Smart Routing of Electric Vehicles for Load Balancing in Smart Grids

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***Abstract*— Electric vehicles (EVs) are expected to be a major component of the smart grid. The rapid proliferation of EVs will introduce an unprecedented load on the existing electric grid due to the charging/discharging behavior of the EVs, thus motivating the need for novel approaches for routing EVs across the grid. In this paper, a novel game-theoretic framework for smart routing of EVs within the smart grid is proposed. The goal of this framework is to balance the electricity load across the grid while taking into account the traffic congestion and the waiting time at charging stations. The EV routing problem is formulated as a repeated noncooperative game. For this game, it is shown that selfish behavior of EVs will result in a pure-strategy Nash equilibrium with the price of anarchy upper bounded by the variance of the ground load induced by the residential, industrial, or commercial users. In particular, it is shown that any achieved Nash equilibrium substantially improves the load balance across the grid. Moreover, the results are extended to capture the stochastic nature of induced ground load as well as the subjective behavior of the owners of EVs as captured by using notions from the behavioral framework of prospect theory. Simulation results provide new insights on more efficient energy pricing at charging stations and under more realistic grid conditions.**

I. Introduction

Electric vehicles (EVs) are rapidly becoming a major component of cities around the world. Based on Bloomberg New Energy Finance, EVs are expected to represent 35 percent of new car sales globally by 2040. Greentech Media Research expects at least 11.4 million electric vehicles (EVs) on the road only in the U.S. in 2025. Due to this rapid proliferation of EVs, an important challenge is to effectively manage and control their integration within the electric power grid [1]. For instance, if too many EVs simultaneously charge their batteries at a charging station, it will substantially reduce the electricity load at that station, which, in turn, will be detrimental to other grid components. However, intelligently routing EVs can turn this challenge into an opportunity by viewing EVs as mobile storage devices which charge/discharge their batteries at high/low power stations. This, in turn, requires introducing an appropriate mechanism design scheme which incentivizes EVs to charge/discharge their batteries at those stations which have extra/shortage of energy, respectively.

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As more EVs join the grid, the waiting time at charging stations along with actual road traffic will constitute a major challenge. Since EVs need to be charged more often than fossil-fueled vehicles [2], charging stations may end up with long queues that can directly impact the comfort of EV owners. This challenge is exacerbated by the fact that EVs require considerably longer periods to charge or discharge, compared to conventional vehicles [3]. To meet these challenges, there is a need for a systematic scheduling of EVs which not only takes into account the distribution of the electricity load but also reduces the traffic congestion and waiting time at charging stations.

There have been several recent works that investigated the challenges of managing EVs in the smart grid [4]–[12]. In [4], the authors propose a vehicle-to-aggregator interaction game and develop a pricing policy and design a mechanism to achieve optimal frequency regulation performance. The works in [5] and [6] propose truthful online auction mechanisms in which agents represent EV owners who bid for energy units and also time slots in which an EV is available for charging/discharging. Similarly, the work in [7] considers a consensus based online mechanism design for EV charging with pre-commitment.

A real-time traffic routing system based on an incentive compatible mechanism design has been considered in [11]. In this system a passenger first reports his maximum accepted travel time, and the mechanism then assigns a path that matches the passenger's preference given the current traffic conditions. In [8] and [10], the authors propose a congestion game model to control the power demand at peak hours, by using dynamic pricing. A similar approach based on congestion games is proposed in [9] for EV charging. A survey on utilizing artificial intelligence techniques to manage EVs over the power grid can be found in [1]. In [13] in which the authors study the effect of the decisions of EV owners on the power and transportation networks by jointly minimizing charge and travel plan costs. However, unlike our game-theoretic framework, the approach in [13] is based on an individual optimization over an extended network. While the earlier literature provides important analytic results for managing EVs in the grid, these works mainly focus on one aspect of smart grid, (e.g., reducing the peak hour demand) without taking into account other important factors such as traffic congestion or waiting time at charging stations which are also crucial in affecting EVs' decisions.

Meanwhile, there is a rich literature on routing games where the traffic congestion is selfishly controlled by vehicle owners who seek to minimize their travel costs [14]–[18]. Depending on whether the traffic flow can be divided among different paths one can distinguish unsplitable and splitable routing games [17]. Moreover, whether each user's contribution to the overall traffic is negligible or not one can distinguish non-atomic and atomic routing games [14]. In this regard, one of the widely used metrics in the literature which measures efficiency and the extent to which a system degrades due to selfish behavior of its agents is the *price of anarchy* (PoA) [15]. It has been shown in [16] that, for a linear latency function, the PoA of a nonatomic routing game is exactly $\frac{4}{3}$. This result has been extended later in [17] to splittable routing game with a slightly different bound on the PoA. Similarly, the authors in [18] have studied the PoA of selfish load balancing in atomic congestion games. Moreover, the price of anarchy of noncooperative demand-response in smart grids with flexible loads/EVs has been studied in [19] and [20]. Recently, in [21]–[23], a so-called "smoothness" condition has been developed under which one can obtain simple bounds on the PoA for a large class of congestion games. However, smoothness requires decoupling in arguments of the social cost function which is not immediately applicable to complex EV models.

Moreover, there is strong evidence [24] that real-world, human decision makers do not make decisions based on expected values of outcomes, but rather based on their perception on the potential value of losses and gains associated with an outcome. Since EVs are owned and operated by humans, the subjective perceptions and decisions of these human owners can substantially affect the grid outcomes. This makes *prospect theory* (PT) [24] a powerful framework that allows modeling real-life human choices, a natural choice for modeling EVs' decision making in smart grids under real behavioral considerations. Applications of PT for energy management by modifying consumers electricity demands have been addressed earlier in [25] and [26]. However, these works do not capture the real-life decision making processes involved in the management of EVs in the smart grid. For other relevant alternative approachs (other than PT) to study risk, uncertainty, and behavioral decisions, we refer to [23] and [27]. For instance, [27] considers a mean-risk model for traffic assignment with stochastic travel times. Moreover, [23] studies the inefficiency of equilibrium in congestion games when behavioral biases lead the players to play a wrong game.

To address the aforementioned challenges, the main contribution of this paper is to develop a comprehensive framework for EV management in smart grids which takes into account the traffic congestion costs, the electricity price and availability, the distributed nature of the system, and the subjective perceptions of the EV owners. Our work differs from prior art in several aspects: 1) It models the interactions between EV using a routing game [14], by taking into account the traffic congestion costs, 2) Factors in the waiting time of EVs at charging stations, 3) Introduces an energy pricing scheme to balance the EV load across the grid, and 4) Incorporates real-life decision behavior of EVs under uncertain energy availability by using PT and studies its deviations from conventional classical game theory (CGT). Our work is motivated by the fact that EVs can be viewed as dynamic storage devices which can move around the grid and balance the load across it. This mandates careful grid designs (e.g., pricing electricity properly at charging stations) that can align the energy needs of selfish EVs with those of the smart grid.

In the studied model, we consider a set of EVs that are traveling from an origin to a destination. Each EV can stop at one of the charging stations along its origin-destination path to charge/discharge its battery. Moreover, each EV can decide on the amount of energy to charge/discharge at that station. Here, the energy price charged at each station for buying or selling depends on the total energy demand at that station as well as the ground load which is induced by other grid components such as residential or industrial users. Therefore, each EV makes a decision by choosing a route, a charging station along that route to join, and the amount of energy to charge/discharge. We formulate the interactions between EVs as a noncooperative game in which each EV seeks to minimize the tradeoff between travel time and energy price. We show that such a game admits a pure-strategy Nash equilibrium (NE) and we show that the PoA of this NE is upper bounded by the ratio of the variance of the ground load to the total number of EVs in the grid. Hence, for a large number of EVs, although each EV selfishly and independently minimizes its own cost, the social cost of all EVs will still be close to its optimal value, i.e., when a central grid authority optimally manages all the EVs. Furthermore, we show that any NE achieved as a result of the EVs' interactions will indeed improve the load balancing across the grid. We then take into account the uncertainty of the ground load and provide a bound on the number of EVs which guarantees a low PoA with high probability. In particular, we extend our model by incorporating the subjective behavior of EVs and study its deviations from CGT.[1] Our simulation results provide new insights on energy pricing at different stations in order to keep the overall performance of the grid, which is measured in terms of the social cost, close to its optimal under more realistic scenarios.

The rest of the paper is organized as follows. In Section II, we introduce our system model. In Section III, we analyze the equilibrium, price of anarchy, and load balancement. We extend our results to a stochastic setting with PT in Section IV. Simulation results are given in Section V, and conclusions are drawn in Section VI.

II. System Model and Problem Formulation

Consider a traffic network modeled as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each edge $e \in \mathcal{E}$ represents a road. This network has a total of n EVs (players) in the set $\mathcal{N}$. We let $n_e \in \mathbb{Z}^{\geq 0}$ be the total number of EVs on road e. We denote the level of battery charge of vehicle i by $b_i \in [b_{\min}, b_{\max}]$, where $b_{\min}$ and $b_{\max}$ denote, respectively, the minimum level of battery charge for an EV to operate, and the maximum capacity of an EV battery (we always have $0 < b_{\min} < b_{\max}$). In this network, we have a total

[1] In CGT, the decisions are made based on expectation of the events.

of m charging stations in the set $\mathcal{M}$ that are located over possibly different roads of the network. Each charging station $j \in \mathcal{M}$ can serve its EVs with a rate of $\sigma_j > 0$.[2] We denote the set of all EVs associated to station j by $\mathcal{Q}_j$. We let $g_j \in \mathbb{R}$ be the the ground electricity load at station j. This load is induced by grid components other than EVs such as residential, industrial, or commercial users. Here, $g_j > 0$ means that station j has extra energy to sell, while $g_j < 0$ indicates a shortage of energy.

We assume that each EV wants to go from its current location $s_i \in \mathcal{V}$ to its destination $t_i \in \mathcal{V}$ over a path (route) P_i. During this route, it can choose to charge/discharge its battery by some amount $l_i \in [b_{\min} - b_i, b_{\max} - b_i]$, at some intermediate station $q_i \in \mathcal{M}$ along that route. Here, $l_i > 0$ means that EV i charges its battery by l_i units of energy, while $l_i < 0$ means it discharges its battery. Therefore, we can denote the action of an EV (player) i by $\boldsymbol{a}_i := (P_i, q_i, l_i)$, where P_i is the path chosen by player i from its source to its destination, q_i is the selected charging station along P_i, and l_i is the amount of electricity that player i decides to charge or discharge at station q_i (Figure 1). Finally, denoting the actions of all the players by $(\boldsymbol{a}_i, \boldsymbol{a}_{-i})$, we can define the cost of EV i as:

$$
\begin{aligned}
C_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) = &\sum_{e \in P_i} c_e(n_e) + \frac{|\mathcal{Q}_{q_i}|}{\sigma_{q_i}} + \ln\Big(\frac{b_{\max}}{b_i + l_i}\Big) \\
&+ \Big(f(-g_{q_i} + \sum_{j \in \mathcal{Q}_{q_i}} l_j) - f(-g_{q_i} + \sum_{j \in \mathcal{Q}_{q_i} \setminus \{i\}} l_j)\Big),
\end{aligned} \tag{1}
$$

where $c_e(\cdot)$ is a latency function that captures the traffic congestion as a function of the total number of EVs over road $e \in \mathcal{E}$, and $f(\cdot)$ is a general energy pricing function determined by the power company. In (1), the first term captures the waiting cost of EV i due to traffic congestion, the second term is the waiting cost for joining station q_i which is proportional to the number of vehicles at station q_i, and the third term is the risk of having an empty battery which grows quickly as the battery level decreases. Here, the choice of a logarithmic function is one way of modeling this risk into the EVs cost functions which is inspired from the log barrier function typically used in optimization theory [28] to model penalty factors. For instance, the cost of $\ln\big(\frac{b_{\max}}{b_i + l_i}\big)$ grows to infinity as l_i approaches $-b_i$ (meaning that EV i discharges all of its battery level b_i and hence cannot complete its trip). Similarly, when l_i approaches $b_{\max} - b_i$, then this risk approaches 0 as now EV i charges its battery to full capacity and can certainly complete its trip.

Finally, the last term in (1) is the energy expense (income) for choosing to charge (discharge) l_i units of electricity at station q_i. In this formulation, the energy price for EV i equals to its marginal energy contribution to station q_i. Note that the last term in (1) can also be negative, which means that EV i can be paid by the system depending on the aggregate load of EVs and ground energy in station q_i. For instance, if the aggregate ground and EVs' load at station q_i is very negative (i.e., there is a substantial shortage of energy in that station), then EV i can sell its battery charge at a very high price at station q_i. This incentivizes EVs who have extra energy in their batteries to join station q_i and discharge their batteries thus balancing the load at that station.

[2] σ_j is the number of served EVs per unit of time at station j.

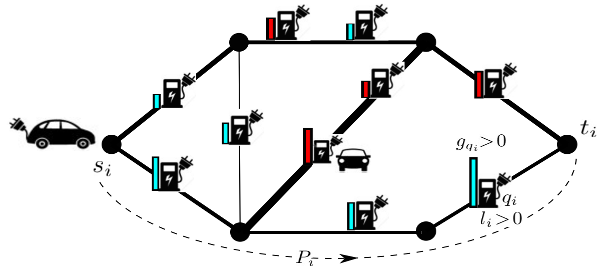

Fig. 1. Illustrative example of the studied model. Each EV wants to move from its origin s_i to its destination t_i. The traffic load on each road is captured by the thickness of that edge (the thicker an edge, the more traffic on that road). The blue bar next to each station shows that that station has extra energy while the red bar shows the shortage level of electricity on that station. Given the current state of the network, it seems most reasonable for EV i to choose the route P_i and stop by station q_i to charge l_i energy units.

Remark 1. The rationale behind using marginal pricing is that when the load in a station is low (e.g., due to EV congestion in that station), marginal pricing becomes more effective and sets a higher price in that station. This disincentivizes more EVs to join that station. It is worth noting that the use of marginal pricing is not specific to our work only and has been extensively justified in economics [6], [29], modeling of electric vehicles [6], [11], as well as engineering literature [11], [30]. For instance, in [6] and [11] the authors utilize marginal payement strategy to design truthful mechanisms for EV charging. Moreover, in [30], a toll payment mechanism based on marginal congestion cost is deployed in order to minimize the social traffic congestion cost in a distributed manner. In fact, by allowing the prices to marginally vary as the players' behavior changes, the planner can ensure that the efficient outcome will be aligned with the equilibrium outcome. Finally, we note that many pricing policies can be implemented as a special case of marginal pricing. For instance, a fixed pricing policy which charges an EV a constant amount of c per unit of electricity usage can be implemented using the linear pricing function $f(x) = cx$.

In fact, one can implement the EV game as a repeated game between EV owners who will, daily, travel the distance between their home (origin) to their work (destination). In particular, at certain peak hours of the day (e.g., 9 am or 5 pm), one may assume that all the drivers are available in the system which subsequently yields the highest traffic congestion or load disturbance in the grid. The information that the EVs require to compute their optimal strategies can be broadcast using a data platform such as a radio station or they can directly be sent to the GPS devices of the EVs using satellite signals. Before starting the trip, the most updated information on the entire network (e.g., road congestion or charging station loads) will be available at each EV's GPS device. Once a driver enters its source-destination to the

GPS, it automatically receives its optimal strategy (i.e., what roads to take or which stations to join) that will be sent immediately by the GPS device to the data station. Then, the data station updates its information which will be again available to all the EVs' GPS devices. Next, we provide two examples for the choice of latency and pricing functions.

Example 1. Given a road $e \in \mathcal{E}$, let b_e be the length of that road. Then, a natural choice for the latency function is the linear latency function given by $c_e(x) = a_e x + b_e$. This means that the travel time of a vehicle that chooses road e depends on the length of that road and linearly increases in terms of the number of other vehicles on that road. In particular, we may assume that the electricity cost of traveling over road e is implicitly captured into this cost function. Otherwise, if an EV incurs $a'_e x + b'_e$ amount of electricity cost due to travel on road e with congestion x, then by defining $c_e(x) := (a_e + \lambda a'_e)x + (b_e + \lambda a'_e)$ we can capture both delay and electricity cost for that EV with λ being the tradeoff rate parameter between these two quantities.

As it can be seen from the definition of EVs' cost functions (1), the incurred cost by an EV depends not only its own action, but also on the other EVs' decisions. This naturally defines a noncooperative game [31] among the EVs having the following key components: A set $\mathcal{N}$ of EVs (players). Each player $i \in \mathcal{N}$ has an action set $\mathcal{A}_i := \mathcal{P}_i \times \mathcal{S}_i \times [b_{\min} - b_i, b_{\max} - b_i]$, where $\mathcal{P}_i$ is the set of all paths between s_i to t_i, and $\mathcal{S}_i$ is the set of all stations along the chosen path by player i. Each player $i \in \mathcal{N}$ takes an action $\boldsymbol{a_i} \in \mathcal{A}_i$ and incurs a cost $C_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i})$ given by (1). In this game, each EV in the grid seeks to select an action which minimizes its own cost, as we study next.

III. Analysis of Equilibrium and PoA

Our first goal is to see whether the proposed game will yield a stable outcome, as captured by the notion of a NE:

Definition 1. *An action profile $(\boldsymbol{a}_i, \boldsymbol{a}_{-i})$ is called a pure-strategy Nash equilibrium (NE) for the EVs' interaction game if $C_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \le C_i(\boldsymbol{a}'_i, \boldsymbol{a}_{-i})$, for all $i \in \mathcal{N}$ and $\boldsymbol{a}'_i \in \mathcal{A}_i$.*

Next, we show that the EVs interaction game admits a pure-strategy NE, meaning that although each EV aims to minimize its own cost, but they collectively will converge to a stable outcome where every EV is satisfied as long as others do not deviate.

Theorem 1. *The EVs game admits a pure-strategy NE.*

Proof. See Appendix I-A. ∎

Theorem 1 shows that a *pure-strategy* NE exists despite the fact that the actions of the players can take both discrete and continuous quantities or they can be highly coupled (e.g. choosing what station to join highly depends on what route to choose). Even though this theorem does not characterize uniqueness or efficiency of the equilibrium, as will be shown later for a large number of EVs, all the equilibrium points will be almost equally efficient in terms of the social cost.

Remark 2. As shown in Appendix I-A, the EVs game is a *potential* game and, thus, every unilateral move by an EV that aims at reducing its cost will bring the entire game one step closer to a NE. The exact characterization of when better/best response dynamics, with or without inertia, converges to a NE was well-studied in the earlier literature [32] and [33] where it was shown that potential games (or weakly acyclic games) often enjoy favorable convergence properties for agents employing various learning dynamics. As a result, if EVs only use their GPS devices on their daily trips, an equilibrium will be reached after a sufficiently long time.

Next, we analyze the price of anarchy of the EVs' game which is an important measure to capture how much the selfish behavior of the EVs can influence the overall optimality of the grid [15].

Definition 2. *For the EVs' interaction game, the PoA is defined as the ratio of the maximum social cost for all Nash equilibria over the minimum (optimal) social cost, i.e.,*

$$PoA = \frac{\max_{\boldsymbol{a} \in NE} \sum_{i=1}^{n} C_i(\boldsymbol{a})}{\min_{\boldsymbol{a}} \sum_{i=1}^{n} C_i(\boldsymbol{a})}.$$

Note that based on Definition 2 we always have $PoA \ge 1$. Here, optimality is measured in terms of EVs' social cost assuming that a network authority with complete information manages the EVs and seeks to minimize the overall social cost. In fact, since EVs are selfish entities whose actions cannot be centrally controlled, modeling EVs' interactions as a game that yields a small PoA is very important. Interestingly, the following theorem shows that for linear latency and quadratic energy pricing, the PoA is bounded above by the averaged sum of the squared ground loads.[3]

Theorem 2. *For a linear latency function $c_e(x) = a_e x + b_e$, and quadratic energy pricing function $f(x) = x^2$, assume that each player incurs at least a unit of cost. Then we have $PoA \le c + 9\big(\frac{\sum_{j=1}^{m} g_j^2}{n}\big)$, where n is number of EVs, m is the number of stations, g_j is the ground load in station j, and $c := 3 + 8b_{\max}^2 + 4\ln(\frac{b_{\max}}{b_{\min}})$ is a constant.*

Proof. See Appendix I-B. ∎

As a result of Theorem 2, if there are many EVs in the grid (i.e., n is large), although every EV minimizes its own cost, the entire grid will still operate close to its optimal state with the minimum social cost and within only a small constant factor c. This allows us to align the selfish EVs' needs with those of the grid and achieve nearly the same optimal social cost when a central grid authority dictates decisions to EVs. It is worth noting that Theorem 2 does not imply that, for a large number of EVs, the players' costs are less (clearly, for higher number of EVs, the traffic congestion and waiting time at charging stations is high). However, it shows that, for a large number of EVs, there is no way to substantially reduce the aggregate cost of all the EVs more than what it is already achieved at a NE.

[3]Here we should mention that our PoA analysis is only valid for instances of EV games with linear latency and quadratic pricing functions; it does not necessarily hold for all other families of such games.

Remark 3. *In the EV game we are assuming* atomic *players and, hence, each individual EV will have an identical unit mass in the system. However, for large number of players n, the influence of each EV in the system is very negligible and, hence, it is reasonable to normalize the mass of each EV by $\frac{1}{n}$ which allows us to compare the PoA between games with different number of players. In this case, there is a strong evidence that the game behaves more similarly to its* nonatomic *counterpart [30], [34] whose PoA by Theorem 2 is expected to be bounded above by $3+8b_{\max}^2+4\ln(\frac{b_{\max}}{b_{\min}})$.*

Next we consider a similar efficiency metric to the PoA, namely the *price of stability* (PoS), which compares the social cost of the "best" NE over the optimal cost, i.e., $PoS=\frac{\min_{\boldsymbol{a}\in\mathcal{NE}}\sum C_i(\boldsymbol{a})}{\min_{\boldsymbol{a}}\sum C_i(\boldsymbol{a})}$. Typically, in real grids one can assume that the minimum social cost is larger than the number of vehicles, i.e., $\min_{\boldsymbol{a}}\sum C_i(\boldsymbol{a})>n$. This simply holds if each player incurs a unit cost in the system (for example we charge each EV \$1 as a toll of using roads or other grid facilities). In this case we can obtain a tighter bound for the PoS as stated in the following theorem:

Theorem 3. *For the linear latency and quadratic pricing function, the PoS of the EVs' interaction game is upper bounded by $PoS\le 2(1+b_{\max}^2)+2\big(\frac{\sum_j g_j^2}{n}\big)$.*

Proof. See Appendix I-D. ■

Next, we show that any NE achieved by the EVs will indeed improve the load balance in the grid. For this purpose, let us fist consider the following definition:

Definition 3. *We refer to a station j as a* good *station if $|g_j|\le\frac{\sqrt{5}}{2b_{\min}}$. Otherwise, we refer to it as a* bad *station. We denote the set of all bad stations by $\mathcal{B}$.*

Based on this definition, the load imbalance of a good station is very small and close to 0 which eliminates the necessity of load balancing in that station. Consider the initial load imbalance of the grid determined by the variance of the initial ground loads at all the bad stations $V_0:=\sum_{j\in\mathcal{B}}g_j^2$. Therefore, we can express the improvement of load balancing at a NE by V_0-V_{NE}, where $V_{NE}:=\sum_{j\in\mathcal{B}}(g_j^{\mathrm{NE}})^2$ denotes the load variance of the bad stations at that achieved NE. The following theorem shows that every achieved NE improves the load balance in all the bad stations without hurting any of the good stations.

Theorem 4. *For the quadratic pricing function, assume that all EVs have the same initial battery level $b_i=b,\forall i$. Then, for any arbitrary NE, all the good stations will remain good while the bad stations become more balanced. In particular, $V_{NE}<V_0-\sum_{j\in\mathcal{B}}\mu_j^2$, where*

$$\mu_j=\begin{cases}|\mathcal{Q}_j|(b_{\min}-b) & \text{if } g_j\le(2|\mathcal{Q}_j|-1)(b_{\min}-b)-\frac{1}{2b_{\min}},\\ |\mathcal{Q}_j|(b_{\max}-b) & \text{if } g_j\ge(2|\mathcal{Q}_j|-1)(b_{\max}-b)+\frac{1}{2b_{\max}},\\ \frac{1}{2}g_j & \text{else},\end{cases}\tag{2}$$

and $|\mathcal{Q}_j|$ denotes the number of EVs at station j at that NE.

Proof. See Appendix I-E. ■

IV. Stochastic Ground Load with Prospect EVs

In this section we consider the EVs' interaction game under a more realistic grid scenario with uncertain ground load environment and study the effect of EVs' behavioral decisions on the overall performance of the smart grid. Toward this goal we assume that the induced ground load at each station g_j, $j\in\mathcal{M}$, which is due to industrial, residential, or commercial users is a random variable with some unknown distribution G_j. Indeed, in a smart grid, a good portion of the energy generated and injected to the grid will stem from renewable resources such as wind turbines or solar panels. Since the amount of such renewable energy highly depends on the environment, such as weather conditions, which is a stochastic phenomenon, the induced renewable energy also changes stochastically at various locations [35]. On the other hand, the energy consumption of residential or industrial users normally follows certain stochastic patterns during specific time slots of a day (e.g., more consumption during early evening hours and less after midnight). Since the ground loads at different stations are mainly influenced by the grid components within their vicinity, for sufficiently distant stations, we can simply assume that the induced ground loads are stochastically independent. Under this independency assumption, we study the optimality of the EVs game under stochastic ground load.

It is worth noting that, in general, the PoA of the EVs game is a function of its underlying parameters such as ground loads or number of EVs. Therefore, in presence of stochastic ground loads the PoA will also be a random variable. Next we provide an estimate for the number of EVs needed to guarantee a low PoA with high probability.

Theorem 5. *Let $G_j, j=1,\dots,m$ be stochastically independent ground loads with support in $[-K,K]$ such that $\mathbb{E}[G_j]=\mu_j$, and $Var[G_j]=\sigma_j^2$. Then, if there are at least $n\ge 9\sum_{j=1}^m(\mu_j^2+\sigma_j^2)+9K\sqrt{m\ln(\frac{1}{\epsilon})}$ electric vehicles in the grid, with probability at least $1-\epsilon$, the PoA is bounded above by the constant $4(1+2b_{\max}^2+\ln(\frac{b_{\max}}{b_{\min}}))$.*

Proof. See Appendix I-F. ■

As it was proposed in [36], the grid authority can use EVs to balance the load on the grid by charging when demand is low and selling power back to the grid (discharging) when demand is high. To this end, Theorem 5 provides an estimate on the required number of EVs to be added into the network in order to keep the grid social cost within a constant factor of its optimal value.

A. Prospect-Theoretic Analysis of the EVs' Game

In this section, we take into account the subjective behavior of EV owners under uncertain energy availability. In this regard, there is a strong evidence [24] that, in the real-world, human decision makers do not make decisions based on expected values of outcomes evaluated by actual probabilities, but rather based on their perception on the potential value of losses and gains associated with an outcome. Indeed, using PT, the authors in [24] showed that human individuals such as EV owners, will often overestimate low probability outcomes and underestimate high probability outcomes. This phenomenon, known as *weighting* effect in PT, reflects the fact that EV owners usually have subjective views on uncertain outcomes such as energy availabiity at the charging stations. Moreover, there is an evidence that in reality humans perceive and frame their losses or gains with respect to a reference point using their own, individual and subjective value function. As an example, risk averse EV owners consider any energy price higher than that when the grid operated in its balanced condition as a loss and overestimate it. This is a consequence of the so-called *loss aversion* behavior which leads different EVs to select different reference points and evaluate their gains/losses according to them. Such reference dependent loss aversion behavior can be explained under the *framing* effect in PT which differs from CGT that assumes players are rational agents who aim to minimize their *expected* losses.

In fact, PT has been successfully applied in many problems with applications both in engineering and economics. For instance, the authors in [37] study humans behavioral decisions in the presence of failure risk in a common-pool resource game. It has been shown in [35] that taking into account the subjective behavior of prosumers (joint prosumer-consumer) in smart grid can substantially change the energy management and distribution pattern compared to the conventional expected utility methods. We refer to [38] and [39] for a comprehensive survey and recent results on PT in economics and other fields.

To capture such behavioral decisions, we use the following definition from PT [24]:

Definition 4. Any EV i has a reference point z_r^i and two corresponding functions $w_i : [0,1] \to \mathbb{R}$ and $v_i(z, z_r^i) : \mathbb{R}^2 \to \mathbb{R}$, known as *weighting* and *valuation* functions. The *expected prospect* of a random variable Z with outcomes $z_1, z_2, \ldots, z_k$, and corresponding probabilities $p_1, p_2, \ldots, p_k$, for electric vehicle i is given by

$$\mathbb{E}^{\text{PT}}[Z] := \sum_{\ell=1}^{k} w_i(p_\ell) v_i(z_\ell, z_r^i).$$

In general, the value function that passes through the reference point is S-shaped and asymmetrical. This means that the value function is steeper for losses than gains indicating that losses outweight gains. Two of the widely used weighting and valuation functions in the PT literature are known as *Prelec* weighting function and *Tversky* valuation function defined by [40], [41],

$$\begin{aligned} v_i(z, z_r^i) &= \begin{cases} (z - z_r^i)^{c_1} & \text{if } \ z \geq z_r^i, \\ -c_2(z_r^i - z)^{c_3} & \text{if } \ z < z_r^i, \end{cases} \\ w_i(p) &= \exp(-(-\ln p)^c), \end{aligned} \tag{3}$$

where $0 < c \leq 1$ is a constant denoting the distortion between subjective and objective probability. Here, $c_1, c_3 \in (0,1)$ determine the curvature of the value function in gains and losses, respectively, and capture humans behaviour as risk averse in gains and risk seeking in losses justified by behavioral economics [39], [41], [42]. On the other hand, loss aversion is typically captured by the parameter $c_2 > 1$ which reflects the fact that human usually perceive losses much more than gains and outweight them.[4] Moreover, we assume that the reference energy price for EV i is given by

$$z_r^i := f(\sum_{j \in \mathcal{Q}_{q_i}} l_j) - f(\sum_{j \in \mathcal{Q}_{q_i} \setminus \{i\}} l_j), \tag{4}$$

which is the price that EV i expects to pay in station q_i given that this station operates in its complete balanced condition (i.e., $g_{q_i} = 0$). In particular, anything above or below this reference price is considered as loss or gain for that EV and is measured by value function $v(z, z_r^i)$.

To formulate the EVs' interaction game using PT, we assume that the ground load at station $j \in \mathcal{M}$ follows a discrete distribution G_j with zero mean and a probability mass function $h_j(\cdot)$. Let Z be the random variable $Z := f(-G_{q_i} + \sum_{j \in \mathcal{Q}_{q_i}} l_j) - f(-G_{q_i} + \sum_{j \in \mathcal{Q}_{q_i} \setminus \{i\}} l_j)$, and $z(\theta)$ be the realization of Z when the ground load at station q_i is θ. Therefore, by Definition 4, the perceived *prospect* gain/loss by EV i equals

$$\mathbb{E}^{\text{PT}}[Z] = \sum_{\theta} w_i(h_{q_i}(\theta)) v_i(z(\theta), z_r^i). \tag{5}$$

On the other hand, as it has been shown in [43], that gains and losses are not all that EVs care about. In other words, not only the sensation of gain or avoided loss does affect the payoff function for an EV i, but so does the actual energy price that EV i pays to satisfy its need. Therefore, in contrast to prior formulation based on a value function defined solely over gains and losses, we take preferences also into the cost functions by assuming that the overall cost to EV i with reference point z_r^i is given by

$$\begin{aligned} C_i^{\text{PT}}(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) = \sum_{e \in P_i} c_e(n_e) + \frac{|\mathcal{Q}_{q_i}|}{\sigma_{q_i}} + \ln\Big(\frac{b_{\max}}{b_i + l_i}\Big) \\ + z_r^i + \mathbb{E}^{\text{PT}}[Z], \end{aligned} \tag{6}$$

where z_r^i and $\mathbb{E}^{\text{PT}}[Z]$ are given by (4) and (5), respectively. Here, each EV i aims to minimize its own prospect cost given by (6) by choosing an appropriate action $\boldsymbol{a}_i$. The following theorem shows that despite extra nonlinearity of the weighting and reference effects in the players' cost functions, the EVs' game under PT still admits a pure NE.

Theorem 6. *For the quadratic pricing $f(x) = x^2$, the EVs' game under PT admits a pure-strategy NE. In particular, the best response dynamics converge to one of such NE points.*

Proof. See Appendix I-G. ∎

Here, we should mention that if we use different pricing functions or assume other sources of uncertainty such as randomness in players actions, then the EVs' game under PT will not necessarily admit a pure-strategy NE. In fact, one of the challenges of analyzing the proposed EV game under

[4]The behavior under $c_2 \in (0,1)$ is often referred to as gain seeking [39].

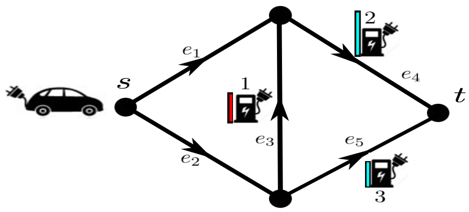

Fig. 2. Network structure and charging station locations for the simulations.

PT is the extra nonlinearities in the players' cost functions which stem from weighting and framing effects. This further complicates the analysis of the PoA under PT. For instance, as opposed to CGT, the PoA of the game with prospect cost functions will now depend on the specific choice of weighting functions and varying reference points. In the next section, we provide some numerical results to study the PoA of the EVs' game under both CGT and PT.

V. Simulation Results

For our simulations, we choose the traffic network to be as in Figure 2 with 5 directed roads, and 3 charging stations. We assume i.i.d Gaussian distributions $G_j \sim N(0, 10)$ for the ground load at different stations. Also, for simplicity, we assume that all the EVs are identical with $b_{\max} = 5, b_{\min} = 0.1$, and $b_i = 3, \forall i$, who want to travel from the origin s to the destination t.

A. PoA under Classical Game Theory

In Figure 3, we illustrate how the PoA under CGT changes as more EVs join the grid and compare the outcomes for different choices of pricing and latency functions. Here, we let the number of EVs increase from $n = 2$ to $n = 9$, and compute the PoA when the nonlinearity of the pricing function increases from $f(x) = x^{2/3}$ to $f(x) = x^{8/3}$. Moreover, we consider the effect of linear latency function $c_e(x) = 5x + 10$ and quadratic latency function $c_e(x) = 5x^2 + 10$ on the PoA. As it can bee seen, joining more EVs monotonically reduces the PoA as was expected by Theorem 2 for the case of linear latency and quadratic pricing functions. However, it turns out that the PoA generally increases as the nonlinearities of the pricing and latency functions increase. In particular, the mismatch between the degree of nonliearity of the pricing and latency functions degrades PoA. Hence, to achieve a high grid performance in terms of social cost, the grid authority should relatively match the energy price with the latency costs.

Figure 4 illustrates the percentage of load balance improvement (i.e., $100\frac{V_0 - V_{NE}}{V_0}\%$) for the worst achieved NE corresponding to each of the cases in Figure 3. It is interesting to see that there is a tradeoff between the PoA and the load balance. For instance, for the linear latency function $c_e(x) = 5x + 10$, the pricing function $f(x) = x^{2/3}$ (red dashed line in both figures) achieves the best PoA and the worst load balancing performance. In fact, for the linear latency function, it can be seen that the quadratic pricing $f(x) = x^2$ (dashed black curve) performs very well both in terms of PoA and load balancing. However, it should be noted that the best performance among the above cases is achieved for the pricing rule $f(x) = x^{4/3}$ (solid blue curve).

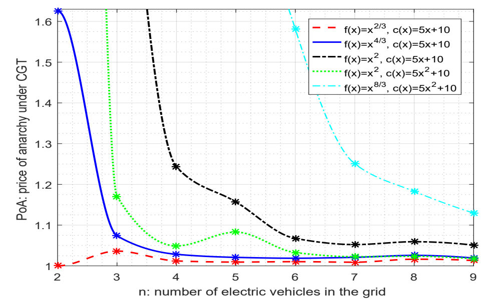

Fig. 3. PoA under CGT for different number of EVs, pricing functions, and latency functions.

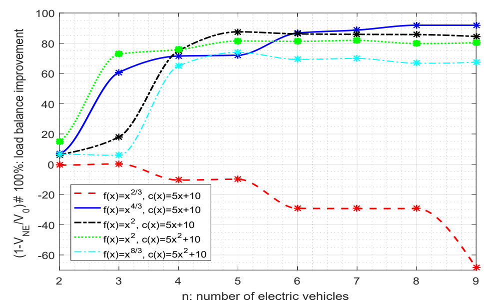

Fig. 4. PoA for different number of EVs under CGT in the smart grid.

TABLE I

Pure NE for $n = 9$ EVs, three paths $P_1 = (e_1, e_4)$, $P_2 = (e_2, e_5)$, $P_3 = (e_2, e_3, e_4)$, and three stations Q_1, Q_2, and Q_3.

n	P_1	P_2	P_3	Q_1	Q_2	Q_3	l_i
1	0	1	0	0	0	1	0.46
2	0	1	0	0	0	1	0.46
3	0	1	0	0	0	1	0.46
4	0	1	0	0	0	1	0.46
5	0	0	1	1	0	0	1.06
6	1	0	0	0	1	0	-1.55
7	1	0	0	0	1	0	-1.55
8	1	0	0	0	1	0	-1.55
9	1	0	0	0	1	0	-1.55
NE	863.53					OPT	821.77

In Table I we have listed the worst NE strategies and social cost, as well as the optimal social cost for $n = 9$ vehicles. As an example the NE strategy for player 1 is to take the route $P_2 = (e_2, e_5)$, join station 3, and charge its battery by $l_1 = 0.79$ energy units. In this table, the initial realized random ground loads at stations Q_1, Q_2, and Q_3 are $g_1 = 0.937$, $g_2 = -11.223$, and $g_3 = 3.061$, respectively. Therefore, the initial load imbalance equals $V_0 = 136.207$, while the ground loads at the NE at these stations are given by $g_1^{NE} = -0.123$, $g_2^{NE} = 5.007$, and $g_3^{NE} = 1.229$. As a result, the load imbalance at this NE equals $V_{NE} = 26.60$ which is substantially lower than the initial load imbalance $V_0 = 136.207$ (84% improvement).

In fact, one of the important features of our model is that, in general, assigning the EVs optimally to balance the load in a centralized manner is computationally very expensive as it requires solving a mixed nonlinear integer program to find the optimal paths, charging stations, and the

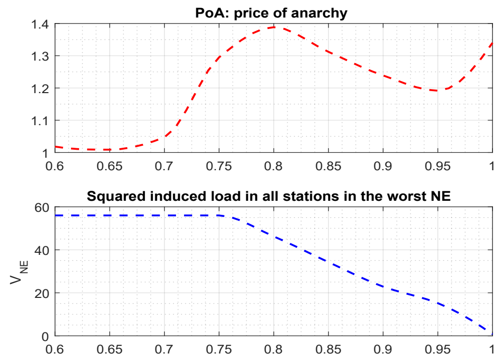

Fig. 5. PoA (red curve) and the total NE induced load (blue curve) for different values of the PT probability weighting function parameters c.

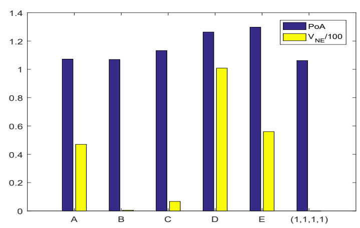

Fig. 6. PoA (blue bar) and total induced load at the worst NE (yellow bar) for different set of PT parameters estimated from behavioral studies.

charge/discharge energy units. However, Theorems 2 and 4 suggest that for large number of EVs the optimal assignment can be approximated within a small constant factor by a solution where each EV selfishly minimizes its own cost. This can be done much more efficiently as now each EV minimizes its cost over only its own strategies.

B. PoA under Prospect Theory

Here, we evaluate the effect of PT on the PoA and load balancing. We set $c_e(x) = 5x + 10$ and $f(x) = x^2$, and consider $n = 6$ EVs over the network of figure 2. Moreover, we assume that all the EVs have the same weighting and valuation functions given by (3) with parameters (c, c_1, c_2, c_3). Figure 5 illustrates the effect of probability distortion parameter c for fixed values of $(c_1, c_2, c_3) = (0.88, 2.25, 0.88)$ which are estimated based on experimental studies on human subjects [42]. As it can be seen from the top figure, the PoA has a complicated nonlinear relation with the distortion probability parameter. One possible reason is that for mid-ranges of the probability distortion parameter, the system has a large number of NEs which results in a worse performance in terms of PoA. However, the induced load in the grid stations monotonically decreases as EVs become more rational (i.e., c approaches to 1). In particular, for small values of c, the EVs start to charge or discharge more aggressively which will fully imbalance the station loads. This is because for very low ranges of c, the EVs behave fully irrational and start to make profit by completely ignoring their travel costs and joining the profitable stations to buy/sell energy at a very low/high price.

Finally, in Figure 6 we have illustrated the effect of different PT parameters (c, c_1, c_2, c_3) estimated from experimental studies [42] on the PoA and total NE induced load in the stations.[5] Here, we set $A = (0.75, 0.68, 2.54, 0.74)$, $B = (0.75, 0.81, 1.07, 0.8)$, $C = (0.75, 0.71, 1.38, 0.72)$, $D = (0.75, 0.86, 1.61, 1.06)$, and $E = (0.75, 0.88, 2.25, 0.88)$. In particular, the last bar corresponds to the selection of $(c, c_1, c_2, c_3) = (1, 1, 1, 1)$ which is for the case of risk neutral EVs. As it can be seen, for the above set of parameters, the grid benefits the most (both in terms of PoA and load balance) when the EVs are risk neutral (as it was the underlying assumption in modeling the EVs interaction game). The worst-case situations occur for the EV owners whose subjective valuation lie in group parameters D and E. This suggest that for such type of EVs, one must modify the pricing rules in order to take into account the negative effects of EVs behavioral decisions.

VI. Conclusions

In this paper, we have studied the interaction of selfish electric vehicles in smart grids. We have formulated a noncooperative game between the EVs and, then, we have shown that the game admits a pure-strategy NE. Then, we have shown that the PoA of the game is bounded above by the "variance" of the ground load divided by the total number of vehicles. This in turn implied that for large number of EVs in the grid, the entire system operates very close to its optimal condition with the minimum social cost, despite the fact that EVs are selfish identities. In particular, we have obtained a tighter upper bound for the PoS of the EVs' interaction game, and showed that for any achieved equilibrium indeed improves the load balancing across the grid. We have extended our results to the case where the ground load is stochastic and incorporated the subjective behavior of EVs using PT into our model. Simulation results showed that, under realistic grid scenarios with subjective EVs, quadratic pricing is more suitable for large number of EVs, while for fewer EVs, exponential pricing would be a better choice.

As a future direction of research, one can extend our model to the case in which there are capacity constraint on the stations. While this can be remedied up to some extent by assuming that the rate of process of each station is proportional to its capacity, however, full analysis of the EV game under hard capacity constraint is an interesting direction of research. Moreover, studying the PoA for other types of latency and pricing functions is interesting and important as such analysis provides new insights on the behavior of EVs under more realistic and accurate grid conditions. Finally, studying the EVs' behavioral decision in the presence of mixed-strategies is very interesting. In such scenarios uncertainty will stem, not only from ground loads but also from EVs' probabilistic decisions. Therefore, one would expect to observe more deviations between PT and CGT as it has been shown in [35] for a different grid setting.

[5] In Figure 6, we have scaled down the total NE induced load V_{NE} by a factor of 0.01.

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Appendix I

A. Proof of Theorem 1

Let $\Phi(\cdot)$ be a potential function defined by

$$\Phi(\boldsymbol{a}_i,\boldsymbol{a}_{-i}) = \underbrace{\sum_{e\in\mathcal{E}}\sum_{x=1}^{n_e} c_e(x)}_{\phi_1(a_i,a_{-i})} + \underbrace{\sum_{\ell=1}^{m}\frac{|\mathcal{Q}_\ell|(|\mathcal{Q}_\ell|+1)}{2\sigma_\ell}}_{\phi_2(a_i,a_{-i})} + \underbrace{\sum_{\ell=1}^{m} f(-g_\ell+\sum_{j\in\mathcal{Q}_\ell} l_j)+\sum_{j=1}^{n}\ln\left(\frac{b_{\max}}{b_j+l_j}\right)}_{\phi_3(a_i,a_{-i})}. \quad (7)$$

We will show that for any two actions $\boldsymbol{a}_i=(P_i,q_i,l_i)$ and $\boldsymbol{a}'_i=(P'_i,q'_i,l'_i)$, we have $\Phi(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\Phi(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})=C_i(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-C_i(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})$. We can write

$$\begin{aligned}
&C_i(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-C_i(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})\\
&=\sum_{e\in P_i\setminus P'_i} c_e(n_e)-\sum_{e\in P'_i\setminus P_i} c_e(n_e+1)\\
&+\frac{|\mathcal{Q}_{q_i}|}{\sigma_{q_i}}-\frac{|\mathcal{Q}_{q'_i}|+1}{\sigma_{q'_i}}+\ln\left(\frac{b_i+l'_i}{b_i+l_i}\right)\\
&+\Big(f(-g_{q_i}+\sum_{j\in\mathcal{Q}_{q_i}} l_j)-f(-g_{q_i}+\sum_{j\in\mathcal{Q}_{q_i}\setminus\{i\}} l_j)\Big)\\
&-\Big(f(-g_{q'_i}+l'_i+\sum_{j\in\mathcal{Q}_{q'_i}\setminus\{i\}} l_j)-f(-g_{q'_i}+\sum_{j\in\mathcal{Q}_{q'_i}\setminus\{i\}} l_j)\Big) \quad (8)
\end{aligned}$$

Next we consider the change in the potential function due to an action change of player i. We can write:

$$\begin{aligned}
&\phi_1(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\phi_1(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})=\sum_{e\in P_i\setminus P'_i} c_e(n_e)-\sum_{e\in P'_i\setminus P_i} c_e(n_e+1),\\
&\phi_2(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\phi_2(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})\\
&\quad=\frac{1}{2}\left(\frac{|\mathcal{Q}_{q_i}|(|\mathcal{Q}_{q_i}|+1)}{\sigma_{q_i}}-\frac{|\mathcal{Q}_{q_i}|(|\mathcal{Q}_{q_i}|-1)}{\sigma_{q_i}}\right)\\
&\quad+\frac{1}{2}\left(\frac{|\mathcal{Q}_{q'_i}|(|\mathcal{Q}_{q'_i}|+1)}{\sigma_{q'_i}}-\frac{(|\mathcal{Q}_{q'_i}|+1)(|\mathcal{Q}_{q'_i}|+2)}{\sigma_{q'_i}}\right)\\
&\quad=\frac{|\mathcal{Q}_{q_i}|}{\sigma_{q_i}}-\frac{|\mathcal{Q}_{q'_i}|+1}{\sigma_{q'_i}},\\
&\phi_3(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\phi_3(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})=\ln\left(\frac{b_{\max}}{b_i+l_i}\right)-\ln\left(\frac{b_{\max}}{b_i+l'_i}\right)\\
&\quad+\Big(f(-g_{q_i}+\sum_{j\in\mathcal{Q}_{q_i}} l_j)+f(-g_{q'_i}+\sum_{j\in\mathcal{Q}_{q'_i}\setminus\{i\}} l_j)\Big)\\
&\quad-\Big(f(-g_{q_i}+\sum_{j\in\mathcal{Q}_{q_i}\setminus\{i\}} l_j)+f(-g_{q'_i}+l'_i+\sum_{j\in\mathcal{Q}_{q'_i}\setminus\{i\}} l_j)\Big).
\end{aligned} \quad (9)$$

Finally, noting that $\Phi(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\Phi(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})=\sum_{k=1}^{3}[\phi_k(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\phi_k(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})]$, and by substituting (9) into this relation, we obtain exactly the same expression as in (8). This shows that $\Phi(\cdot)$ is an exact potential function for the game, and hence, it admits a pure-strategy NE [44].

B. Proof of Theorem 2

Denote an arbitrary but fixed NE profile by $\{\boldsymbol{a}_i=(P_i,q_i,l_i,)\}_{i=1}^{n}$, and the optimal action profile which minimizes the sum of the costs $\sum_{i=1}^{n} C_i(\cdot)$ by $\{\boldsymbol{a}^*_i=(P^*_i,q^*_i,l^*_i,)\}_{i=1}^{n}$. Moreover, let us denote the cost of this NE and the optimal cost by NE, and OPT, respectively, i.e., $NE:=\sum_{i=1}^{n} C_i(\boldsymbol{a}_i,\boldsymbol{a}_{-i})$, and $OPT:=\sum_{i=1}^{n} C_i(\boldsymbol{a}^*_i,\boldsymbol{a}^*_{-i})$. By definition of NE, for all $i\in[n]$ we have

$$\begin{aligned}
&C_i(\boldsymbol{a}_i,\boldsymbol{a}_{-i})\le C_i(\boldsymbol{a}^*_i,\boldsymbol{a}_{-i})\\
&=\sum_{e\in P^*_i\setminus P_i} c_e(n_e+1)+\sum_{e\in P^*_i\cap P_i} c_e(n_e)+\frac{|Q_{q^*_i}\setminus\{i\}|+1}{\sigma_{q^*_i}}\\
&+f(-g_{q^*_i}+\sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k+l^*_i)-f(-g_{q^*_i}+\sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k)+\ln\left(\frac{b_{\max}}{b_i+l^*_i}\right).
\end{aligned}$$

Summing all the above inequalities for $i\in[n]$ we obtain

$$\begin{aligned}
&NE\le\sum_{i=1}^{n} C_i(\boldsymbol{a}^*_i,\boldsymbol{a}_{-i})\\
&=\sum_{i=1}^{n}\Big(\sum_{e\in P^*_i\setminus P_i} c_e(n_e+1)+\sum_{e\in P^*_i\cap P_i} c_e(n_e)\Big)+\sum_{i=1}^{n}\frac{|Q_{q^*_i}\setminus\{i\}|+1}{\sigma_{q^*_i}}\\
&+\sum_{i=1}^{n}\Big[f(-g_{q^*_i}+\sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k+l^*_i)-f(-g_{q^*_i}+\sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k)+\ln\left(\frac{b_{\max}}{b_i+l^*_i}\right)\Big].
\end{aligned} \quad (10)$$

Next we upper bound each of the three summands in (10). To this end, let OPT_1 and NE_1 denote the traffic congestion costs for the optimal solution and the NE, respectively. i.e.,

$$\begin{aligned}
&OPT_1:=\sum_{i=1}^{n}\sum_{e\in P^*_i} c_e(n^*_e)=\sum_{e\in\mathcal{E}} n^*_e c_e(n^*_e)=\sum_{e\in\mathcal{E}} n^*_e(a_e n^*_e+b_e)\\
&NE_1:=\sum_{i=1}^{n}\sum_{e\in P_i} c_e(n_e)=\sum_{e\in\mathcal{E}} n_e c_e(n_e)=\sum_{e\in\mathcal{E}} n_e(a_e n_e+b_e).
\end{aligned} \quad (11)$$

where n_e and n^*_e are the number of vehicles on edge $e\in\mathcal{E}$ induced by the Nash equilibrium and the optimal solution.[6] To find a bound for the first sum in (10) we use a similar method as in [17, Theorem 3.1]. We can write

$$\begin{aligned}
&\sum_{i=1}^{n}\Big(\sum_{e\in P^*_i\setminus P_i} c_e(n_e+1)+\sum_{e\in P^*_i\cap P_i} c_e(n_e)\Big)\\
&\le\sum_{i=1}^{n}\sum_{e\in P^*_i} c_e(n_e+1)=\sum_{e\in\mathcal{E}} n^*_e c_e(n_e+1)\\
&=\sum_{e\in\mathcal{E}} a_e n^*_e n_e+\sum_{e\in\mathcal{E}} n^*_e(a_e+b_e)\\
&\le\sqrt{\sum_e a_e n_e^2\sum_e a_e(n^*_e)^2}+\sum_e n^*_e(a_e n^*_e+b_e)\\
&\le\sqrt{\sum_e (a_e n_e^2+b_e)\sum_e (a_e(n^*_e)^2+b_e)}+\sum_e n^*_e(a_e n^*_e+b_e)\\
&=\sqrt{NE_1\times OPT_1}+OPT_1.
\end{aligned} \quad (12)$$

Next, define OPT_2 and NE_2 to be

[6] Note that OPT_1 and NE_1 are *not* the equilibrium and optimal costs if we restrict our utility functions into the first term only, and they are only a cost portion that vehicles incur in Nash equilibrium and optimal allocation.

$$OPT_2 := \sum_{i=1}^{n} \frac{|Q^*_{q^*_i}|}{\sigma_{q^*_i}} = \sum_{j=1}^{m} \sum_{k\in Q^*_j} \frac{|Q^*_j|}{\sigma_j} = \sum_{j=1}^{m} \frac{|Q^*_j|^2}{\sigma_j},$$
$$NE_2 := \sum_{i=1}^{n} \frac{|Q_{q_i}|}{\sigma_{q_i}} = \sum_{j=1}^{m} \sum_{k\in Q_j} \frac{|Q_j|}{\sigma_j} = \sum_{j=1}^{m} \frac{|Q_j|^2}{\sigma_j}.$$

As before, we can bound the second term in (10) by

$$\sum_{i=1}^{n} \frac{|Q_{q^*_i} \setminus \{i\}|+1}{\sigma_{q^*_i}} \le \sum_{i=1}^{n} \frac{|Q_{q^*_i}|+1}{\sigma_{q^*_i}} = \sum_{j=1}^{m} |Q^*_j| \frac{|Q_j|+1}{\sigma_j} \le \sqrt{OPT_2 \times NE_2} + OPT_2, \quad (13)$$

where the equality follows by the fact that exactly $|Q^*_j|$ of the players in the second summand of (10) will change their station from some q_i to j, and the last inequality is due to the Cauchy-Schwarz inequality.

Finally, we proceed to bound the last summand in (10). For this purpose, let us define $L_j := \sum_{k\in Q_j} l_k$ and $L^*_j := \sum_{k\in Q^*_j} l^*_k$ be the aggregate load induced by Nash equilibrium and optimal solution in station j. We can write

$$\begin{aligned}
&OPT_3 := \\
&= \sum_{i=1}^{n}\Big[f(-g_{q^*_i} + \sum_{k\in Q^*_{q^*_i}} l^*_k) - f(-g_{q^*_i} + \sum_{k\in Q^*_{q^*_i}\setminus\{i\}} l^*_k) + \ln\big(\frac{b_{\max}}{b_i+l^*_i}\big)\Big] \\
&= \sum_{i=1}^{n}\Big((l^*_i)^2 + 2l^*_i\big(-g_{q^*_i} + \sum_{k\in Q^*_{q^*_i}\setminus\{i\}} l^*_k\big) + \ln\Big(\frac{b_{\max}}{b_i+l^*_i}\Big)\Big) \\
&= \sum_{j=1}^{m}\Big(2\big(\sum_{k\in Q^*_j} l^*_k\big)^2 - \sum_{k\in Q^*_j}(l^*_k)^2\Big) - 2\sum_{i=1}^{n} g_{q^*_i} l^*_i + \sum_{i=1}^{n}\ln\Big(\frac{b_{\max}}{b_i+l^*_i}\Big) \\
&= \sum_{j=1}^{m}\Big(2(L^*_j)^2 - \sum_{k\in Q^*_j}(l^*_k)^2\Big) - 2\sum_{j=1}^{m} g_j L^*_j + \sum_{i=1}^{n}\ln\Big(\frac{b_{\max}}{b_i+l^*_i}\Big) \\
&\ge 2\sum_{j=1}^{m}(L^*_j)^2 - \sum_{i=1}^{n}(l^*_i)^2 - 2\sqrt{\sum_{j=1}^{m} g_j^2 \sum_{j=1}^{m}(L^*_j)^2} + \sum_{i=1}^{n}\ln\Big(\frac{b_{\max}}{b_i+l^*_i}\Big) \\
&\ge 2\sum_{j=1}^{m}(L^*_j)^2 - nb_{\max}^2 - 2\sqrt{\sum_{j=1}^{m} g_j^2 \sum_{j=1}^{m}(L^*_j)^2} - n\ln\Big(\frac{b_{\max}}{b_{\min}}\Big).
\end{aligned} \quad (14)$$

Let $A := \sqrt{\sum_{j=1}^{m}(L^*_j)^2}$ and $\eta = b_{\max}^2 + \ln(\frac{b_{\max}}{b_{\min}})$. We get $A^2 - \sqrt{\sum_{j=1}^{m} g_j^2}A - \frac{1}{2}(\eta n + OPT_3) \le 0$. Therefore, we must have $\Delta^* := \sum_{j=1}^{m} g_j^2 + 2(\eta n + OPT_3) \ge 0$. Otherwise, the above quadratic polynomial is always nonnegative which is a contradiction. Solving this relation for A, we get that $A \le \frac{1}{2}(\sqrt{\sum_{j=1}^{m} g_j^2} + \sqrt{\Delta^*})$. Therefore, we have

$$\sqrt{\sum_{j=1}^{m}(L^*_j)^2} \le \frac{1}{2}\Big(\sqrt{\sum_{j=1}^{m} g_j^2} + \sqrt{\Delta^*}\Big). \quad (15)$$

Using the same procedure for Nash equilibrium we obtain

$$\sqrt{\sum_{j=1}^{m} L_j^2} \le \frac{1}{2}\Big(\sqrt{\sum_{j=1}^{m} g_j^2} + \sqrt{\Delta}\Big), \quad (16)$$

where $\Delta := \sum_{j=1}^{m} g_j^2 + 2(\eta n + NE_3)$. Now, to bound the third term in (10), we can write

$$\begin{aligned}
&\sum_{i=1}^{n}\Big[f(-g_{q^*_i} + \sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k + l^*_i) - f(-g_{q^*_i} + \sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k) + \ln\big(\frac{b_{\max}}{b_i+l^*_i}\big)\Big] \\
&= 2\sum_{i=1}^{n} l^*_i \sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k + \sum_{i=1}^{n}(l^*_i)^2 - 2\sum_{i=1}^{n} g_{q^*_i} l^*_i + \sum_{i=1}^{n}\ln\Big(\frac{b_{\max}}{b_i+l^*_i}\Big) \\
&= 2\sum_{i=1}^{n} l^*_i \sum_{k\in Q_{q^*_i}\setminus\{i\}} l_k - 2\sum_{i=1}^{n} l^*_i \sum_{k\in Q^*_{q^*_i}\setminus\{i\}} l^*_k + OPT_3 \\
&= 2\sum_{i=1}^{n} l^*_i\big(L_{q^*_i} - l_i\mathbf{1}_{q_i=q^*_i} - (L^*_{q^*_i} - l^*_i)\big) + OPT_3 \\
&= 2\sum_{i=1}^{n} l^*_i(l^*_i - l_i\mathbf{1}_{q_i=q^*_i}) + 2\sum_{j=1}^{m} L^*_j(L_j - L^*_j) + OPT_3 \\
&\le 4nb_{\max}^2 + 2\sum_{j=1}^{m} L^*_j L_j + OPT_3 \\
&\le 4nb_{\max}^2 + 2\sqrt{\sum_{j=1}^{m}(L^*_j)^2 \sum_{j=1}^{m} L_j^2} + OPT_3 \\
&\le 4nb_{\max}^2 + \frac{1}{2}\Big(\sqrt{\sum_{j=1}^{m} g_j^2} + \sqrt{\Delta}\Big)\Big(\sqrt{\sum_{j=1}^{m} g_j^2} + \sqrt{\Delta^*}\Big) + OPT_3 \\
&= 4nb_{\max}^2 + \frac{1}{2}\sqrt{\Delta\Delta^*} + \frac{1}{2}\sqrt{\sum_{j=1}^{m} g_j^2}(\sqrt{\Delta}+\sqrt{\Delta^*}) + \frac{1}{2}\sum_{j=1}^{m} g_j^2 + OPT_3 \\
&= 4nb_{\max}^2 + \sqrt{(\gamma+NE_3)(\gamma+OPT_3)} \\
&\quad + \delta(\sqrt{\gamma+NE_3} + \sqrt{\gamma+OPT_3}) + \delta^2 + OPT_3,
\end{aligned} \quad (17)$$

where $\gamma := \frac{\sum_{j=1}^{m} g_j^2 + 2\eta n}{2}$, $\delta := \sqrt{\frac{\sum_{j=1}^{m} g_j^2}{2}}$, and the last inequality is due to (15) and (16). Replacing (17), (13), and (12) into (10) we obtain

$$\begin{aligned}
NE &\le \sqrt{OPT_1 \times NE_1} + \sqrt{OPT_2 \times NE_2} \\
&\quad + \sqrt{(\gamma+NE_3)(\gamma+OPT_3)} + (OPT_1 + OPT_2 + OPT_3) \\
&\quad + \delta(\sqrt{\gamma+NE_3} + \sqrt{\gamma+OPT_3}) + \delta^2 + 4nb_{\max}^2 \\
&\le \sqrt{(\gamma+OPT_1+OPT_2+OPT_3)(\gamma+NE_1+NE_2+NE_3)} \\
&\quad + OPT + \delta(\sqrt{\gamma+NE_3} + \sqrt{\gamma+OPT_3}) + \delta^2 + 4nb_{\max}^2 \\
&= \sqrt{(\gamma+OPT)(\gamma+NE)} + OPT \\
&\quad + \delta(\sqrt{\gamma+NE_3} + \sqrt{\gamma+OPT_3}) + \delta^2 + 4nb_{\max}^2 \\
&\le \sqrt{(\gamma+OPT)(\gamma+NE)} + OPT \\
&\quad + \delta(\sqrt{\gamma+NE} + \sqrt{\gamma+OPT}) + \delta^2 + 4nb_{\max}^2,
\end{aligned} \quad (18)$$

where the first inequality holds because for any four positive numbers a_1, a_2, a_3 and a_4 we have $\sqrt{a_1a_2} + \sqrt{a_3a_4} \le \sqrt{(a_1+a_3)(a_2+a_4)}$. Moreover, the last inequality stems from the fact that $NE_3 \le NE$, and $OPT_3 \le OPT$. Dividing both sides of (18) by OPT and setting $x = \frac{NE}{OPT}$ and assuming that $OPT \ge n$ (this will happen if we charge each player \$1 for using the network), we get

$$x \leq \sqrt{(\frac{\gamma}{n}+1)(\frac{\gamma}{n}+x)}+1 + \frac{\delta}{\sqrt{n}}(\sqrt{\frac{\gamma}{n}+x}+\sqrt{\frac{\gamma}{n}+1})+\frac{\delta^2}{n}+4b_{\max}^2, \quad (19)$$

This in view of Lemma 1 completes the proof.

C. Auxiliary Lemma for Bounding PoA

Lemma 1. *Let* $\eta = b_{\max}^2 + \ln(\frac{b_{\max}}{b_{\min}})$, $\gamma := \frac{\sum_{j=1}^m g_j^2 + 2\eta n}{2}$, *and* $\delta := \sqrt{\frac{\sum_{j=1}^m g_j^2}{2}}$. *Then if x satisfies* (19), *we must have* $x \leq 3 + 8b_{\max}^2 + 4\ln(\frac{b_{\max}}{b_{\min}}) + 9\frac{\delta^2}{n}$.

Proof. Let $p := \sqrt{\frac{\gamma}{n}+1} + \frac{\delta}{\sqrt{n}}$, and $q := 1 + \frac{\delta^2}{n} + 4b_{\max}^2 + \frac{\delta}{\sqrt{n}}\sqrt{\frac{\gamma}{n}+1}$. Then we can rewrite (19) as $x - q \leq p\sqrt{\frac{\gamma}{n}+x}$. Squaring both sides and solving for x we obtain

$$x \leq \frac{1}{2}(p^2 + 2q + p\sqrt{p^2 + 4q + 4\frac{\gamma}{n}}). \quad (20)$$

Now since $p^2 + 4q + 4\frac{\gamma}{n} \leq (p + \frac{2q}{p} + \frac{2\gamma}{pn})^2$, replacing this into (20) we get $x \leq p^2 + 2q + \frac{\gamma}{n}$. Replacing the expressions for p, q, and γ into this relation and simplifying we obtain

$$x \leq 3 + 8b_{\max}^2 + 3\frac{\delta^2}{n} + 4\frac{\delta}{\sqrt{n}}\sqrt{1+\frac{\gamma}{n}} + 2\frac{\gamma}{n}. \quad (21)$$

Replacing the identity $\frac{\gamma}{n} = \frac{\delta^2}{n} + \eta$ into (21) and noting that $1 + \frac{\gamma}{n} \leq (\frac{\delta}{\sqrt{n}} + \frac{\eta\sqrt{n}}{2\delta})^2$ we get

$$\begin{aligned} x &\leq 3 + 8b_{\max}^2 + 3\frac{\delta^2}{n} + 4\frac{\delta^2}{n} + 2\eta + 2\frac{\delta^2}{n} + 2\eta \\ &= 3 + 8b_{\max}^2 + 4\ln(\frac{b_{\max}}{b_{\min}}) + 9\frac{\delta^2}{n}. \end{aligned}$$

■

D. Proof of Theorem 3

To bound the PoS, we use the potential function method as in [45, Theorem 19.13] to show that the social cost $C(\boldsymbol{a}) := \sum_i C_i(\boldsymbol{a})$ has pretty much the same structure as the potential function $\Phi(\boldsymbol{a})$. To do so, by using the linear latency function $c_e(x) = a_e x + b_e$, and the quadratic energy pricing $f(x) = x^2$ in the potential function (7), we get

$$\begin{aligned} \Phi(\boldsymbol{a}) = \frac{1}{2}\sum_e \big(a_e n_e^2 + (a_e + 2b_e)n_e\big) + \sum_{j=1}^m \frac{|Q_j|(|Q_j|+1)}{2\sigma_j} \\ + \sum_{j=1}^m g_j^2 + \sum_{j=1}^m L_j^2 - 2\sum_{j=1}^m g_j L_j + \sum_{i=1}^n \ln\Big(\frac{b_{\max}}{b_i + l_i}\Big). \end{aligned} \quad (22)$$

On the other hand, the social cost equals

$$\begin{aligned} C(\boldsymbol{a}) = \sum_e \big(a_e n_e^2 + b_e n_e\big) + \sum_{j=1}^m \frac{|Q_j|^2}{\sigma_j} - \sum_{j=1}^m \sum_{k\in Q_j} l_k^2 \\ + 2\sum_{j=1}^m L_j^2 - 2\sum_{j=1}^m g_j L_j + \sum_{i=1}^n \ln\Big(\frac{b_{\max}}{b_i + l_i}\Big). \end{aligned} \quad (23)$$

Comparing (22) and (23), we can write

$$\begin{aligned} \frac{1}{2}C(\boldsymbol{a}) \leq \Phi(\boldsymbol{a}) &\leq C(\boldsymbol{a}) + \sum_{j=1}^m \sum_{k\in Q_j} l_k^2 + \sum_{j=1}^m g_j^2. \\ &\leq C(\boldsymbol{a}) + nb_{\max}^2 + \sum_{j=1}^m g_j^2. \end{aligned}$$

Now let $\hat{\boldsymbol{a}}$ be the NE which minimizes the potential function $\Phi(\cdot)$, and $\boldsymbol{a}^*$ be the optimal action profile, i.e., $C(\boldsymbol{a}^*) = C^*$. We have,

$$C(\hat{\boldsymbol{a}}) \leq 2\Phi(\hat{\boldsymbol{a}}) \leq 2\Phi(\boldsymbol{a}^*) \leq 2[C(\boldsymbol{a}^*) + nb_{\max}^2 + \sum_{j=1}^m g_j^2].$$

Therefore, dividing both sides by $C^* \geq n$, we get $PoS \leq \frac{C(\hat{\boldsymbol{a}})}{C(\boldsymbol{a}^*)} \leq 2\Big(1 + b_{\max}^2 + \frac{\sum_j g_j^2}{n}\Big)$.

E. Proof of Theorem 4

Consider an arbitrary but fixed station j. Note that for any arbitrary Nash equilibrium $\{(P_i, q_i, l_i)\}_{i\in\mathcal{N}}$, the load components of all the players who join station j, i.e., $\{l_i : i \in \mathcal{Q}_j\}$ must form a Nash equilibrium if the players' costs are restricted to only the load portion of their costs. In other words, for every $i \in \mathcal{Q}_j$, if we consider $|\mathcal{Q}_j|$ players with cost functions

$$\begin{aligned} \hat{C}_i(l'_i, l'_{-i}) &= (-g_j + \sum_{j\in\mathcal{Q}_j} l'_j)^2 - (-g_j + \sum_{j\in\mathcal{Q}_j\setminus\{i\}} l'_j)^2 + \ln\big(\frac{b_{\max}}{b_i + l'_i}\big) \\ &= (l'_i)^2 + 2l'_i(\sum_{j\in\mathcal{Q}_j\setminus\{i\}} l'_j - g_j) + \ln\big(\frac{b_{\max}}{b_i + l'_i}\big), \end{aligned} \quad (24)$$

then, $\{l_i : i \in \mathcal{Q}_j\}$ must be a Nash equilibrium for this restricted game.[7] Since, for every $i, k \in \mathcal{Q}_j$ we have $\frac{\partial}{\partial^2 l'_i}\hat{C}_i = 2 + \frac{1}{(b+l'_i)^2}$, and $\frac{\partial}{\partial l'_i \partial l'_k}\hat{C}_i = 2$, the restricted game with cost functions (24) is diagonally strictly convex and admits a unique pure-strategy Nash equilibrium [46, Theorem 2], given by $\{l_i : i \in \mathcal{Q}_j\}$. In addition, since by assumption $b_i = b, \forall i$, all the players have the same cost function. As a result the restricted game is a symmetric convex game which means that its unique equilibrium is symmetric [47, Theorem 3]. Thus $l_i = l, \forall i \in \mathcal{Q}_j$, for some $l \in [b_{\min} - b, b_{\max} - b]$. As a result, the load costs for all the players $i \in \mathcal{Q}_j$ at the Nash equilibrium are the same and equal to

$$\hat{C}_i(l) = (2|\mathcal{Q}_j| - 1)l^2 - 2lg_j + \ln\big(\frac{b_{\max}}{b + l}\big). \quad (25)$$

In particular, the equilibrium load l must be the unique minimizer of (25) in the feasible range $[b_{\min} - b, b_{\max} - b]$, which is given by

$$l = \begin{cases} b_{\min} - b & \text{if } g_j \leq (2|\mathcal{Q}_j| - 1)(b_{\min} - b) - \frac{1}{2b_{\min}}, \\ b_{\max} - b & \text{if } g_j \geq (2|\mathcal{Q}_j| - 1)(b_{\max} - b) - \frac{1}{2b_{\max}}, \\ \frac{2g_j - \Psi + \sqrt{\Psi^2 + 4|\mathcal{Q}_j| - 2}}{4|\mathcal{Q}_j| - 2} & \text{otherwise,} \end{cases}$$

[7] Note that this property only holds for a fixed charging station and the load components across different stations do not necessarily form an NE. This is because, it is possible that a player can save in its load cost if he joins a different station but at the same time looses more in his traffic cost.

where $\Psi := (2|\mathcal{Q}_j|-1)b+g_j$.

Next, we compute the equilibrium load reduction in station j given by $(g_j^{NE})^2-g_j^2=(-g_j+|\mathcal{Q}_j|l)^2-g_j^2$ for each of the above three possibilities:

Case I: If $g_j \le (2|\mathcal{Q}_j|-1)(b_{\min}-b)-\frac{1}{2b_{\min}}$, we have

$$\begin{aligned}|\mathcal{Q}_j|^2l^2-2|\mathcal{Q}_j|lg_j &= (|\mathcal{Q}_j|(b_{\min}-b))^2-2|\mathcal{Q}_j|(b_{\min}-b)g_j\\ &\le(-3|\mathcal{Q}_j|^2+2|\mathcal{Q}_j|)(b_{\min}-b)^2+\frac{|\mathcal{Q}_j|(b_{\min}-b)}{b_{\min}}\\ &\le(-3|\mathcal{Q}_j|^2+2|\mathcal{Q}_j|)(b_{\min}-b)^2\le-|\mathcal{Q}_j|^2(b_{\min}-b)^2,\end{aligned}$$

where the first inequality is by the upper bound on g_j, and the second inequality is because $b_{\min}-b\le 0$.

Case II: If $g_j \ge (2|\mathcal{Q}_j|-1)(b_{\max}-b)+\frac{1}{2b_{\max}}$, we have

$$\begin{aligned}|\mathcal{Q}_j|^2l^2-2|\mathcal{Q}_j|lg_j &= (|\mathcal{Q}_j|(b_{\max}-b))^2-2|\mathcal{Q}_j|(b_{\max}-b)g_j\\ &\le(-3|\mathcal{Q}_j|^2+2|\mathcal{Q}_j|)(b_{\max}-b)^2-\frac{|\mathcal{Q}_j|(b_{\max}-b)}{b_{\max}}\\ &\le(-3|\mathcal{Q}_j|^2+2|\mathcal{Q}_j|)(b_{\max}-b)^2\le-|\mathcal{Q}_j|^2(b_{\max}-b)^2,\end{aligned}$$

where the first inequality is by the lower bound on g_j.

Case III: If g_j does not belong to Cases I and II, then $l=\frac{2g_j-\Psi+\sqrt{\Psi^2+4|\mathcal{Q}_j|-2}}{4|\mathcal{Q}_j|-2}$, which is the unique root of the derivative of (25), and hence it satisfies $l=\frac{1}{2|\mathcal{Q}_j|-1}(g_j+\frac{1}{2(b+l)})$. We can write

$$\begin{aligned}&|\mathcal{Q}_j|^2l^2-2|\mathcal{Q}_j|lg_j\\ &=(\frac{|\mathcal{Q}_j|}{2|\mathcal{Q}_j|-1})^2(g_j+\frac{1}{2(b+l)})^2-\frac{2|\mathcal{Q}_j|}{2|\mathcal{Q}_j|-1}g_j(g_j+\frac{1}{2(b+l)})\\ &=(\frac{|\mathcal{Q}_j|}{2|\mathcal{Q}_j|-1})^2\Big[-g_j^2(3-\frac{2}{|\mathcal{Q}_j|})-\frac{g_j}{b+l}(1-\frac{1}{|\mathcal{Q}_j|})+\frac{1}{4(b+l)^2}\Big]\end{aligned}$$

Now, one can easily see that, if $|g_j|>\frac{1}{b_{\min}}$, then $g_j\ge\frac{1}{b+l}$ or $g_j\le\frac{-1}{b+l}$, and the quadratic expression inside of the above brackets is always less than $-g_j^2$. Thus,

$$|\mathcal{Q}_j|^2l^2-2|\mathcal{Q}_j|lg_j\le-(\frac{|\mathcal{Q}_j|}{2|\mathcal{Q}_j|-1})^2g_j^2\le-(\frac{g_j}{2})^2.$$

On the other hand, if $|g_j|\le\frac{1}{b_{\min}}$, then the quadratic expression inside of the above brackets can be at most $(\frac{2|\mathcal{Q}_j|-1}{|\mathcal{Q}_j|})^2\frac{1}{4b_{\min}^2(3-\frac{2}{|\mathcal{Q}_j|})}$ which implies that:

$$(g_j^{NE})^2-g_j^2=|\mathcal{Q}_j|^2l^2-2|\mathcal{Q}_j|lg_j\le\frac{1}{4b_{\min}^2(3-\frac{2}{|\mathcal{Q}_j|})}\le\frac{1}{4b_{\min}^2}.$$

Therefore we have $(g_j^{NE})^2\le\frac{5}{4b_{\min}^2}$ which means that station j remains to be a good station in the Nash equilibrium.

Finally, using I, II, and III, we have $(g_j^{NE})^2-g_j^2\le-\mu_j^2$ for all the bad stations $j\in\mathcal{B}$ where μ_j is given by (2). Summing this inequality over all the bad stations we get the desired result.

F. Proof of Theorem 5

Since $\{G_j, j\in\mathcal{M}\}$ are independent, so are their squares $\{G_j^2\}$, and we have $\mathbb{E}[\frac{\sum_{j=1}^m G_j^2}{m}]=\frac{\sum_{j=1}^m(\mu_j^2+\sigma_j^2)}{m}$. Using Hoeffding bound for independent and non-identical random variables we have $\mathbb{P}\Big[\sum_{j=1}^m G_j^2-\sum_{j=1}^m(\mu_j^2+\sigma_j^2)>mt\Big]\le\exp\big(-\frac{2mt^2}{K^2}\big)$. Since $PoA\le c+9(\frac{\sum_{j=1}^m G_j^2}{n})$, where $c=3+8b_{\max}^2+4\ln(\frac{b_{\max}}{b_{\min}})$, by choosing $t=\frac{\frac{n}{9}-\sum_{j=1}^m(\mu_j^2+\sigma_j^2)}{m}$, we can write

$$\begin{aligned}\mathbb{P}[PoA\ge c+1]&\le\mathbb{P}\Big[\frac{\sum_{j=1}^m G_j^2}{n}>1\Big]\\ &=\mathbb{P}\Big[\frac{\sum_{j=1}^m G_j^2}{n}>\frac{\sum_{j=1}^m(\mu_j^2+\sigma_j^2)+mt}{n}\Big]\\ &\le\exp\big(-\frac{2mt^2}{K^2}\big).\end{aligned}\tag{26}$$

Now in order the probability in (26) to be less than ϵ, we need to have $t\ge K\sqrt{\frac{\ln(\frac{1}{\epsilon})}{2m}}$. Finally, replacing the expression for t in this inequality and solving for n, we get

$$n\ge 9\sum_{j=1}^m(\mu_j^2+\sigma_j^2)+9K\sqrt{\frac{m\ln(\frac{1}{\epsilon})}{2}}.$$

G. Proof of Theorem 6

For the *quadratic* pricing function $f(x)=x^2$, we have

$$z(\theta)-z_r^i=(-\theta+\sum_{j\in\mathcal{Q}_{q_i}}l_j)^2-(-\theta+\sum_{j\in\mathcal{Q}_{q_i}\setminus\{i\}}l_j)^2-z_r^i=l_i\theta.$$

Substituting this relation into (6), we obtain

$$\begin{aligned}C_i^{\mathrm{PT}}(\boldsymbol{a}_i,\boldsymbol{a}_{-i})&=\sum_{e\in P_i}c_e(n_e)+\frac{|\mathcal{Q}_{q_i}|}{\sigma_{q_i}}+\ln\Big(\frac{b_{\max}}{b_i+l_i}\Big)\\ &\quad+z_r^i+\underbrace{\sum_{\theta}w_i(h_{q_i}(\theta))\hat{v}_i(l_i\theta)}_{\tilde{C}^{\mathrm{PT}}(\boldsymbol{a})},\end{aligned}$$

where $\hat{v}_i(x)=x^{c_1}$ if $x\ge0$, and $\hat{v}_i(x)=-c_2|x|^{c_3}$, otherwise. Now consider the function $\Psi(\cdot)$ defined by

$$\begin{aligned}\Psi(\boldsymbol{a}_i,\boldsymbol{a}_{-i})=&\sum_{e\in\mathcal{E}}\sum_{x=1}^{n_e}c_e(x)+\sum_{\ell=1}^m\frac{|\mathcal{Q}_\ell|(|\mathcal{Q}_\ell|+1)}{2\sigma_\ell}+\sum_{i=1}^n\ln\Big(\frac{b_{\max}}{b_i+l_i}\Big)\\ &+\sum_{\ell=1}^m(\sum_{j\in\mathcal{Q}_\ell}l_j)^2+\underbrace{\sum_{i=1}^n\sum_{\theta}w_i(h_{q_i}(\theta))\hat{v}_i(l_i\theta)}_{\tilde{\Psi}(\boldsymbol{a})}.\end{aligned}$$

We argue that this function is an exact potential function for the EVs' game under PT. In fact, if we did not have the prospect terms $\tilde{C}^{\mathrm{PT}}(\boldsymbol{a})$ and $\tilde{\Psi}(\boldsymbol{a})$ in the structure of $C_i^{\mathrm{PT}}(\boldsymbol{a})$ and $\Psi(\boldsymbol{a})$, the proof would immediately follow by the same lines of argument as in the proof of Theorem 1. However, for the quadratic pricing, since the term $\sum_\theta w_i(h_{q_i}(\theta))\hat{v}_i(l_i\theta)$ is a player specific function which *only* depends on action of player i and is uncorrelated from $\boldsymbol{a}_{-i}$, we easily get $\tilde{C}^{\mathrm{PT}}(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\tilde{C}^{\mathrm{PT}}(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})=\tilde{\Psi}^{\mathrm{PT}}(\boldsymbol{a}_i,\boldsymbol{a}_{-i})-\tilde{\Psi}^{\mathrm{PT}}(\boldsymbol{a}'_i,\boldsymbol{a}_{-i})$. This shows that $\Psi(\cdot)$ is indeed an exact potential function for the EVs' game under PT and quadratic pricing. As a result, any minimizer of $\Psi(\cdot)$ is a pure-strategy NE of the EVs' game. In particular, since the action set of players are compact in their own ambient space, this immediately implies that the sequence of best responses of EVs will converge to a pure-strategy NE [44].