**	**10.4**
CV	0.1	324	4.3	2.3	0.4	2.3	7.6	5.5	2.2	5.1*	162	13.9	7.4	1.9	7.8*	14.5	11.4	5.8	10.6*
	0.5	324	6.6	2.1	0.3	3.0	7.1	4.5	2.1	4.6*	162	16.4	6.7	1.6	8.3*	15.6	10.2	4.9	10.2*
Q	1	216	3.7	2.1	0.5	2.1*	5.1	4.1	2.2	3.8*	108	10.9	6.4	5.0	7.4	13.0	10.9	7.1	10.3
	3	216	6.3	2.2	0.3	2.9*	8.5	5.5	2.2	5.4	108	17.3	7.4	0.2	8.3	16.1	10.7	4.4	10.4
	5	216	6.4	2.3	0.3	3.0*	8.5	5.5	2.1	5.4	108	17.3	7.4	0.2	8.3	16.1	10.7	4.5	10.4
sc_j	100	216	6.8	3.0	0.5	3.4	7.7	4.9	2.0	4.9*	108	20.1	8.3	2.0	10.1	16.8	11.1	5.6	11.2
	250	216	5.5	2.6	0.4	2.9	7.7	4.8	2.5	5.0	108	14.2	8.7	1.7	8.2	16.0	10.6	5.8	10.8
	500	216	4.1	1.0	0.2	1.8*	6.7	5.4	1.8	4.6	108	11.3	4.1	1.7	5.7*	12.4	10.7	4.5	9.2*
h^r_i	0.25	324	1.3	0.5	0.1	0.7*	3.0	2.2	0.7	2.0*	-	-	-	-	-	-	-	-	-
	1	324	9.6	3.9	0.6	4.7*	11.8	7.8	3.9	7.8*	-	-	-	-	-	-	-	-	-
h^f_j	2	324	6.9	3.4	0.6	3.6*	10.4	7.6	3.4	7.1*	162	15.1	9.0	2.0	8.7*	19.1	14.8	6.4	13.5*
	5	324	4.0	1.1	0.1	1.7*	4.4	2.4	0.9	2.6*	162	15.3	5.1	1.6	7.3*	11.0	6.8	4.3	7.3*
tc^{var}	2	216	4.7	1.4	0.1	2.1*	5.6	3.1	1.2	3.3*	108	9.6	4.3	1.9	5.3*	9.3	5.5	3.3	6.0*
	5	216	6.0	2.5	0.4	3.0	7.6	5.2	2.3	5.0*	108	14.9	6.5	1.8	7.7*	14.3	10.0	5.0	9.8*
	10	216	5.7	2.7	0.5	3.0	8.9	6.8	3.0	6.2*	108	21.0	10.4	1.7	11.0*	21.6	16.9	7.7	15.4*
tc^{fix}	200	216	5.2	2.2	0.3	2.6	6.7	4.9	2.5	4.7	108	12.6	5.6	1.1	6.4*	12.5	9.7	5.0	9.0*
	500	216	5.5	2.2	0.3	2.7	7.4	4.9	2.1	4.8	108	14.9	6.9	1.8	7.9*	14.7	10.6	5.2	10.2*
	1,000	216	5.6	2.2	0.4	2.7	8.0	5.2	1.9	5.0	108	18.1	8.6	2.5	9.7*	18.0	12.2	5.8	12.0*

**according to the Wilcoxon signed-rank test with $p<0.01$, the mean rank significantly differs from the mean rank of the other (two) related sample(s)*

Tests were performed on a Linux cluster equipped with Intel Xeon E5-2687W Processors 3.1 GHz and 256 GB RAM using the commercial MIP-solver IBM ILOG CPLEX 12.5 limited to a single thread. The computation time was uniformly set for all (sub-) problems to one hour. Due to the small problem size, we were able to obtain optimal solutions for the majority of the test instances using CPLEX. In order to improve the solution quality, we added valid inequalities proposed for the IRP (see Appendix A.2 Valid inequalities). All test instances of the smaller data set A were solved to optimality. For the larger data set B, solutions of the CLSVRP and CLSVRP^{JIT} show an average integrality gap of 2.1% and 0.9%, respectively.

The average relative cost savings ΔTC for scenario I and II are summarized in Table 5. Results are broken down according to the respective problem parameters to emphasize their impact on ΔTC. In order to verify whether the observed impact is statistically significant, the Wilcoxon signed-rank (WSR) test, a nonparametric test for paired data, is applied. If the WSR test yields a p-value smaller than 1%, the parameter is considered as a significant influence factor for ΔTC which is indicated by an asterisk (*). More precisely, we asterisk the mean of a parameter if the observations for ΔTC of that parameter group significantly differ from all observations of the other parameter group(s). For instance, when studying the impact of the vehicle capacity parameters $Q = 1$, $Q = 3$, $Q = 5$, an asterisk for $Q = 1$ implies that the mean of parameter group $Q = 1$ differs significantly from the mean of the other parameter groups $Q = 3$ and $Q = 5$.

Let us first discuss Scenario I.

Problem size (A and B)

Comparing the averaged results of the smaller data set A with those of the larger data set B, we clearly see that the value of coordination grows as problem size grows. Increasing problem size goes along with higher flexibility, and thus, improves the possibility to establish a more efficient supply-production plan. The positive correlation between problem size and ΔTC is supported by the WSR test signifying that the average ΔTC of data set B is larger than of data set A.

Production capacity (utilization 50%, 70%, 90%)

A similar explanation as for problem size applies to the production capacity. If capacity constraints become loose, it is more likely to find a better inventory-routing plan that is feasible for production. The manufacturer can decide more freely whether to keep inventories of input materials or end items when finding a more efficient supply schedule. The WSR test confirms the strong interdependence between ΔTC and production capacity.

Vehicle capacity (Q)

In many cases, cost savings result from consolidating supply quantities into full truckload shipments. Vehicle capacity represents an upper limit stating to what extend freight consolidation is maximum possible. Thus, ΔTC by tendency increase as capacity increases. Nevertheless, if vehicle capacity exceeds a certain amount no longer restricting the optimal solution (in terms of vehicle capacity), no further improvements can be realized. This explains why in our experiments an increment from factor 3 to 5 achieves only non-significant improvements.

Demand variation (CV)

We observed that setups occur less frequent if demand is high (CV of 50%) as if demand is stable (CV of 10%) over time. Again, less frequent setups promote freight consolidation in the integrated and sequential solution approach likewise such that gains from integrated planning are smaller. Therefore, coordination seems less beneficial if demand variation is high. Except for the small data set A, the WSR test supports our hypothesis that ΔTC tend to be higher if the demand is constant. In data set A, the planning horizon is only 5 periods which might explain the insignificance of the CV for this particular case.

Setup cost (sc_j)

Increasing setup costs commonly lead to a reduction of setup operations in the optimal solution. Less frequent setups imply less frequent supply which contributes to the consolidation of shipments even in sequential planning. Consequently, ΔTC exhibit a decreasing trend if setup costs rise (e.g. data set A). Simultaneously, we observe an opposing effect, i.e. ΔTC increase if setup costs rise (data set B). This is particularly true if holding costs of end items h_j^f are high. Table 6 underlines the predominance of h_j^f over the setup costs when evaluating ΔTC. Since an integrated approach benefits from a reduction of setup operations, ΔTC might be small if the number of setups in the sequential and integrated plan do not noticeably differ. Given high holding costs h_j^f but low setup costs, a reduction of setups might not be desirable even for the integrated approach unless setup costs become predominant. The WSR test corroborates our assumption of opposing effects.

Table 6: Cost savings ΔTC (in %) subject to setup and unit holding cost of end items

Avg. ΔTC % (Scenario I)		*sc* 100	250	500	*Avg. ΔTC^{JIT} % (Scenario II)*		*sc* 100	250	500
hc^f	2	7.1	5.7	3.2	hc^f	2	15.6	11.6	6.1
	5	1.1	2.2	3.2		5	5.7	7.5	8.8

Holding cost (h^r_i, h^f_j)

Substantial cost savings can be expected if the ratio between holding costs of input materials and end items $h_i^r : h_j^f$ is high, which means that the storage of input materials is relatively expensive compared to end items (see Table 7). Given that h_i^r is high, stocking larger amounts of input materials is not desirable. In combination with low h_j^f, a coordinated plan balancing transportation and system holding costs, i.e. by shifting inventories downstream the supply chain, offers promising cost advantages. If, by contrast, h_i^r is low, the sequential approach may yield satisfactory results since stock-keeping improves the chance to operate vehicles at full capacity. The WSR test fortifies the strong influence of holding costs for both input materials and end items on ΔTC.

Table 7: Cost savings ΔTC (in %) subject to the holding cost ratio $h^r : h^f$ (Scenario I)

Avg. ΔTC % (Scenario I)	*h^r / h^f* 0.05	0.125	0.2	0.5
Data set A	0.5	2.2	3.9	8.5
Data set B	0.7	3.2	4.6	10.9

Transportation cost (tc^{var}, tc^{fix})

Though interpretation is not straightforward, ΔTC show an upward trend if the proportion of transportation costs on overall costs grows. As the sequential approach determines an optimal production plan first, coordination gains relevance if costs accounting for the supply part outweigh those of the production part. Table 8 breaking down average cost savings subject to variable and fixed transportation cost highlights the positive correlation between tc^{var} and ΔTC. The results of tc^{fix} are not as obvious indicating a second, opposing effect. An explanation might be that there is lower bound on the number of trips (i.e. at some point, no further improvement is possible even if transportation costs further increase). Also in sequential planning, the number of trips can be reduced by accepting larger input material inventories. Now, if $tc^{var}$and tc^{fix} outweigh setup and system holding costs, total costs increase continuously with tc^{fix} such that the relative cost savings decrease. Beyond that, the number of trips does not only depend on tc^{fix} but also tc^{var}, i.e. the impact of tc^{fix} can be minor. The WSR test confirms our hypothesis that there is no strict unidirectional effect visible for tc^{fix}.

Table 8: Cost savings ΔTC (in %) subject to fixed and variable transportation cost

Avg. ΔTC % (Scenario I)		tc^{fix}			*Avg. ΔTC^{JIT} % (Scenario II)*		tc^{fix}		
		200	500	1,000			200	500	1,000
	2	2.0	2.7	3.4		2	3.4	5.4	8.2
tc^{var}	5	3.9	4.0	4.1	tc^{var}	5	7.3	8.5	10.5
	10	4.9	4.7	4.3		10	12.6	13.1	13.9

Let us now focus on Scenario II. According to test results gathered in Table 5, we clearly see that potential gains from integrated planning are remarkably higher if the manufacturer adopts JIT. In Scenario I, ΔTC amount to 2.7% (set A) and 4.9% (set B) while in Scenario II, ΔTC^{JIT} amount to 8.0% (set A) and 10.4% (set B) indicating a strong relation between the expected benefits and the underlying scenario which is not surprising. In a JIT environment, supply and production processes are fully dependent. Changes in the production plan will directly affect the supply schedule causing inefficiencies if supply is ignored in the lot sizing step.

The effect of problem size, production capacity, setup costs, holding costs and transportation costs from Scenario I apply to Scenario II likewise, but effects are reinforced. With JIT supply, transportation costs have a relatively larger impact on ΔTC^{JIT} compared to ΔTC as the manufacturer has no opportunity to respond to rising transportation costs by accepting larger inventories. This might explain why there is a systematic positive correlation between transportation costs, whether tc^{var} or tc^{fix}, and the savings potential which is not visible for Scenario I (see Table 8). Another difference between Scenario I and II is the relevance of vehicle capacity which is not significant as in Scenario I. Given JIT, the supply of raw materials occurs more frequent than in Scenario I, thus, batch sizes tend to be smaller. If we would test on smaller vehicle capacities, e.g. a factor of 0.25 and compare to 0.5, there might be a significant effect recognizable.

5.3 Intermediate scenarios

In addition to Scenario I and II, contrasting uncapacitated and zero inventories of input materials, we now investigate the case of capacitated input material inventories. By doing so, five different capacity levels have been tested on the larger data set B resulting in 5*1,944 additional problems. The maximum stock-level is expressed a percentage of the average demand of input materials has been set to 25%, 50%, 100%, 200%, and 300%. Figure 5 demonstrates ΔTC for intermediate scenarios as well as for Scenario I (stock-level of 900%) and Scenario II (stock-level of 0%), we see that allowing small stocks covering the average dependent demand of two periods, ΔTC reach a steady-state coincident with Scenario I. Hence, even if there is only a limited opportunity to store input materials, under certain conditions (see parameter discussion for Scenario I) the expected benefits of an integrated approach can be low.

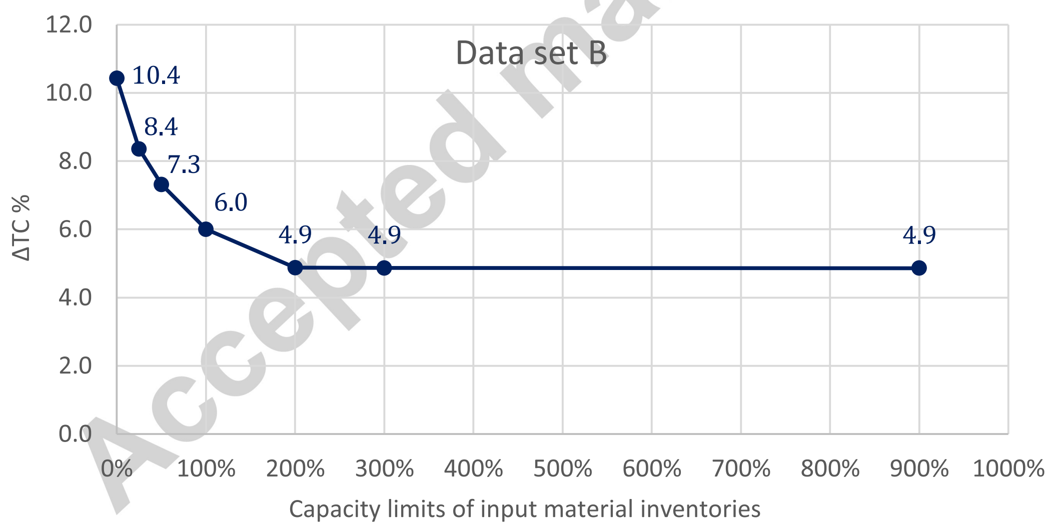

Figure 5: Cost savings ΔTC (in %) of data set B subject to different capacity levels of input material inventories expressed as a percentage of the average dependent demand per period

5.4 Discussion of results with respect to related work

We now compare our findings with Kuhn and Liske (2011) and with those ones reported in papers addressing the production-distribution problem in order to see whether our results are coherent or whether our observations are problem specific. In fact, papers in this domain studying the influence of problem parameters on the value of coordination are rare. Table 9 indicates that our results are to a large extent consistent with the ones obtained by Kuhn and Liske (2011). Due to the extended model used in the present work, we are able to analyze the behavior of the results with respect to additional parameters. Our results show considerably fewer savings than those reported by Kuhn and Liske (2011) and might be due to the simpler modeling approach used.

Table 9: Effects of increasing parameter values on relative cost savings ΔTC

Problem	Problem size	Prod. capacity	Demand var.	Vehicle capacity	Setup cost	Holding cost ratio	Transp. cost	Savings ΔTC
Supply & Production								
CLSVRP	**ΔTC ↑**	**ΔTC ↑**	**ΔTC ↓**	**ΔTC ↑**	**ΔTC ↓**	**ΔTC ↑**	**ΔTC ↑**	**0 - 12%**
Kuhn & Liske (2011)					✓	✓	✓	5 - 90%
Production & Distribution								
Chandra & Fisher (1994)	✓	✓		✓	✓	✓	✓	3 - 20%
Fumero & Vercellis (1999)	✓	✓		✓		✓	✓	8 - 12%

Comparing the results with those reported for production-distribution problems we see a coherent picture, i.e., most of the findings for production-distribution problems seem to be valid for supply-production as well: cost savings increase as problem size, capacity limits and holding cost decrease. Apparently, the effect of setup costs are not as self-evident as argued by Kuhn and Liske (2011) and Chandra and Fisher (1994). Though we agree that there is a tendency visible, i.e., ΔTC decrease as setup costs decrease, we recommend to additionally take holding cost parameters into account when assessing expected benefits. A similar reasoning applies to transportation costs. The positive correlation between ΔTC and rising transportation costs is apparent. Still, it should be noted, that if the opportunity of consolidating shipments is exhausted, a further cost increase, particularly of tc^{fix}, inflates the total costs such that the relative cost advantage diminishes.

6 Conclusion

In this paper we have developed and investigated an integrated model for the supply routing and production planning problem where transportation costs result from vehicle-routing. The combination of IRP and CLSP yields the Capacitated Lot Sizing and Supply Side Vehicle Routing Problem (CLSVRP) for which A MIP formulation has been suggested. For our analysis we distinguished between two scenarios: in Scenario I, raw material inventories are taken into account, while in Scenario II, the storage of raw materials is not possible which is in line with the JIT principle.

Based on the results of 3,888 test instances we could show that an integrated planning approach offers remarkably cost advantages, even with raw material inventories involved. In 72% of the test cases the integrated approach produced better solutions (ΔTC larger than 0) compared to the sequential one, though the magnitude of ΔTC varies greatly among single instances. Since cost advantages mainly result from freight consolidation and enhanced routing, relaxed capacity constraints of production resources and vehicles are prerequisites for finding a more efficient supply schedule that is feasible. Besides, the holding cost ratio $h_i^r : h_j^f$ turns out to be a crucial factor for the savings potential. If the ratio is high, i.e. holding of input materials is relatively costly, they should be already considered in the lot sizing step.

The second key finding is that the cost advantage from joint optimization varies greatly between Scenario I and II. Companies following the JIT principle may expect much higher gains from coordinated planning (here: 200-300% more) compared to Scenario I where material storage serves as buffer. Under Scenario II, in 82% of the test cases integrated planning is superior (ΔTC^{JIT} larger than 0). This outcome stresses the importance of cross-functional cooperation for production planning in JIT environments. Furthermore, with JIT supply, transportation cost parameters gain relevance due to the direct impact on ΔTC^{JIT}. Our experiments emphasize that with or without raw material inventory, an integrated planning approach has a great potential to cut cost. This is coherent as already pointed out in various papers tackling production-distribution planning.

On the basis of small to medium sized test instances, this paper managed to show under which circumstances an integrated planning approach offers remarkably cost advantages and when it may not. Still, an efficient algorithm is needed to be able to solve real-world problems. Clustering techniques to simplify the supply routing problem, a two-phase solution procedure, or other heuristic approaches already developed and successfully applied to IRPs and PRPs might be worth to examine. After having found an efficient algorithm, the next step will be to investigate a real life case to support our findings.

Appendix

A.1 Model formulation with asymmetric transportation cost

The supply-production network of the asymmetric variant, named CLSVRP$_{asym}$, is represented by a complete graph $\mathcal{G} = (\mathcal{V} \cup \{0\}, \mathcal{A})$, where every pair of vertex set $\mathcal{V} \cup \{0\} = \{0,1, \ldots, S\}$ is connected via a unique arc of set $\mathcal{A} = \{(s, s'): s, s' \in \mathcal{V}, s \neq s'\}$. Every arc of set $\mathcal{A}$ is associated with transportation costs $tc_{ss'}$ which may depend on the direction of how the arc is traversed.

The modified notation for routing variables of the CLSVRP$_{asym}$ is as follows:

$x^k_{ss't}$ equals 1, if vehicle k travels from vertex s to s' in period t, 0 otherwise

u^k_{st} auxiliary variable needed for the subtour constraint

Model CLSVRP$_{asym}$:

$$\min \ \sum_{j \in \mathcal{N}} \sum_{t \in \mathcal{T}} sc_j z_{jt} + \sum_{j \in \mathcal{N}} \sum_{t \in \mathcal{T}} h^f_j I^f_{jt} + \sum_{i \in \mathcal{M}} \sum_{t \in \mathcal{T}} h^r_i \, I^r_{it} + \sum_{s,s' \in \mathcal{V} \cup \{0\}} \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} tc_{ss'} \, x^k_{ss't} \tag{A.1}$$

subject to (2) - (7), (12)

$$\sum_{i \in \mathcal{M}} b_{is} \, q^k_{it} \ \leq Q \sum_{\substack{s' \in \mathcal{V} \\ s' \neq s}} x^k_{ss't} \qquad \forall \, t, k, s \tag{A.2}$$

$$\sum_{s' \in \mathcal{V} \cup \{0\}} x^k_{0s't} = \sum_{s \in \mathcal{V} \cup \{0\}} x^k_{s0t} = 1 \qquad \forall \, t, k \tag{A.3}$$

$$\sum_{\substack{s' \in \mathcal{V} \cup \{0\} \\ s' \neq s}} x^k_{ss't} = \sum_{\substack{s' \in \mathcal{V} \cup \{0\} \\ s' \neq s}} x^k_{s'st} \qquad \forall \, t, k, s \tag{A.4}$$

$$u^k_{s't} \geq u^k_{st} + 1 - S(1 - x^k_{ss't}) \qquad \forall \, t, k, s, s' \tag{A.5}$$

$$0 \leq \ u^k_{st} \ \leq \ S \qquad \forall \, t, k, s \tag{A.6}$$

$$z_{jt}, x^k_{ss't} \in \{0,1\} \qquad \forall \, j, t, k, s, s' \tag{A.7}$$

The objective function (A.1) as well as the production part stated in equations (2) - (7) and (12) are identical to the symmetric formulation. Constraints (A.2) ensure that vehicle capacity is not violated. Constraints (A.3) and (A.4) reflect common VRP flow conservation constraints. Inequalities (A.5) known as Miller-Tucker-Zemlin subtour elimination constraint (Miller et al., 1960; Desrochers and Laporte, 1991) prohibit trucks to return to supplier s if already visited on the same tour. The auxiliary decision variable u^k_{st} defined in (A.6) infers the position of vertex s in the tour. Finally, binary decision variables are defined in (A.7).

A.2 Valid inequalities

$\sum_{k \epsilon \mathcal{K}} q_{it}^{k} \leq \sum_{j \epsilon \mathcal{N}} \sum_{\tau=t}^{T-1} a_{ij}\, d_{j,\tau+1}$ $\quad \forall\, i, t$ (A.8)

$x_{0st}^{k} \leq 2 v_{st}^{k}$ $\quad \forall\, t, k, s$ (A.9)

$x_{ss't}^{k} \leq v_{st}^{k}$ $\quad \forall\, t, k, s, s'$ (A.10)

$v_{st}^{k} \leq v_{0t}^{k}$ $\quad \forall\, t, k, s$ (A.11)

$v_{0t}^{k} \leq v_{0t}^{k-1}$ $\quad \forall\, t, k \in \mathcal{K} \setminus \{1\}$ (A.12)

$v_{st}^{k} \leq \sum_{\substack{s' \epsilon \mathcal{V} \\ s' < s}} v_{s't}^{k}$ $\quad \forall\, t, k \in \mathcal{K} \setminus \{1\}, s$ (A.13)

Constraints (A.8) limit the total amount supplied in period t to the aggregated secondary demand for the remainder of the planning horizon. Constraints (A.9) - (A.13) represent logical inequalities established for the Multi Vehicle Inventory Routing Problem (MIRP) by Coelho and Laporte (2013). Constraints (A.9) require that if the depot is the successor (or predecessor) of supplier s on vehicle k's route then s must be visited by the same vehicle. Analogously, constraints (A.10) apply to two successive suppliers s and s'. Constraints (A.11) state that supplier s is only visited by vehicle k in period t if vehicle k leaves the depot in period t. The symmetry breaking constraints (A.12) and (A.13) impose that vehicle k cannot leave the depot in period t unless vehicle $k-1$ is used in t, and that supplier s cannot be visited by vehicle k in period t, if no supplier with a smaller index has been visited by vehicle $k-1$ in t.

References

Absi, N., Archetti, C., Dauzère-Pérès, S., Feillet, D., 2014. A two-phase iterative heuristic approach for the production routing problem. Transportation Science 49, 784–795.

Adulyasak, Y., Cordeau, J.-F., Jans, R., 2013. Formulations and Branch-and-Cut Algorithms for Multivehicle Production and Inventory Routing Problems. INFORMS Journal on Computing 26, 103–120.

Adulyasak, Y., Cordeau, J.-F., Jans, R., 2015. The production routing problem: A review of formulations and solution algorithms. Computers & Operations Research 55, 141–152.

Amorim, P., Meyr, H., Almeder, C., Almada-Lobo, B., 2013. Managing perishability in production-distribution planning: a discussion and review. Flexible Services and Manufacturing Journal 25, 389–413.

Archetti, C., Bertazzi, L., Laporte, G., Speranza, M.G., 2007. A Branch-and-Cut Algorithm for a Vendor-Managed Inventory-Routing Problem. Transportation Science 41, 382–391.

Bard, J.F., Nananukul, N., 2010. A branch-and-price algorithm for an integrated production and inventory routing problem. Computers & Operations Research.

Buschkühl, L., Sahling, F., Helber, S., Tempelmeier, H., 2010. Dynamic capacitated lot-sizing problems: A classification and review of solution approaches. OR Spectrum 32, 231–261.

Campbell, A., Clarke, L., Kleywegt, A., Savelsbergh, M., 1998. The Inventory Routing Problem, in: Crainic, T.G., Laporte, G. (Eds.), Fleet Management and Logistics. Springer US, pp. 95–113.

Chandra, P., Fisher, M.L., 1994. Coordination of production and distribution planning. European Journal of Operational Research 72, 503–517.

Coelho, L.C., Cordeau, J.-F., Laporte, G., 2014. Thirty Years of Inventory Routing. Transportation Science 48, 1–19.

Coelho, L.C., Laporte, G., 2013. The exact solution of several classes of inventory-routing problems. Computers & Operations Research 40, 558–565.

Desrochers, M., Laporte, G., 1991. Improvements and extensions to the Miller-Tucker-Zemlin subtour elimination constraints. Operations Research Letters 10, 27–36.

Díaz-Madroñero, M., Peidro, D., Mula, J., 2015. A review of tactical optimization models for integrated production and transport routing planning decisions. Computers & Industrial Engineering 88, 518–535.

Elmaghraby, S.E., 1978. The Economic Lot Scheduling Problem (ELSP): Review and Extensions. Management Science 24, 587–598.

Erengüc, S.S., Simpson, N.C., Vakharia, A.J., 1999. Integrated production/distribution planning in supply chains: An invited review. European Journal of Operational Research 115, 219–236.

Fahimnia, B., Farahani, R.Z., Marian, R., Luong, L., 2013. A review and critique on integrated production–distribution planning models and techniques. Journal of Manufacturing Systems 32, 1–19.

Fischetti, M., González, J.J.S., Toth, P., 1998. Solving the Orienteering Problem through Branch-and-Cut. INFORMS Journal on Computing 10, 133–148.

Fumero, F., Vercellis, C., 1999. Synchronized Development of Production, Inventory, and Distribution Schedules. Transportation Science 33, 330–340.

Ghiani, G., Laporte, G., Musmanno, R., 2004. Introduction to Logistics Systems Planning and Control. John Wiley & Sons, Ltd.

Kuhn, H., Liske, T., 2011. Simultaneous supply and production planning. International Journal of Production Research 49, 3795–3813.

Kuhn, H., Liske, T., 2014. An exact algorithm for solving the economic lot and supply scheduling problem using a power-of-two policy. Computers & Operations Research 51, 30–40.

Laporte, G., 1992. The vehicle routing problem: An overview of exact and approximate algorithms. European Journal of Operational Research 59, 345–358.

Lee, C.-G., Bozer, Y.A., White III, C.C., 2003. A heuristic approach and properties of optimal solutions to the dynamic inventory routing problem. Working Paper. University of Toronto, Faculty of Applied Science and Engineering.

Lei, L., Liu, S., Ruszczynski, A., Park, S., 2006. On the integrated production, inventory, and distribution routing problem. IIE Transactions 38, 955–970.

Miller, C.E., Tucker, A.W., Zemlin, R.A., 1960. Integer Programming Formulation of Traveling Salesman Problems. J. ACM 7, 326–329.

Quadt, D., Kuhn, H., 2008. Capacitated lot-sizing with extensions: a review. 4OR 6, 61–83.

Sarmiento, A.M., Nagi, R., 1999. A review of integrated analysis of production-distribution systems. IIE Transactions 31, 1061–1074.

Tempelmeier, H., Derstroff, M., 1996. A Lagrangean-based heuristic for dynamic multilevel multi-item constrained lotsizing with setup times. Management Science 42, 738–757.

Thomas, D.J., Griffin, P.M., 1996. Coordinated supply chain management. European Journal of Operational Research 94, 1–15.

Toth, P., Vigo, D., 2002. The Vehicle Routing Problem. SIAM Monographs on Discrete Mathematics, Philadelphia.

Vidal, C.J., Goetschalckx, M., 1997. Strategic production-distribution models: A critical review with emphasis on global supply chain models. European Journal of Operational Research 98, 1–18.

Wagner, H.M., Whitin, T.M., 1958. Dynamic version of the economic lot size model. Management Science 5, 89–96.