

EPC Linear Model.

Let us recall the input data for the **EPC** problem:

Vehicle related input

M : number of stations (*Depot* excluded)
 $\Gamma = (Depot = 0, 1, \dots, M, Depot = M + 1)$: vehicle tour (without refueling)
 $TMax$: maximal time for the vehicle to achieve its tour
 C^{Veh} : vehicle tank capacity
 E_0 : initial vehicle hydrogen load
 For $j = 0, \dots, M$, t_j : required time to go from station j to station $j + 1$
 For $j = 0, \dots, M$, d_j : required time to go from station j to the micro-plant
 For $j = 0, \dots, M$, d_j^* : required time to go from the micro-plant to station j
 For $j = 0, \dots, M$, e_j : required energy to go from station j to station $j + 1$
 For $j = 0, \dots, M$, ε_j : required energy to go from station j to the micro-plant
 For $j = 0, \dots, M$, ε_j^* : required energy to go from the micro-plant to station j

Micro-plant production related input

C^{MP} : micro-plant tank capacity
 N : number of production periods
 p : duration (in time units) of one production period
 H_0 : initial micro-plant hydrogen load
 $Cost^F$: activation cost
 For $i = 0, \dots, N - 1$, $P_i = [p.i, p.(i + 1)[$: time interval related to production period i
 For $i = 0, \dots, N - 1$, R_i : production rate related to period i
 For $i = 0, \dots, N - 1$, $Cost^V_i$: production cost related to period i

Table 1. Input data for the EPC problem

I. An Integrated Mathematical Programming (MP) Model

MP is not well-fitted to **EPC**. Still, we may use it in order to formulate our problem in an unambiguous way, based upon 3 main variables:

- **Production variables:**

- $z = (z_i, i = -1, \dots, N - 1)$, with $\{0, 1\}$ values: $z_i = 1 \sim$ the micro-plant is active during period i ($i = -1$ corresponds to a fictitious period);
- $y = (y_i, i = 0, \dots, N - 1)$, with $\{0, 1\}$ values: $y_i = 1 \sim$ the micro-plant is activated at the beginning of i ;
- $V^{Tank} = (V^{Tank}_i, i = 0, \dots, N)$, with non negative integer values: V^{Tank}_i is the hydrogen load of the micro-plant tank at the beginning of period i ; We involve here a fictitious period N in order to express the fact that the load of micro-plant tank at the end of the process should be at least equal to H_0 ;
- $\delta = (\delta_i, i = 0, \dots, N - 1)$, with $\{0, 1\}$ values: $\delta_i = 1 \sim$ the vehicle is refueling during period i ;
- $L^* = (L^*_i, i = 0, \dots, N - 1)$, with non negative integer values: in case $\delta_i = 1$, L^*_i is the quantity of hydrogen loaded by the vehicle during period i ; else, L^*_i may take any non negative value.

- **Vehicle variables:**
 - $x = (x_j, j = 0, \dots, M)$, with $\{0, 1\}$ values: $x_j = 1 \sim$ the vehicle refuels while traveling from station j to station $j + 1$;
 - $L = (L_j, j = 0, \dots, M)$, with non negative integer values: if $x_j = 1$, L_j = hydrogen quantity loaded by the vehicle while traveling from j to $j + 1$; else it may take any non negative value;
 - $T = (T_j, j = 0, \dots, M + 1)$, with non negative integer values: T_j = time when the vehicle arrives at j ;
 - $T^* = (T^*_j, j = 0, \dots, M + 1)$, with non negative integer values: if $x_j = 1$, T^*_j = time when the vehicle starts refueling while traveling from j to $j + 1$; else it may take any non negative value;
 - $V^{Veh} = (V^{Veh}_j, j = 0, \dots, M + 1)$, with non negative integer values: V^{Veh}_j = hydrogen load of the vehicle tank when the vehicle arrives in j .
- **Synchronization variables:** $U = (U_{ij}, i = 0, \dots, N - 1, j = 0, \dots, M)$ with $\{0, 1\}$ values: $U_{ij} = 1 \sim$ the vehicle is going to refuel during period i while traveling from j to $j + 1$.

Constraints come as follows (for a better understanding, we use here a logical formulation, easy to linearize through *Big M* technique):

- **Objective function:** Minimize

$$\sum_{i=0, \dots, N-1} (Cost^F.y_i + Cost^V.z_i) + \alpha.T_{M+1}.$$
- **Production constraints:**
 - For any $i = 0, \dots, N - 1$: $y_i = 1 \rightarrow (z_i = 1 \wedge z_{i-1} = 0)$;
 - For any $i = 0, \dots, N - 1$: $z_i + \delta_i \leq 1$;
 - $z_{-1} = 0$;
 - $V^{Tank}_0 = H_0$; $V^{Tank}_N \geq H_0$;
 - For any $i = 0, \dots, N - 1$: $V^{Tank}_i \leq C^{MP}$;
 - For any $i = 0, \dots, N - 1$: $V^{Tank}_{i+1} = V^{Tank}_i + z_i.R_i - \delta_i.L^*_i$.
- **Vehicle Constraints:**
 - $T_0 = 0$; $V^{Veh}_0 = E_0$; $V^{Veh}_{M+1} \geq E_0$;
 - For any $j = 1, \dots, M + 1$: $V^{Veh}_j \leq C^{Veh}$;
 - For any $j = 0, \dots, M$: $V^{Veh}_j \geq \varepsilon_j$; (E1)
(E1) means that at any time, the vehicle must be able to go to the *micro-plant* and refuel, and relies on the *Triangle Inequality* for energy coefficients e_j and ε_j ;
 - For any $i = 0, \dots, M$: $L_j \leq C^{Veh} + \varepsilon_j - V^{Veh}_j$; (E2)
(E2) expresses the fact that the vehicle cannot refuel more than the space which remains inside its tank;
 - For any $j = 0, \dots, M$: $T_{j+1} \geq (1 - x_j).(T_j + t_j) + x_j.(T^*_j + p + d^*_{j+1})$; (E3)
 - For any $j = 0, \dots, M$: $T^*_j \geq T_j + d_j$; (E4)
 - For any $j = 0, \dots, M$: $x_j = 0 \rightarrow V^{Veh}_{j+1} = V^{Veh}_j - e_j$;
 - For any $j = 0, \dots, M$: $x_j = 1 \rightarrow V^{Veh}_{j+1} = V^{Veh}_j - \varepsilon_j - \varepsilon^*_{j+1} + L_j$;
 - $T_{M+1} \leq TMax$.
- **Synchronization constraints:**
 - For any $j = 0, \dots, M$: $\sum_{i=0, \dots, N-1} U_{ij} = x_j$; (E5)
 - For any $i = 0, \dots, N - 1$, $\delta_i = \sum_{j=0, \dots, M} U_{ij}$; (E6)

$$\circ \text{ For any } j = 0, \dots, M, x_j = 1 \rightarrow T^*_{j} = \sum_{i=0, \dots, N-1} p.i. U_{i,j}; \quad (\text{E7})$$

$$\circ \text{ For any } i = 0, \dots, N-1: L^*_i \leq V^{Tank}_i; \quad (\text{E8})$$

(E8) expresses that load L^*_i cannot exceed the current load of the micro-plant tank;

$$\circ \text{ For any } j = 0, \dots, M: L_j = \sum_{i=0, \dots, N-1} U_{i,j} L^*_i. \quad (\text{E9})$$

We may state:

Theorem 1: Solving above **MP_EPC** model also solves our **EPC** problem.

Proof: Checking that a feasible solution (y, x, T, L) of **EPC** can be turned into a feasible solution of above linear model with the same cost comes in a straightforward way. We only need to follow the trajectory induced by (y, δ, x, L) and compute $z, L^*, T, T^*, V^{Tank}, V^{Tank}, U$, accordingly.

Conversely, let us consider some feasible solution $(y, \delta, x, T, L, z, L^*, T^*, V^{Tank}, V^{Tank}, U)$ of above linear model. The key point is that vector U defines a matching $i \rightarrow j(i)$ between $I^\circ = \{i \in 0, \dots, N-1, \text{ such that } \delta_i = 1\}$ and $J^\circ = \{j \in 0, \dots, M, \text{ such that } x_j = 1\}$ and that this matching is consistent with standard linear ordering: if $i_1 < i_2$ then $j(i_1) < j(i_2)$. The first point is contained into equations (E5, E6). The second point derives from equations (E7), which fixes values $T^*_{j(i)}$ and inequalities (E3, E4): if i_1, i_2 are consecutive in I° and such that $j(i_1) > j(i_2)$, then we get, by propagating (E3, E4), $T^*_{j_1} \geq T^*_{j_2}$ and a contradiction with (E7).

It comes that, if $(y, \delta, x, T, L, z, L^*, T^*, V^{Tank}, V^{Tank}, U)$ is optimal, we see that (E3, E4) are going to give rise to equalities, which means that T and T^* are going to follow the EPC trajectory induced by (y, δ, x, L) . But we also see that related load $L_{j(i)} = L^*_i$ (because of (E9)) are going to be feasible in the sense that they should exceed neither the load of the micro-plant tank at the beginning of period i , nor the difference between the capacity of vehicle tank and its current load when its arrive to the micro-plant, while moving from j to $j+1$, because (E2) and (E8)). We conclude that our solution $(y, \delta, x, T, L, z, L^*, T^*, V^{Tank}, V^{Tank}, U)$ may be interpreted as an **EPC** trajectory, with the same value. \square

Let us pay attention now to the linearization **Linear-EPC**, through *Big M* techniques, of above **MP_EPC** model, and its rational relaxation. Let us suppose that we reformulate any implications:

- $X = 0 \rightarrow Y \geq 0$
- $X = 1 \rightarrow Y \leq 0$

as:

- $X + Y/Big_M \geq 0$
- $X + Y/Big_M \leq 1$

where *Big_M* is a very large number.

Then we see that:

Proposition 1: According to this hypothesis, the optimal value of the rational relaxation of **Linear-EPC** is null.

Proof: It is enough to check that, if *Big_M* is choosen large enough, then we get a feasible solution of the rational relaxation of **Linear_EPC** by setting:

- $x_j = 1/2$ for every j ; $\delta_i = (M+1)/2N$ for any i ;
- $U_{i,j} = 1/2N$ for any i, j ;

- $z_i = y_i = 0$ for any i ;
- $L_j = L^*_j = 0$ for any j ;
- $V^{Tank}_i = H_0$ for any i ; $V^{Veh}_j = E_0$ for any j ;
- $L_j = 0$ and $T^*_j = d_j$ for any j .

This solution clearly yields a null value. \square

Still, we may enhance the quality of such a relaxation by noticing that several additional constraints may be inserted to **MP_EPC**:

- For any j , $T_{j+1} \geq T_j + t_j$;
- $\sum_j L_j \geq \sum_j e_j$.

II. Additional Constraints.

For any $j = 1, \dots, M$, we set:

- $D_j = \sum_{k=0, \dots, j-1} t_k + d_j$;
- $D^*_j = \sum_{k=j+1, \dots, M} t_k + d^*_{j+1}$;
- $\text{Min}_j = \lceil D_j/p \rceil$;
- $\text{Max}_j = N - 1 - \lceil D^*_j/p \rceil$;
- For any pair $j1, j2$, $j1 < j2$, $\mu_{j1, j2} = \sum_{j1 \leq j < j2} e_j + \epsilon_{j2}$.
- For any pair $j1, j2$, $j1 < j2$, $\text{INT}_{j1, j2} = \lceil (\sum_{j1+1 \leq j < j2} t_j + d^*_{j1+1} + d_{j2})/p \rceil$;

We say that 2 pairs $(i1, j1)$ and $(i2, j2)$ are *antagonistic* iff $i1 < i2$ and $j1 > j2$. A collection Λ of pairwise *antagonistic* pairs (i, j) is called an *antagonistic clique*.

We say that 2 pairs $(i1, j1)$ and $(i2, j2)$ are *time-inconsistent* iff $(i2 - i1) \leq \text{INT}_{j1, j2}$. A collection Λ of pairwise *time-inconsistent* pairs (i, j) is called an *time-inconsistent clique*.

II.1. Simple Time Constraints.

- For any $j = 0, \dots, M$: $T_{j+1} \geq T_j + t_j + x_j \cdot (d_j + d^*_j - t_j)$;

II.2. Energy Constraints.

We introduce additional variable $F_j \geq 0$, $j = 0, \dots, M+1 = 0$, with the meaning that F_j means the energy used by the vehicle from 0 to j .

- For any $j = 0, \dots, M$: $F_{j+1} \geq F_j + e_j + x_j \cdot (\epsilon_j + \epsilon^*_j - e_j)$;
- For any $j = 0, \dots, M$: $F_j - E_0 \geq \sum_{k < j} L_k$;
- For any $j = 0, \dots, M$: $\sum_{0 \leq i \leq \text{Max}_j} R_{i, Z_i} \geq F_j - E_0 - H_0$;
- $\sum_{0 \leq i \leq N-1} R_{i, Z_i} \geq F_{M+1}$;
- For any $j = 0, \dots, M$: $\sum_{0 \leq i \leq \text{Max}_j} y_i \geq (F_j - E_0 - H_0)/C^{\text{MP}}$;
- $\sum_{0 \leq i \leq N-1} y_i \geq (F_{M+1})/C^{\text{MP}}$;
- For any $j1, j2$, K such that $\mu_{j1, j2} > C^{\text{Veh}} + K \cdot C^{\text{MP}}$: $\sum_{\text{Min}_{j1} \leq i \leq \text{Max}_{j2}} y_i \geq K$;
- For any $j1, j2$ such that $\mu_{j1, j2} > C^{\text{Veh}}$: $\sum_{j1 \leq j \leq j2-1} x_j \geq 1$.

II.3. Structural Constraints.

- For any j , any i such that $i < \text{Min}_j$ or $i > \text{Max}_j$: $U_{ij} = 0$;
- For any $i, j1, j2$: $U_{ij1} + U_{i+1,j2} \leq 1$;
- For any *antagonistic clique* Λ : $\sum_{(i,j) \in \Lambda} U_{ij} \leq 1$.
- For any *time-inconsistent clique* Λ : $\sum_{(i,j) \in \Lambda} U_{ij}$

III. Separating the Structural Constraints.

III.1. Separating the Antagonistic Cliques.

SEPARATE_ANTAGO:

Input: Current vector U , rational.

Output: An antagonistic clique Λ , which violates the antagonistic clique constraint. In case Λ is undefined, then no such a clique exists.

$j \leftarrow 0$; $i \leftarrow N-1$;

While $j \leq M+1$ do

$\Pi(i, j) \leftarrow -0$;

 For $j1 \leq j$ do

 For $i1 \geq i$ do

 If $(i1 \neq i)$ OR $(j1 \neq j)$ then

 If $U_{ij} + \Pi(i1, j1) > \Pi(i, j)$ then

$\Pi(i, j) \leftarrow U_{ij} + \Pi(i1, j1)$;

$\text{Arg}(i, j) \leftarrow (i1, j1)$.

 If $i \geq 1$ then $i \leftarrow i - 1$

 Else

$i \leftarrow N-1$;

$j \leftarrow j+1$;

If $\Pi(0, M+1) > 1$ then $\Lambda \leftarrow \text{Reconstruction through Arg}(0, M+1)$ else $\Lambda \leftarrow \text{Undefined}$;

III.2. Separating the Time-Inconsistent Cliques.

Preliminary: Weakening the time-inconsistency.

Such as it has been defined, the time-inconsistency constraints seems difficult to separate. So we simplify them as follows:

- For any pair $j1, j2$, $j1 < j2$, we set $\text{INT}^*_{j1,j2} = \lfloor (\sum_{j1 \leq j < j2} t_j) / p \rfloor$; If $j1 = j2$ then $\text{INT}^*_{j1,j2} = 0$.

So we separate the clique in the sense of INT^* (instead of INT), while making the assumption that:

- $\inf_j t_j + \inf_j d_j + \inf_j d^*_j \leq p$.

In case this assumption is not satisfied, then our process works as an approximation.

Please also notice that constraint: For any $i, j1, j2$: $U_{ij1} + U_{i+1,j2} \leq 1$, is a specific case of the time-inconsistency constraint.

SEPA_RE_INCONSISTENT:**Input:** Current vector U , rational.**Output:** A time inconsistent clique Λ , which violates the time-inconsistency clique constraint.In case Λ is undefined, then no such a clique exists. $j \leftarrow 0; i \leftarrow 0;$ While $j \leq M+1$ do $\Pi(i, j) \leftarrow -0;$ For $j_1 \geq j$ doFor $i_1 = i, \dots, i + \text{INT}^*_{j-i_1} \leftarrow j_1 + 1$ doIf $(i_1 \neq i) \text{ OR } (j_1 \neq j)$ thenIf $U_{i_1 j_1} + \Pi(i_1, j_1) > \Pi(i, j)$ then $\Pi(i, j) \leftarrow U_{i_1 j_1} + \Pi(i_1, j_1);$ $\text{Arg}(i, j) \leftarrow (i_1, j_1).$ If $i \leq N-2$ then $i \leftarrow i + 1$

Else

 $i \leftarrow 0;$ $j \leftarrow j+1;$ If $\Pi(N-1, M+1) > 1$ then $\Lambda \leftarrow \text{Reconstruction through Arg}(N-1, M+1)$ else $\Lambda \leftarrow \text{Undefined};$