The Hybrid Vehicle Routing Problem

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Abstract

In this paper the Hybrid Vehicle Routing Problem (HVRP) is introduced and formalized. This problem is an extension of the classical VRP in which vehicles can work both electrically and with traditional fuel. The vehicle may change propulsion mode at any point of time. The unitary travel cost is much lower for distances covered in the electric mode. An electric battery has a limited capacity and may be recharged at a recharging station (RS). A limited number of RS are available. Once a battery has been completely discharged, the vehicle automatically shifts to traditional fuel propulsion mode. Furthermore, a maximum route duration is imposed according to contracts regulations established with the driver. In this paper, a Mixed Integer Linear Programming formulation is presented and a Large Neighborhood Search based Matheuristic is proposed. The algorithm consists into destroying, starting from an initial solution, at each iteration a small number of routes, letting unvaried the other ones, and reconstructing a new feasible solution running the model on only the subset of customers involved in the destroyed routes. This procedure allows to completely explore a large neighborhood within very short computational time. Computational tests that show the performances of the matheuristics are presented. The method has also been tested on a simplified version of the HVRP already presented in the literature, the Green Vehicle Routing Problem (GVRP), and very good results have been obtained.

Keywords:
Routing, Hybrid Vehicles, Refueling station, Matheuristic, Mixed Integer Linear Programming

1. Introduction

In the last few years, the greenhouse effect has become a hot political topic throughout the world and laws and regulations have been adopted to reduce pollution emissions in all of the highly developed countries. Such political decisions have had an important effect on the logistics industry. Many logistics companies have adopted *Green Logistics* projects, to reduce CO2 emissions. A reduction in pollution can be obtained in two different ways: through a better exploitation current resources, and by using new, environmental friendly technologies. The first step in this process is to make a better use of available resources. This could be obtained by applying more efficient and sophisticated routing planning optimization methods and adopting smart distribution systems, [16], [17], which would help to decrease the traveling distance of vehicles and hence emissions. However, this generally results in a decline of emissions of only a few percent and the emission level of trucks and vans remains high. A more promising strategy is the use of *zero emission* electric battery vehicles. Although this emerging technology is very attractive from an environmental point of view, it has not been adopted extensively due the limited capacity of the batteries, which only allow very short driving ranges. In most cases, vehicles need to be recharged along the delivery route, with a consequent loss of time. Furthermore, recharging stations (RS) are frequent on road networks; therefore, recharging stops should be a priori included in the routing planning in order to prevent drivers from coming to a standstill, without the minimum level of battery necessary to reach the nearest recharging station. As a result, long deviations from the original path could be necessary to include recharging stops, thus significantly increasing the total distance traveled by the vehicle and the route duration. This limitation could be overcome using hybrid vehicles, which can work both with traditional fuel and electric propulsion. The electric battery could be exploited on short routes (which can be performed without recharging the battery) or in zones in which recharging stations may be easily reached, while traditional fuel propulsion could be used in cases in which a visit to a recharging station implies a long deviation, or when the electric battery would allow almost the whole route to be covered. In fact, in this cases, it would be more advantageous to cover a few kilometers with a traditional fuel engine than to plan a battery recharging stop. The use of hybrid vehicles could constitute a fair compromise between economic interest and environmental issues.

2. Literature Review

In the last few years, increasing attention has been paid to Green Logistics, which involves the integration of environmental aspects in logistics. Many papers concerning Operations Research applications to Green Logistics have been proposed in the literature. A wide set of issues has been addressed, such as intermodal transportation, mode choice models, fleet choice and exploitation, smart distribution systems and fuel choice. For a complete survey on this subject, the readers may refer to [6].

One emerging research area concerns pollution emission minimization. In [3], the authors introduced the Pollution-Routing Problem (PRP), an extension of the classical Vehicle Routing Problem with Time Windows, which consists in routing a number of vehicles to serve a set of customers, and determining their speed on each route segment in order to minimize a function that includes fuel, emission and driving costs. The same problem has been addressed in [7] where an Adaptive Large Neighborhood Search based heuristics approach is proposed. A time-dependent version of the PRP has been addressed in [11], while [8] introduced the bi-level pollution routing problem.

In the nineties and the first decade of the 21st century, refineries focused on removing lead additives from gasoline, in order to preserve air quality. Biofuels based on organic waste can easily be mixed with standard gasoline. However, this technologies involves the necessity of adapting engines, which can be quite rather expensive. Electric vehicles are environmentally friendly, since their engines emit almost no emissions. However, due their limited autonomy, they are more popular for the movements of in-city goods than for medium-long range freight transport. In order to compensate for their short range, a dense power re-supply network would need to be set-up, possibly in conjunction with the possibility of changing batteries. Unfortunately, the present re-supply network is still very limited. Several works have been conducted related to power recharging and to location optimization of supply stations, ([20], [13, 14], [23], [15] and [2], while some earlier works may be found in the literature, that focused on military applications and considered issues related to the limited capacity of fuel tanks ([18], [19], [24] and [25]). Only a few papers have actually dealt with VRPs pertaining alternative fuel vehicles. In [12], the authors dealt with a VRP concerning Pickup and Delivery (VRPPD) with a mixed fleet that consisted of both electric and traditional fueled vehicles. The objective was to minimize the total costs, which consist of vehicle related fixed and variable costs. They considered time and capacity constraints and simply added an extra time for recharging the electric vehicles batteries, where needed. However, they did not explicitly consider recharging stations location in their model. Therefore, the problem resulted in a mixed fleet VRPPD with an additional distance-dependent time variable. In [5] the recharging vehicle routing problem (RVRP) was introduced, in which vehicles were allowed to recharge directly at customer locations, adding a time penalty to the route duration. Erdogan and Miller-Hooks, [9] were the first to combine a VRP with the possibility of refueling a vehicle at a station along the route. They considered a limited number of refueling infrastructures located along the network and a fixed recharging time (independent of the remaining level of battery charge). The proposed Green Vehicle Routing Problem (GVRP), considers a maximum route duration and fuel constraint. Fuel is consumed with a given rate per traveled distance and is tank is totally replenished at recharging stations. Schneider et al., [22], present the Electric Vehicle Routing Problem with Time Windows and Recharging Stations (E-VRPTW), which can be seen as an extension of the GVRP in which time windows are considered. An extension of the E-VRPTW, where partial recharging are allowed, has been studied in [4]. Finally, multiple recharging technologies, characterized by different recharging times and costs, have been introduced in [10].

3. Problem description

The Hybrid Vehicle Routing Problem (HVRP) consists in visiting a set of customers, I, starting from a depot v_0. The available fleet is composed of M identical vehicles. Since multiple trips are not allowed, at most M routes may be scheduled. Each vehicle must start and end its route at the depot. Each node j is characterized by a service time p_j. Each pair of customers, (i, j), is associated with a travel time t_{ij} and a distance d_{ij}. The Travel speeds are assumed to be constant over a link. In addition, no limit is set on the number of stops that can be made for refueling, but each refueling station may be visited only once. When refueling is undertaken, it is assumed that the tank is filled up to capacity. The routes start and end at the depot and must contain visits to refueling stations, if they are necessary. Each route may be covered using only the electric engine, or a portion of it can be covered using traditional fuel propulsion. The distance covered by a vehicle in traditional fuel mode, over the link connecting to customer j is defined as

W_j. A distance unitary additional penalty cost, ρ, is added when traditional fuel propulsion is used in order to dissuade this mode and to promote the use of the electric engine. A limit on the duration of the routes, T_{max}, is imposed.

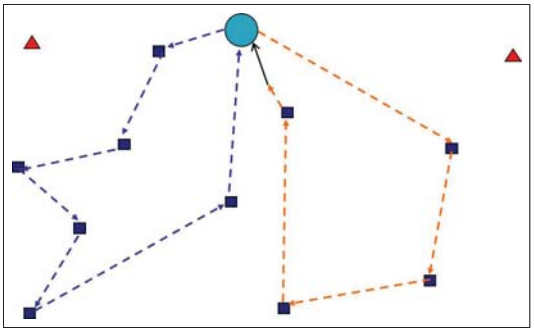
An example on how the optimal routes of a VRP, GVRP and HVRP may differ (from each other) is reported in Figure 1. More in detail, given its limited autonomy, the electric vehicle must perform a very long deviation from the optimal VRP route, in order to reach a refueling station, while the hybrid vehicle may cover the same route as in the VRP, but must cover a small portion of the path with traditional fuel propulsion, thus paying an extra cost which is still much lower than the cost of reaching the refueling station.

The additional notations used in formulating the HVRP are defined hereafter.

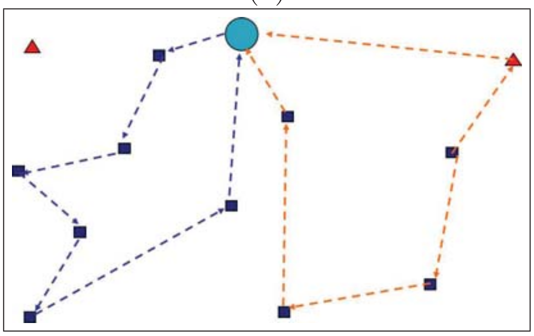
- I: Set of customers
- I_0: Set of customers and the depot $I \cup v_0$
- F: Set of refueling stations
- V: Set of nodes without the depot $I \cup F$
- V_0: Set of nodes $I_0 \cup F$
- r: Battery charge consumption rate (for km) (for electric propulsion)
- Q: Battery capacity
- ρ: Distance unitary penalty for using traditional fuel propulsion
- M: Maximum number of routes
- μ_j: Minimum distance from j to the nearest refueling stations or to the depot

The variables of the problem are now described.

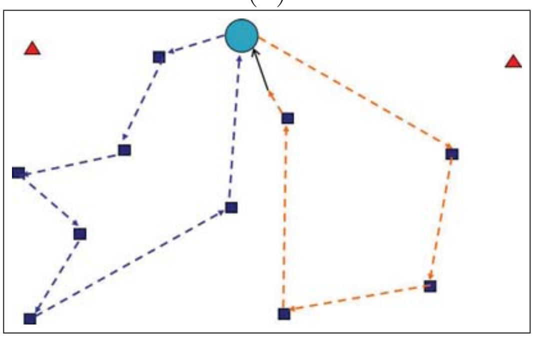
- X_{ij}: Binary variable equal to 1 if a vehicle travels from node i to node j and 0 otherwise
- Y_j: Battery level upon arrival at node j



(a)



(b)



(c)

Figure 1: Optimal Routes for VRP (a) ,GVRP (b) and HVRP (c)

- T_j: Arrival time at node j

The mathematical formulation of the HVRP is the following:

$$\min \sum_{i \in V_0} \sum_{j \in V_0} d_{ij} X_{ij} + \sum_{j \in V_0} \rho W_j \tag{1}$$

s.t.

$$\sum_{j \in V j \neq i} X_{ij} = 1 \quad \forall\, i \, \in \, V \tag{2}$$

$$\sum_{j \in V_0 j \neq i} X_{ij} \leq 1 \quad \forall\, i \, \in \, F \tag{3}$$

$$\sum_{i \in V_0 i \neq j} X_{ji} = \sum_{i \in V_0 i \neq j} X_{ij} \quad \forall\, j \, \in \, V_0 \tag{4}$$

$$\sum_{j \in V_0 j \neq 0} X_{0j} \leq M \tag{5}$$

$$\sum_{j \in V_0 j \neq 0} X_{j0} \leq M \tag{6}$$

$$T_j \geq T_i + (t_{ij} + p_j) X_{ij} - T_{max}(1 - X_{ij}) \quad \forall\, i \, \in \, V_0 \, j \, \in \, V \, and \, i \neq j \tag{7}$$

$$0 \leq T_0 \leq T_{max} \tag{8}$$

$$t_{0j} \leq T_j \leq T_{max} - (t_{j0} + p_{ij}) \quad \forall\, j \, \in \, V \tag{9}$$

$$Y_j \leq Y_i - r \cdot d_{ij} + Q(1 - X_{ij}) \quad \forall j \, \in \, I \, i \, \in \, V_0 \, and \, i \neq j \tag{10}$$

$$Y_j = 0 \quad \forall j \, \in \, F \cup v_0 \tag{11}$$

$$Y_j \geq r * (\mu_j - W_j) \quad \forall j \, \in \, I \tag{12}$$

$$X_{ij} \in 0,1 \quad \forall i \, \in \, V_0 \forall j \, \in \, V_0 \tag{13}$$

The objective function is defined in (1). Constraint (2) implies that each customer is visited exactly once, while each refueling station may be visited only once, as stated in (3). Route continuity is guaranteed by constraint (4) while a maximum number of routes is imposed by constraints (5) and (6). The arrival time at each node is tracked by constraint (7). Constraints (8) and (9) make certain that each vehicle returns to the depot no later than a given timelimit, T_{max}. The battery charging level, upon arrival at each node, is ruled by constraint (10). The time and battery charge level tracking constraints, constraints (7) and (10), respectively, exclude the possibility of subtours. Constraint (11) set the value of battery charge equal to Q at the departure from the depot and resets it up to Q upon a visit to a recharging station. Constraint (12) implies that if the remaining battery charge is not enough to reach the depot, or a recharging station, the remaining distance is covered at a higher cost, by means of traditional fuel propulsion. Since the autonomy of traditional fuel tank is much larger than that of electric batteries, and since fuel stations are located frequently along the road network, no capacity restriction is considered for fuel tanks. Finally, the domain of the variables is defined by constraint (13).

4. A Large Neighborhood Search based Matheuristics for the HVRP

Large Neighborhood search heuristics, (LNS), belongs to the class of heuristics known as Very Large Scale Neighborhood search (VLSN) algorithms, as stated in [1]. All VLSN algorithms are based on the observation that searching a large neighborhood results in finding local optima of high quality, and hence a VLSN algorithm may return better solutions overall. However, searching a large neighborhood is very time consuming, hence various filtering techniques are used to limit the search. In VLSN algorithms, the search is usually restricted to a subset of the solutions belonging to the neighborhood which can be efficiently explored. Unlike what happens in other VLSN, the neighborhood is implicitly defined, in LNS, by the moves used to destroy and repair an incumbent solution. For a detailed survey on LNS applications to routing problems the reader may refer to [21]. The destroy operators may be defined in different ways. For routing problems, for instance, a destroy operator could consist in breaking k routes leaving the others unvaried, or in removing a fixed percentage of the arcs in the current solution. A random (or randomized) component is used to select the arcs that have to be removed. The repair method rebuilds a feasible solution starting from the partially destroyed one. Generally, a greedy construction heuristic is used to rebuild the solution. This is a very fast but not always very accurate method, since only a sample solution is analyzed in the neighborhood. The innovative aspect of the LNS proposed in this paper concerns the possibility of addressing the whole neighborhood in reasonably short computational time. In fact, the large neighborhood search is exploited directly by the model. In this way, it is possible to obtain the local minimum with respect to the addressed neighborhood, which renders the intensification phase of the algorithm more powerful and precise. A detailed explanation of how the proposed matheuristic works is given in the following subsection.

4.1. Algorithm description

The proposed Matheuristic, from now on called MH, works as follows. The procedure starts from a feasible solution. Two routes are destroyed at each iteration and th model is run again, with a short time-limit (i.e. 10 seconds) only on those customers who/that were previously involved in those routes, while the other routes are left unvaried. As the number of customer involved is small, the model is able to solve the problem to optimality (or almost to optimality) in a very short time. The procedure terminates when no further improvement can be found from destroying only two routes, or when a maximum number of iterations is reached.

The idea is to apply a classical destroy operator while exploiting the model to efficiently explore a large neighborhood of solutions instead of reconstructing a new feasible solution with a greedy procedure, as is normal practice in LNS algorithms. In this way, the reconstructed solution quality results to be much higher, because there is the possibility of exhaustively (or almost) exploring the neighborhood within a few seconds and of easily reaching the local minimum in the neighborhood, while in classical LNS algorithms, only a small part of the neighborhood is explored. The destroy operator implies a strong perturbation on the solution to ensure diversification in the search process.

Let us introduce additional notations:

- $R = r_1, r_2, ...r_n$: set of routes in the current solution
- $R_{best} = r_1, r_2, ...r_n$: set of routes in the best solution
-

A pseudocode of the algorithm is reported hereafter:

Algorithm 1 **A Large neighborhood search matheuristics for the HVRP**

set iter=0
set $S_b = S_0$
for $a = 0; a \leq n; a++$ **do**
  for $b = 0; b \leq n; b++$ **do**
    if $iter \leq MAXITER$ **then**
      set $R = R_{best}$
      destroy routes r_a and r_b
      run the model with 2 vehicles and considering as customers only customers that were involved in r_a and r_b
      obtain two new routes r'_a and r'_b
      if the sum of the cost of r'_a and $r'_b \leq$ the sum of the cost of r_a and r_b **then**
        substitute r_a and r_b with r'_a and r'_b in R
        set $R_{best} = R$
        set $a = 0$ and $b = 0$
      end if
      set iter=iter+1
    end if
  end for
end for

5. Computational results

In order to prove the efficiency of the proposed algorithm, it has been tested on a special case of the HVRP, the Green Vehicle Routing Problem (GVRP), for which benchmark results may be found in the literature.

5.1. A particular case: the GVRP

The GVRP can be considered a special case of the HVRP in which the use of traditional fuel propulsion is not allowed, i.e. the fleet is composed of pure electric propulsion vehicles. This further constraints can be easily added to the HVRP model by forcing all the W_j variables to zero. This problem has been introduced in [9], where two constructive heuristics were proposed. The first one is a Modified Clarke and Wright Savings heuristic, while the second one is a Density-Based Clustering Algorithm. In the following, the heuristics are referred to as *Erdogan1* and *Erdogan2*, respectively. In [22], the authors have presented a Variable Neighborhood search heuristic combined with a Tabu Search while a non-deterministic Simulated Annealing has been proposed in [10]. Although these heuristics were originally proposed for two different extensions of the GVRP, they were also tested on the GVRP instances introduced by [9]; therefore it is possible to compare the MH results with those obtained by them.

Computational tests have been carried out on 4 instance sets proposed in [9]:

- S1: uniform customer distribution - 10 randomly generated instances of 20 uniformly distributed customers with 3 recharging station locations
- S2: clustered customer distribution - 10 randomly generated instances of 20 clustered customers with 3 recharging station locations
- S3: Impact of the spatial recharging station configuration - 10 instances, half selected from S1 and half from S2, each instance with 6 recharging stations randomly generated
- S4: Impact of station density - 10 instances, half of which have been created from one instance of S1 and half from one instance of S2, while gradually increasing the number of recharging station from 2 to 10 in increments of 2

The initial solution used as the starting point for the MH is the best solution that can be obtained when the model is run with a timelimit of 5 seconds.

Details of the results obtained with MH on the above described sets are reported in Table 1-4. The tables are organized as follows. The first column report the name of the instance while columns 2 and 3 report the objective function obtained with *Erdogan1* and *Erdogan2*. Columns 4 and 5 report, report the objective function and computational time (expressed in seconds) for *Schneider* while the same data, for *Felipe* and for the MH proposed in this paper, are reported in columns 6-7 and 8-9 respectively. The second last row reports the average values, while the percentage gap between MH and the other heuristics is reported in the last row. The gap is computed as $(S_h - S_{MH})/S_{MH}$, where S_h represents the value of the objective function of the solution obtained with the h_{th} heuristic and S_{MH} the value of the one obtained by MH, where h varies in $[1, 4]$. In this way, positive

INSTANCE	Erdogan1	Erdogan2	Schneider	TIME	Felipe	TIME	MH	TIME
20c3sU1	1818.35	1797.51	1797.49	41	1805.41	1.64	1797.49	11
20c3sU2	1614.15	1613.53	1574.77	38	1574.78	1.48	1574.77	30
20c3sU3	1969.64	1964.57	1704.48	38	1704.48	1.47	1704.48	20
20c3sU4	1508.41	1487.15	1482	39	1482	1.53	1482	4
20c3sU5	1752.73	1752.73	1689.37	40	1689.37	2.15	1689.37	23
20c3sU6	1668.16	1668.16	1618.65	40	1618.65	1.9	1618.65	3
20c3sU7	1730.45	1730.45	1713.66	38	1713.67	2.09	1713.9	20
20c3sU8	1718.67	1718.67	1706.5	40	1722.78	1.76	1738.04	13
20c3sU9	1714.43	1714.43	1708.81	40	1708.82	2.14	1708.81	14
20c3sU10	1309.52	1309.52	1181.31	38	1181.31	1.44	1181.31	0.2
AVG	1680.451	1675.672	1617.704	39.2	1620.127	1.76	1620.882	13.82
	3.68%	3.38%	-0.20%		-0.05%			

Table 1: GVRP: Comparison of results on set S1

gaps indicate that MH is outperforming the related heuristics. Erdogan and Miller-Hooks, [9], do not provide explicit data about the computational time for each instances but they declare that they are of the order of seconds. All the algorithms have been run on machines with similar computational performances.

As shown in the resume reported in Table 5, MH clearly outperforms *Erdogan1* and *Erdogan2*, obtaining better solutions for most instances and providing an average saving of around 12%, while a very similar performance is observed with respect to *Schneider* and *Felipe*.

5.2. The general case: The HVRP

Since the HVRP has been introduced for the first time in this paper, no benchmark instances are available from the literature. Therefore, a new set of instances has been generated. This set, named SH, is composed of instances of 30, 50 and 75 customers. For each size of customers 3 instances have been constructed, with different spatial location of the refueling stations:

- RS located within the customers area
- RS located around the customers area

INSTANCE	Erdogan1	Erdogan2	Schneider	TIME	Felipe	TIME	MH	TIME
20c3sC1	1300.62	1300.62	1173.57	37	1178.97	1.5	1178.97	20
20c3sC2	1553.53	1553.53	1539.97	35	1539.97	1.3	1567.82	21
20c3sC3	1083.12	1083.12	880.2	15	880.2	0.58	898.32	6
20c3sC4	1135.9	1091.78	1059.35	32	1059.35	1.11	1109.73	13
20c3sC5	2190.68	2190.68	2156.01	36	2156.01	1.42	2156.01	13
20c3sC6	2883.71	2883.71	2758.17	43	2758.17	1.17	2758.17	30
20c3sC7	1701.4	1701.4	1393.99	11	1393.99	0.28	1393.99	1
20c3sC8	3319.74	3319.74	3139.72	37	3139.72	1.28	3139.72	55
20c3sC9	1811.05	1811.05	1799.94	36	1799.94	1.2	1799.94	24
20c3sC10	2648.84	2644.11	2583.42	27	2583.42	1.11	2583.42	11
AVG	1962.859	1957.974	1848.43	30.84	1848.97	1.10	1858.609	19.4
	5.61%	5.35%	-0.55%		-0.52%			

Table 2: GVRP: Comparison of results on set S2

INSTANCE	Erdogan1	Erdogan2	Schneider	TIME	Felipe	TIME	MH	TIME
S1_2i6s	1614.15	1614.15	1578.12	43	1578.12	1.53	1578.12	30
S1_4i6s	1561.3	1541.46	1397.27	45	1413.97	1.64	1397.27	9
S1_6i6s	1616.2	1616.2	1560.49	44	1571.3	1.73	1597.31	15
S1_8i6s	1902.51	1882.54	1692.32	44	1692.33	1.73	1692.32	14
S1_10i6s	1309.52	1309.52	1173.48	43	1173.48	1.5	1173.48	3
S2_2i6s	1645.8	1645.8	1633.1	45	1645.8	1.64	1645.8	25
S2_4i6s	1505.06	1505.06	1532.96	53	1505.07	1.34	1544.98	5
S2_6i6s	3115.1	3115.1	2431.33	47	2660.49	2.42	2431.33	25
S2_8i6s	2722.55	2722.55	2158.35	34	2175.66	1.15	2175.66	16
S2_10i6s	1995.62	1995.62	1958.46	37	1585.46	1.11	1635.78	9
AVG	1898.781	1894.8	1711.588	43.38	1700.17	1.58	1687.205	15.1
	12.54%	12.30%	1.45%		0.77%			

Table 3: GVRP: Comparison of results on set S3

INSTANCE	Erdogan1	Erdogan2	Schneider	TIME	Felipe	TIME	MH	TIME
S1_4i2s	1582.2	1582.2	1582.21	38	1598.91	1.56	1582.21	14
S1_4i4s	1580.52	1580.52	1460.09	41	1483.19	1.58	1460.09	40
S1_4i6s	1561.29	1541.46	1397.27	45	1413.97	1.64	1397.27	9
S1_4i8s	1561.29	1561.29	1397.27	49	1397.27	1.62	1397.27	19
S1_4i10s	1536.04	1529.73	1396.02	51	1396.02	1.67	1396.02	50
S2_4i2s	1135.89	1117.32	1059.35	31	1059.35	1.12	1059.35	6
S2_4i4s	1522.72	1522.72	1446.08	36	1446.08	1.31	1446.08	8
S2_4i6s	1786.21	1730.47	1434.14	41	1434.14	1.37	1434.14	5
S2_4i8s	1786.21	1786.21	1434.14	45	1434.14	1.39	1434.14	25
S2_4i10s	1783.63	1729.51	1434.13	47	1434.13	1.4	1434.13	40
AVG	1583.6	1568.143	1404.07	42.36	1409.72	1.47	1404.07	21.6
	12.79%	11.69%	0.00%		0.40%			

Table 4: GVRP: Comparison of results on set S4

SET	Erdogan1	Erdogan2	Schneider	Felipe	MH
S1	1680.45	1675.67	1617.70	1620.13	1620.88
S2	1962.86	1957.97	1848.43	1848.97	1858.61
S3	1898.78	1894.80	1711.59	1700.17	1687.21
S4	1583.60	1568.14	1404.07	1409.72	1404.07
	12.79%	11.69%	0.00%	0.40%	

Table 5: Resume of results on the GVRP

NAME	RS LOCATION	CUSTOMERS	VEHICLES	RS	Tmax
HVRP30-1	WITHIN	30	5	5	270
HVRP30-2	AROUND	30	5	5	270
HVRP30-3	FAR	30	5	5	540
HVRP50-1	WITHIN	50	8	8	270
HVRP50-2	AROUND	50	8	8	270
HVRP50-3	FAR	50	10	10	540
HVRP75-1	WITHIN	75	10	9	540
HVRP75-2	AROUND	75	15	9	540
HVRP75-3	FAR	75	20	20	540

Table 6: HVRP instances layout

- RS located far from the customers area

Table 6 reports a resume of the characteristics of each instance. The battery capacity Q is fixed equal to 1000 for all the instances and the kilometric consumption rate is fixed equal to 10, which means that each vehicle has a maximum autonomy of 100 Kms. Tmax is expressed in minutes. The service time is equal to 10 minutes for the customers and to 60 minutes for the refueling stations. The initial solution timelimit is fixed equal to 10,30 and 60 seconds for instances with 30,50 and 75 customers, respectively. The kilometic cost for traditional fuel covered distances is 3 time greater than the kilometric cost for electric propulsion.

Results obtained by the model with a timelimit of 10800 seconds are reported in Table 7, while a comparison between the model and the matheuristic is reported in Table 8. As shown in the table, MH strongly outperforms the model results (32.1% of improvement) even starting from a very poor quality initial solution (52.9% of improvement with respect to the initial solution) which is clear a proof of the robustness of the method. The average computational time is around 100 seconds for the 30 customers instances, 200 seconds for the 50 customers ones and rises up to 1000 seconds on the 75 customers ones, which is still one order of magnitude less than those implied by the model. Therefore, MH can be considered a powerful tool, both in terms of efficiency and effectiveness, to address the HVRP.

NAME	CUSTOMERS	UB	LB	GAP
HVRP30-1	30	669.88	127.18	81.01%
HVRP30-2	30	899.52	192.45	78.61%
HVRP30-3	30	1628.84	441.82	72.88%
HVRP50-1	50	504.02	147.52	70.73%
HVRP50-2	50	731.56	230.65	68.47%
HVRP50-3	50	374.53	177.01	52.74%
HVRP75-1	75	141.1	120.22	14.80%
HVRP75-2	75	327.6	240.52	26.58%
HVRP75-3	75	3128.46	602.5	80.74%

Table 7: UB and LB obtained by the model with a timelimit of 10800 seconds

6. Conclusions and Future Developments

In this paper a new emerging Green Vehicle Routing Problem has been presented, the Hybrid VRP (HVRP), which is an extension of the Green VRP (GVRP) presented in the literature a few years ago, in which vehicles may operate with electric propulsion or a traditional fuel engine, paying a higher unitary cost for distances traveled in the latter mode, which can be seen as a penalty for introducing more pollution into the air other than a higher operational cost, given the fact that traditional fuel is much more expensive than electric propulsion. The main objection transportation companies make about electric vehicles usage is that, given their limited autonomy, visits to refueling stations must be planned along the routes, which implies long deviations from the original path with a consequent increase in traveled distances and route duration. This issue becomes more critical when the number of refueling stations along the road network is very limited. This problem may be overcome if hybrid vehicles are used, as they offer the possibility of covering some parts of the routes under traditional fuel propulsion in order to avoid long deviations to reach a refueling stations. A Large Neighborhood Search based Matheuristic (MH) for the HVRP has been proposed, in which neighborhoods are explored by means of the mathematical model, which is able to find the local minimum even for a large neighborhood, in a short computational time. This method is much more effective and efficient than classical Large Neighborhood Search approaches in which simple

NAME	MODEL	INIT SOL	MH
HVRP30-1	669.88	970.11	610.2
HVRP30-2	899.52	899.52	666.52
HVRP30-3	1628.84	1940.13	616.76
HVRP50-1	504.02	1107.96	266.37
HVRP50-2	731.56	769.07	515.16
HVRP50-3	374.53	2612.23	374.53
HVRP75-1	141.1	283.63	143.52
HVRP75-2	327.6	408.32	293.46
HVRP75-3	3128.46	3128.46	2221.21
AVG	933.95	1346.60	634.19
IMPROVEMENT	32.10%	52.90%	

repair operators often produce lower quality solutions, and fail to reach the local minimum of the neighborhood, while more complex operators take very long computational times to reconstruct a high quality solution. The MH performances have been validated on the GVRP, for which benchmark results are available in the literature, and the method has obtained the same average performances as the best heuristic from the literature, and, in most of the cases, has reached the best known solution. When applied to the HVRP, MH shows high quality performances, and greately improves results obtained with the model. The computational times are at least one order of magnitude shorter, even when starting from a very poor quality solution. This high robustness is an important point of strength of the algorithm.

Future developments in this field could address the definition of extensions of this problem in which Traffic Limited Zones, in which it is forbidden to use traditional fuel propulsion are considered. Other developments could address the possibility of partial battery recharging and the usage of the so called *regenerative breaking* which allows EVs to recover a percentage of their battery charge while traveling on downhill roads. Moreover, multiple recharging technologies, with different recharging times, costs and availability could be considered, as in the work by [10] on the pure electric propulsion case. From a methodological point of view, the proposed innovative approach could be extended to other Vehicle Routing Problems.

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