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Integrated production planning with sequence-dependent family setup times

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ABSTRACT

This paper proposes an integrated optimization model of aggregate production planning (APP), family disaggregation planning, and family scheduling problems in hierarchical production planning (HPP) systems considering sequence-dependent family setup times. The model obtains the optimal production plan for each product type and product family in each period, together with the globally optimal production sequence of product families in all planning periods. The proposed model is tested with randomly generated experimental data consistent with what is prevalent in the manufacturing industry and its results are compared with those of the traditional HPP models. Our results show that the integrated model realizes greater cost savings.

1. Introduction

Production planning problems are generally formulated in one of two ways: monolithic and hierarchical. The monolithic approach (Manne, 1958; Lasdon and Terjung, 1971) formulates the problem as a mixed integer linear programming model for all items. However, this level of detail in the formulation of the problem requires demand data that is difficult to forecast accurately and is expensive to implement (Bitran et al., 1981).

On the other hand, the hierarchical approach (Hax and Meal, 1975) decomposes the problem into several layers of sub-problems corresponding to different product structures, including APP for product type, family disaggregation planning for product families, and item disaggregation planning for items. Product types are groups of items having similar unit costs, direct costs (excluding labor), holding costs per unit per period, productivities, and seasonalities, while product families are groups of items pertaining to the same type and sharing similar setups (Bitran et al., 1981). Since the sub-problems can be solved much more easily, the hierarchical approach may meet managers' needs for a quick solution than a monolithic approach. Moreover, the fact that there are usually a small number of types justifies the use of sophisticated forecasting techniques that may be expensive to employ for thousands of items (Bitran et al., 1981).

However, the hierarchical planning approach suffers from several weaknesses. First, its hierarchical sub-problems should correspond to the organizational and decision-making echelons in the firm, resulting in increased interaction between the planning system and the decision-makers at each level (Graves, 1982). But organizational structures are becoming increasingly flatter, calling into question the utility of HPP models with rigid structures.

Second, the family setup costs and times, determined at the second level family disaggregation models but influencing the top level models, are not taken into account in the aggregate decisions. Consequently, the resulting production plans are not necessarily optimal and susceptible to loss of cost savings opportunities, since a near-optimal aggregate plan may lead to much lower setup costs than an optimal one. Also the obtained production plans may be infeasible because the family setup times in the family disaggregation model inevitably consume the production capacity, which should have been assigned to produce the types in APP decisions. Therefore, when family setup costs and times are not trivial (Qiu et al., 2001; Yalcin and Boucher, 2004; Omar and Teo, 2007; Pastor et al., 2009) as in some manufacturing industries, their influences on the optimality and feasibility of the aggregate plans, and consistency when the aggregate plans are disaggregated, have to be considered.

Third, the total family setup time in the planning horizon, usually calculated by summing up the optimal setup time in each period, is not necessarily globally optimal when the sequence dependencies between different families are considered. Due to the sequence dependency, the optimal schedule in the planning horizon is not only determined by the families produced in each period, but by the initial machine status in each period.

To alleviate these weaknesses we propose a model that integrates APP problem and family disaggregation planning and scheduling problems with sequence dependent setup times and costs, and considers the global family scheduling optimization in

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the planning horizon. First, the model is robust enough to fit different organizational structures, providing flexibility for the decision maker. This is because it cannot only plan the production and scheduling of types for top-level managers, but also decompose the aggregate plans into detailed production plans of families for mid-level managers, and even provide scheduling information for the scheduling staff on the shop floor. When the organizational structures become flatter, managers at each layer can easily extract the relevant information (production plans of types or families) they need from the model. Second, the infeasibility of aggregate plans due to capacity consumption by family setup times can be eliminated. The model also maintains consistency when aggregate plans are disaggregated, and ensures the optimality of the production plans and schedules, or as close to optimal as possible. Third, the globally optimal family schedules in all periods of the planning horizon are concatenated to form an optimal schedule, rather than optimizing them period by period. The latter sequential family scheduling is more likely to be trapped in a local optimum, since the schedule in one period may result in an inferior schedule in the subsequent period.

The remainder of this paper is organized as follows. In Section 2, the related literature is reviewed. In Section 3 a representative traditional HPP system by Ozdamar et al. (1998) is presented. Section 4 formulates the proposed integrated model. In Section 5, the corresponding benchmark problems of the proposed integrated approach are presented. The performance of the integrated model is validated with randomly generated experimental data in Section 6, together with some managerial implications of the study. We discuss the conclusions and future research directions in Section 7.

2. Literature review

When evaluating HPP systems, two approaches are often used. First, three crucial factors are of interest. *Consistency* requires that the constraints from upper-level plans imposed on the lower-level plans be satisfied. *Feasibility* requires that the plan at each level be practical. And, *optimality* requires that the total cost of the obtained production plans and schedules in all layers be globally optimal. Second, five elements (anticipation, instruction, reaction, implementation, and ex post feedback) proposed by Schneeweiss (1995) to evaluate the hierarchical interdependence within an organization, can be employed to assess HPP systems, since their APP and family disaggregation planning interact in a hierarchical way. Schneeweiss and Zimmer (2004) apply the hierarchical coordination mechanisms to the global optimization of the supply chain between a producer and a supplier.

Due to the appealing features of the hierarchical approach, a number of authors have studied the efficacy of HPP systems, after the pioneers Hax and Meal (1975). Bitran et al. (1981) present a typical HPP approach to plan and schedule production for single stage manufacturing environments. Axsater (1986) describes constraints on the aggregate level types that will guarantee the existence of a feasible disaggregation at the lower level. Bowers and Jarvis (1992) propose a three-tiered hierarchical model with a product grouping procedure that is designed to implicitly minimize sequence-dependent setup time. McKay et al. (1995) review prior research and provide a guide to configure different HPP systems for different real-life environments. This guide facilitates the development of hierarchical planning and its wide use in practice. Ozdamar et al. (1998) propose a decision support system for HPP in order to facilitate the production planning task for end-users by providing an easy-to-use tool. Yalcin and Boucher (2004) propose a continuous-time algorithm to disaggregate the aggregate production plans so that individual families are scheduled for production on shared resources. Neureuther et al. (2004) propose a three-tiered HPP for a make-to-order steel fabrication facility in order to develop a production plan and master schedule for a set of the so-called "product archetypes", classified by class as industrial or commercial, or by type as angle, beam or plate, or by weight as light or heavy.

The above approaches make sense only when setup costs and times are negligible. Therefore, they cannot guarantee global optimality and feasibility when family setup times are considered. From the perspective of the hierarchical interdependence, *instruction*, *implementation*, and *ex post feedback* are achieved, while *anticipation* and *reaction* are not considered.

At the APP level, Qiu and Burch (1997) introduce the concept of expected setup cost (ESC) to anticipate the sequence-dependent setup costs at the family disaggregation level. Although the ESCs are not necessarily optimal, the influence of the anticipation element is recognized in the model to some extent. When tested with actual plant data, Qiu et al. (2001) find that the HPP–ESC model outperforms prior HPP models, especially the model proposed by Bowers and Jarvis (1992). Some improved degree of optimality brought about by the ESC and consistency are ensured in both models. However, the setup times and their influence on production capacity are still not considered, which may challenge the feasibility of the obtained aggregate plans. Moreover, the suboptimal setup costs may also negatively affect optimality of the aggregate plans. From the interdependence point of view, the only drawback of the above two models is that they ignore the reaction element. In the HPP system proposed by Rohde (2004), the anticipation function provides knowledge of the base level to the top-level in terms of approximate setup times per period. The effect of setup times from the base level is considered and therefore the feasibility of the HPP system is ensured to some degree. However, the HPP system does not consider setup costs in the top-level planning model, so the optimality of the entire system cannot be guaranteed.

Graves (1982) integrates the family disaggregation model with sequence independent setup costs into the APP model, but setup times are not considered. Omar and Teo (2007) propose a hierarchical model for multi-product batch chemical plants, integrating the corresponding setup times and costs into the capacity constraints and objective function of the APP model in order to ensure feasibility and optimality of the production plans. However, sequence dependencies between families are not considered. Moreover, it is questionable whether their hierarchical system can be applied to other industries.

Soman et al. (2004) propose a conceptual HPP framework for MTO (make-to-order) and MTS (make-to-stock) production situations, where only the MTO products with the same color are aggregated into a family to reduce solution complexity. However, the influence of lower level setup costs and setup times is not considered in mid-term production plans.

In the recent past, interest has focused on the integration of production planning and other manufacturing features such as maintenance (Wienstein and Chung, 1999; Aghezzaf et al., 2007; Dehayem Nordem et al., 2009), scheduling (Jozefowska and Zimniak, 2008), plant capacity planning (Hsu and Li, 2009), product safety related traceability factor (Wang et al., 2010), process planning (Li et al., 2010), and cell formation (Ah kioon et al., 2009; Safaei and Tavakkoli-Moghaddam, 2009). The integration brings about huge computational efforts partly due to the monolithic production planning for all items.

3. Traditional HPP model

Ozdamar et al. (1998) propose a Hierarchical Decision Support System for production planning, encompassing aggregate planning (type planning level) and family disaggregation before considering end item disaggregation and item scheduling. According to the hierarchical decision logic, the APP model is solved first for the optimal production plans of all types in the planning horizon. Then, the aggregate plan in the first period is decomposed by the family disaggregation model.

3.1. APP model

The parameters and variables for the general APP model are shown in Table 1.

At the type level, the APP model considers most mid-term decisions, including production, inventory, subcontracting, and backordering levels, regular time and overtime, and labor employment and dismissal plans. The APP model is formulated as follows:

$$\min \sum_{t=1}^{T} \left[ch_t H_t + cf_t F_t + \sum_{m=1}^{M} (tc_{mt} X_{mt} + h_{mt} I_{mt} + cs_{mt} S_{mt} + cb_{mt} B_{mt} + cr_{mt} R_{mt} + co_{mt} O_{mt}) \right] \quad (1)$$

$$\text{s. t.} \quad I_{m,t-1} + X_{mt} + S_{mt} - I_{mt} + B_{mt} - B_{m,t-1} = d_{mt} \quad \forall m,t \quad (2)$$

$$\sum_{m=1}^{M} ut_{mt} X_{mt} \le (AR_t + AO_t)A \quad \forall t \quad (3)$$

$$ut_{mt} X_{mt} = R_{mt} + O_{mt} \quad \forall m,t \quad (4)$$

$$\sum_{m=1}^{M} O_{mt} \le AO_t = poAR_t \quad \forall t \quad (5)$$

$$S_{mt} \le CAS_{mt} \quad \forall m,t \quad (6)$$

$$B_{mt} \le CAB_{mt} \quad \forall m,t \quad (7)$$

$$AR_t - AR_{t-1} = H_t - F_t \quad \forall t \quad (8)$$

$$\sum_{m=1}^{M} ua_m I_{mt} \le OS \quad \forall t \quad (9)$$

$$X_{mt}, I_{mt}, B_{mt}, S_{mt}, R_{mt}, O_{mt}, H_t, F_t \ge 0 \quad \forall m,t \quad (10)$$

Table 1
Parameters and variables of the APP model.

Indices: $m=1,2,\ldots,M$, index of type $t=1,2,\ldots,T$, index of period

Parameters
tc_{mt}, ut_{mt} = unit production cost (materials+overhead) and processing time for type m in period t
h_{mt}, cs_{mt}, cb_{mt} = unit inventory carrying, subcontracting, and backordering costs for type m in period t
cr_{mt}, co_{mt} = regular time and overtime costs per man-hour for type m in period t
ch_t, cf_t = cost of hiring one man-hour (including some overhead costs, like advertising, interviewing and training costs) and laying off one man-hour in period t
d_{mt} = net demand for type m in period t
A = capacity allowance percentage (used for allowing machine breakdowns, earlier due dates, etc.)
po = permitted percentage of overtime to available regular time
CAS_{mt} = maximum subcontracting capacity for type m in period t
CAB_{mt} = maximum backordering quantity permitted for type m in period t
ua_m = space occupied by each unit inventory of type m
OS = total available space for inventory storage
Variables: X_{mt}, I_{mt} = production and inventory level of type m in period t
S_{mt}, B_{mt} = subcontracting and backordering quantity of type m in period t
H_t, F_t = man-hours of regular time hired and laid off in period t
Auxiliary variables: R_{mt}, O_{mt} = regular time and overtime hours consumed by type m in period t
AR_t, AO_t = available regular time and overtime hours in period t

Table 2
The family disaggregation model.

Indices: $i=1,2,\ldots,I$, index of family
Parameters:
$J(m)$ = set of families pertaining to type m
δ_i = setup cost of family i
lb_{it}, ub_{it} = lower bound and upper bound of the lot size of family i in period t, where $lb_{it}=d_{it}-I_{i,t-1}$ (safety stock is not considered), and $ub_{it}=os_{it}-I_{i,t-1}$ (os_{it} are overstock limits) (Bitran et al., 1981)
Decision variables: l_{it} = lot size of family i in period t

In this model, the objective is to minimize the total relevant costs in the planning horizon. Constraints (2) are the production–inventory balance equations. Constraints (3) are the capacity limits. In constraints (4), the total capacity consumed by each type in each period is defined. Constraints (5) ensure that the total overtime in a period will not exceed available overtime, calculated as a pre-assigned proportion of available regular time. Constraints (6) are subcontracting capacity limits and constraints (7) are backordering limits. Constraints (8) are workforce balance equations. Constraints (9) are the inventory storage space limits.

3.2. Family disaggregation model

The objective of the family disaggregation model is to minimize setup costs subject to the total quantity allocated to all families equal to the type quantity specified in the aggregate plan in the current period. The relevant notations for the family disaggregation model are shown in Table 2, and the resulting model is

$$\min \sum_{i=1}^{I} \delta_i ub_{it} / l_{it} \quad (11)$$

$$\text{s.t.} \quad \sum_{i \in J(m)} l_{it} = X_{mt} \quad \forall m,t \quad (12)$$

$$lb_{it} \le l_{it} \le ub_{it} \quad \forall i,t \quad (13)$$

The above HPP system is widely applied in many industries, such as tile (Liberatore and Miller, 1985), fiberglass (Aull and Burch, 1990) and steel manufacturing (Qiu et al., 2001; Neureuther et al., 2004).

4. The integrated model

The family disaggregation planning and scheduling model considering sequence-dependent setup times and costs, is integrated into the APP model to ensure the consistency, feasibility and optimality of the obtained production plans. In the integrated model, the family setup costs are optimized globally in the planning horizon to ensure the optimization of the obtained family schedules. The common constraints imposed on the family disaggregation model ensure consistency of the disaggregation process. The optimized family setup times are added to the production capacity constraints to ensure the feasibility of all layers of plans. The obtained production plans are implemented on a rolling horizon basis.

The integrated model also meets the five key elements of the hierarchical interdependence (Schneeweiss, 1995). Anticipation and reaction are achieved by considering setup costs and times in the objective function and production capacity constraints, while instruction is achieved by imposing type level constraints at the family level. Since the two layers of models agree on a decision once an optimal solution is obtained, they are implemented on the object system. The ex post feedback element is satisfied through the rolling horizon.

The integrated model assumes that (1) sufficiently precise demand data for types are available for all planning periods, (2) safety stocks are not taken into account, and (3) the production quantity of each family in each planning period is positive. Assumption (3) makes sense for the following reasons. First, the demand for each family cannot be satisfied with inventory alone since there is often large total demand for the products within a family and limited storage space for each family. Second, holding excess inventory for a family should be avoided in that it will consume capital and storage resources and reduce the inventory turnover. Finally, production can balance the costs of holding excessive inventory and the shortage costs due to lower inventory.

4.1. Notations

Besides the notations in Section 3, the specific notations for the integrated model are shown in Table 3.

4.2. Mathematical formulation

The integrated model, denoted as *Model* p^0, can be formulated as follows:

$$\min \sum_{t=1}^{T}\left[ch_tH_t+cf_tF_t+\sum_{i=1}^{I}\left(sc\sum_{j=1}^{I}\sum_{k=0}^{I}st_{ij}Y_{ijkt}+c_{it}P_{it}\right)+\sum_{m=1}^{M}(h_{mt}I_{mt}+cs_{mt}S_{mt}+cb_{mt}B_{mt}+cr_{mt}R_{mt}+co_{mt}O_{mt})\right] \quad (14)$$

s.t. (2), (5)–(10)

$$\sum_{m=1}^{M}ut_{mt}X_{mt}+\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{I}st_{ij}Y_{ijkt}\le(AR_t+AO_t)A \quad \forall t \quad (3')$$

$$ut_{mt}X_{mt}+\sum_{i=1}^{I}\sum_{j\in J(m)}\sum_{k=1}^{I}st_{ij}Y_{ijkt}=R_{mt}+O_{mt} \quad \forall m,t \quad (4')$$

$$\sum_{i\in J(m)}P_{it}=X_{mt} \quad \forall m,t \quad (15)$$

$$\alpha_{it}X_{mt}\le P_{it}\le\beta_{it}X_{mt}, \quad i\in J(m) \quad \forall m,t \quad (16)$$

Table 3
Specific notations for the integrated model.

Indices:
$i, j=1,2,\ldots,I$, index of family
$k=0,1,\ldots,I$, index of processing position of each family in a period where the 0th position in a period means the last position in the preceding period

Parameters:
c_{it}=average production cost (labor cost excluded) of products within family i in period t
α_{it},β_{it}=lower and upper proportions of family i to its type in period t (α_{it} and β_{it} are experts' judgment obtained from the historical production data of each family and its type)
st_{ij}=sequence-dependent setup time from family i to family j ($st_{ij}=0$, if $i=j$)
sc=cost per unit setup time (a constant parameter to convert setup time into setup cost)

Decision variables:
P_{it}=production level of family i in period t
$Y_{ijkt}=1$, if family i is produced at the kth position and family j at the $(k+1)$th position in period t; and 0 otherwise
$\eta_{ikt}=1$, if family i is produced at the kth position in period t, and 0 otherwise
$\eta_{jkt}=1$, if family j is produced at the kth position in period t, and 0 otherwise

$$\sum_{k=1}^{I}\eta_{ikt}=1 \quad \forall i,t \quad (17)$$

$$\sum_{i=1}^{I}\eta_{ikt}=1 \quad \forall k,t \quad (18)$$

$$\eta_{i0t}=\eta_{iI,t-1}, \quad t=2,3,\ldots,T \quad \forall i \quad (19)$$

$$\eta_{ikt}\eta_{j,k+1,t}=Y_{ijkt}, \quad k=0,1,\ldots,I-1 \quad \forall i,j(\ne i),t \quad (20)$$

$$P_{it}=0,1,\ldots \quad \forall i,t \quad (21)$$

$$Y_{ijkt},\eta_{ikt}(\eta_{jkt})\in\{0,1\} \quad \forall i,j,k,t \quad (22)$$

In the model, the objective is to minimize the total cost with sequence-dependent setup cost. Constraints (3′) are the capacity limits, and constraints (4′) define the regular time and overtime capacity consumed by the production and setups. Constraints (15) ensure that the total production quantity of the families pertaining to the same type is equal to the quantity of the type. Constraints (16) set the upper and lower proportions' limits of the production quantity of each family to the corresponding type. Constraints (17) and (18) ensure that each family can only be produced at one position and only one family is produced at each position in each period, respectively. Constraints (19) indicate that the family produced at the 0th position in a period is also the family produced at the last position in the preceding period. For the first period, a dummy family is assumed to be produced at the 0th position and its setup times to other families are set to be constant. Therefore, the optimal solution of the family scheduling problem is not affected by the initial machine condition. Constraints (20) indicate that the setup between family i and family j exists ($Y_{ijkt}=1$), if and only if family i is produced at the kth position and family j at the $(k+1)$th position ($\eta_{ikt}=1$and $\eta_{j,k+1,t}=1$); otherwise, there is no setup between them.

4.3. Linearization

Model p^0 is nonlinear due to the quadratic term $\eta_{ikt}\eta_{j,k+1,t}$ in constraints (20). The nonlinear model can be converted into an equivalent linear model through a binary variable, along with two constraints for each such variable (Watters, 1967). Y_{ijkt} is employed to replace $\eta_{ikt}\eta_{j,k+1,t}$, subject to

$$\eta_{ikt}+\eta_{j,k+1,t}-Y_{ijkt}\le 1, \quad k=0,1,\ldots,I-1 \quad \forall i,j,t \quad (23)$$

$$-\eta_{ikt}-\eta_{j,k+1,t}+2Y_{ijkt}\le 0, \quad k=0,1,\ldots,I-1 \quad \forall i,j,t \quad (24)$$

The linearized integer programming model (called *Model* P^*) is as follows:

$$\min \sum_{t=1}^{T}\left[ch_tH_t+cf_tF_t+\sum_{i=1}^{I}\left(sc\sum_{j=1}^{I}\sum_{k=1}^{I}st_{ij}Y_{ijkt}+c_{it}P_{it}\right)+\sum_{m=1}^{M}(h_{mt}I_{mt}+cs_{mt}S_{mt}+cb_{mt}B_{mt}+cr_{mt}R_{mt}+co_{mt}O_{mt})\right]$$

s.t. (2), (3′), (4′), (5)–(10), (15)–(19), (31)–(24)

4.4. Features and solution procedure of model P^*

Proposition 1. *The globally optimal family schedule in all periods of the planning horizon is at least as good as the family schedule obtained by concatenating the separate optimal family schedule in each period under assumption* (3).

Proof. The proposition is obvious according to traditional HPP models. □

Proposition 2. *The integrated model reaches its optimal value only when the family schedule is optimal under assumption* (3).

Proof. Suppose the optimal solution of the integrated model is (H,F,Y,P,I,S,B,R,O), with the objective function of $f(H,F,Y,P,I,S,B,R,O)$. Assume Y, the total setup time of the family schedule in the planning horizon obtained from the integrated model, is not optimal, and Y^* is the total setup time of the optimal family schedule.

When Y is reduced to Y^*, the direct influence is due to constraints (4′). That is, the sum of consumed regular time and overtime is reduced by $(Y-Y^*)$. Consequently, the consumed labor time is reduced by $(Y-Y^*)$, among which x units of time is assumed to be assigned to regular time and $(Y-Y^*-x)$ to overtime.

Assume the values of the other variables do not change, and that the current solution is $(H,F,Y^*,P,I,S,B,R-x,O-(Y-Y^*-x))$. Subsequently, the feasibility of the new solution is validated through checking whether all constraints are met.

In constraints (3′), $\sum_{m=1}^{M} ut_{mt}X_{mt}+Y^* < \sum_{m=1}^{M} ut_{mt}X_{mt}+Y \le (AR_t+AO_t)A$, therefore the new solution satisfies constraints (3′).

In constraints (4′), $ut_{mt}X_{mt}+Y=R_{mt}+O_{mt}$. Subtracting $(Y-Y^*)$ from both sides, gives

$$\begin{aligned} ut_{mt}X_{mt}+Y^* &= R_{mt}+O_{mt}-(Y-Y^*) \\ &= R_{mt}-x+O_{mt}-(Y-Y^*-x) \end{aligned}$$

Therefore, the new solution satisfies constraints (4′). Due to $O-(Y-Y^*-x)<O$, we have $O-(Y-Y^*-x)\le AO_t=poAR_t$ and constraints (5) are satisfied.

In conclusion, the new solution $(H,F,Y^*,P,I,S,B,R-x,O-(Y-Y^*-x))$ satisfies all constraints in the integrated model and is feasible. Therefore,

$$f(H,F,Y^*,P,I,S,B,R-x,O-(Y-Y^*-x)) < f(H,F,Y,P,I,S,B,R,O)$$

The inequality means that if the family scheduling problem does not reach its optimal value, there is decreasing space for the objective function of the integrated model. That is, the integrated model reaches its optimal value only when the family schedule is optimal. □

According to Proposition 2, we get the simplified solution procedure of *Model* P^*. First, solve the family scheduling model in the planning horizon and acquire the globally optimal family schedule. The linearized family scheduling model can be formulated as follows:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{I} sc\,st_{ij}Y_{ijkt} \quad \text{s.t.(17)-(19), (22)-(24)}$$

Second, the resulting optimal family schedule in the planning horizon is substituted for the corresponding variables in *Model* P^*. Third, *Model* P^* is therefore simplified substantially with known family schedule and can be solved with much lower computational complexity.

5. Benchmark problem

In order to validate the performance of the integrated model we compare it with the traditional HPP models in Section 3 considering sequence dependency between families and globally optimal family scheduling in the planning horizon within the family disaggregation model. The family disaggregation model is formulated as follows:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}\left(sc\sum_{j=1}^{I}\sum_{k=0}^{I} st_{ij}Y_{ijkt}+c_{it}P_{it}\right)$$
$$\text{s.t. } (15)-(19),\ (22)-(24)$$

When comparing the total cost of *Model* P^* with the benchmark model, it is inappropriate to add up the costs of the APP and family disaggregating models because the aggregate plan may be infeasible. Therefore, the first step is to eliminate the infeasibility of aggregate plan so that the resulting adjustment cost is also considered and included in the total cost. In order to reduce the extent of infeasibility or eliminate the infeasibility of aggregate plans due to the capacity consumption of setup times, there are three possible ways to adjust the obtained aggregate plans.

First, anticipate the family setup times (Qiu and Burch, 1997; Qiu et al., 2001) optimized in the lower-level plans and incorporate them into the APP model. However, the expected family setup times are not necessarily accurate. Therefore the method may reduce the extent of infeasibility with the accuracy of anticipation but cannot eliminate it. Consequently, the method is not considered in this paper.

Second, backlog the products that cannot be produced due to the capacity utilization. When the aggregate plan is determined by the APP model, it will be implemented strictly as planned. Since setup times consume production capacity, some products scheduled for manufacturing are backlogged to the next period. Although the infeasibility of aggregate plans can be removed, the backlogged products result in high backordering costs, even lost sales and loss of reputation. However, the heuristic rule is easy to implement for practitioners.

Third, adjust the aggregate plan in the first period with the actual result of family setup times. Besides backlogging, there are other possible options to adjust the aggregate plans and eliminate the infeasibility, such as the available capacity that is not fully consumed in production, or available subcontracting capacity. Xue et al. (2009) integrate all possible options to adjust the aggregate plan into the family disaggregating planning model in the first period, and the aggregate plan is optimally adjusted.

The second and third methods are considered in the form of two adjustment rules:

Rule I: maintain the assigned capacity in the aggregate plan of the first period and backlog the types whose production capacity is used by setup times on the rolling horizon basis.

Rule II: use the idle capacity, namely the difference between available capacity and consumed capacity, to absorb the setup times. Besides, some families planned to be produced internally are subcontracted or backlogged to the following period in order to release capacity for the setup times (Xue et al., 2009).

6. Experimental analysis

6.1. Case description

The plant we investigated designs and manufactures small and mid-size molds for numerical control (NC) machines. The 68 kinds of NC molds produced in the plant are aggregated into 5 families and 2 types. Type 1 is comprised of Families 1–3, and Type 2 is comprised of Families 4 and 5. Each family contains multiple products. The planning horizon is 12 months. The peak season for both types is from March to June and the off-season is from July to February. In January the family disaggregation plan and schedule are employed to adjust the obtained aggregate plan according to *Rule* I.

Demands for the 2 types can be forecasted accurately according to production planners' expertise and are summarized, for each month of the current year, in Table 4. All costs and time parameters can be collected from the plant. The sequence-dependent setup times among the families are shown in Table 5.

A significant number of experiments with randomly generated data are used to validate the performance of *Model* P^* based on

Table 4
Actual demands of Types 1 and 2 in the planning horizon.

Month	January	February	March	April	May	June	July	August	September	October	November	December
Type 1	75	125	175	200	240	275	120	150	105	117	142	103
Type 2	31	45	50	70	80	100	30	60	48	42	36	22

Table 5
Sequence-dependent family setup times (h).

Family	1	2	3	4	5
1	0	8	5	18	18
2	5	0	3	12	15
3	6	7	0	9	11
4	15	18	15	0	8
5	12	14	12	5	0

Table 6
Average total cost of the integrated model and the benchmark models.

		No seasonality	Moderate seasonality	High seasonality
Average demand	Type 1	149.4	148.9	151.8
	Type 2	51.27	50.69	50.73
Low[a] ($1,000,000)	Integrated model	44.77	48.16	58.24
	Rule I	45.29	48.75	58.93
	Rule II	45.07	48.54	58.73
Medium ($1,000,000)	Integrated model	45.11	48.47	58.57
	Rule I	45.63	49.05	59.25
	Rule II	45.40	48.84	59.05
High ($1,000,000)	Integrated model	45.64	49.04	59.19
	Rule I	46.17	49.63	59.87
	Rule II	45.94	49.42	59.67

[a] The low, medium, and high mean low, medium, and high unit inventory cost, respectively.

the background of the above case. The average demand of ten randomly generated problem sets with no seasonality, moderate seasonality, and high seasonality, respectively, are used to validate the influence of varying unit inventory costs on the total cost of the integrated model and traditional HPP models. Normal distributions are widely applied in both research and practice (Zeng and Hayya, 1999), and the demands generated in this paper also follow normal distributions with different means and standard deviations determined by the actual data in Table 4.

The low, medium, and high unit inventory costs are assumed to be about 5%, 10%, and 20% of the unit cost of each type, respectively, while cost per unit setup time is assumed to be 50% of regular time cost per man-hour.

6.2. Solution results and comparison

Model P^* is applied in the plant and solved with LINGO 8. The number of variables, integer variables, and constraints of the integrated model are 3207, 3099, and 4103, respectively, while for the APP and family disaggregation models in the benchmark HPP system proposed in Section 5 they are 3279, 3099, and 4103, respectively. Therefore, the integrated model with more elaborate features can be solved with less computational effort than the traditional HPP model considering sequence-dependent setup times. The average time to solve the integrated model with known family schedule is about 20 h with an optimality gap of 0.05% on a DELL OptiPlex GX-620 computer with 2.0 GHz RAM, W8400 Processor 80,547, Pentium 4 Prescott Dt 630. In HPP systems, the average time to solve the APP model is about 4 s while that for the family disaggregation model is about 30 h with an optimality gap of 0.67%.

According to the above data and the solution procedure proposed in Section 4.4, the average total costs of the integrated model and benchmark model, obtained by averaging the total costs of the ten problem sets with different randomly generated demands and the same seasonality and unit inventory cost, are shown in Table 6. For Type 1, the varying rates of the generated average demands for medium and high seasonality are 0.3347% and 1.606%, respectively, while those for Type 2 are 1.131% and 1.053%. The differences are so small that their influence on the results can be neglected. The average total cost differences between the two adjustment rules of benchmark models and the integrated model are shown in Tables 7 and 8. Based on the solution of 90 problem sets with different seasonalities and unit inventory costs, the following conclusions can be reached:

(1) The integrated model outperforms the HPP models because all its costs are lower than those of the benchmark models as shown in Table 6. In this problem, the total setup time of 356 h, obtained from the global family scheduling in 12 months, is 12 h or 3% less than the 368 h of the sequential scheduling optimization obtained on a month by month basis. Since all families are produced in each period the setup cost savings are the same in each problem set. In the current global competitive environment where capacity and labor costs are at a premium, the 3% reduction in setup time and cost might help enterprises to set up cost advantages over competitors, especially for products with low profit margins. Besides reducing setup cost, the model may result in lower subcontracting or backordering quantities, which most managers would prefer.
(2) Adjustment *Rule* II outperforms *Rule* I, since all its average total costs are lower than those of *Rule* I, as shown in Table 6.
(3) When unit inventory cost is fixed, the total cost increases with seasonality, and the range is relatively large, even after eliminating the influence of slightly different average demands. Although the average demands of Types 1 and 2 with moderate seasonality are lower than those with no seasonality, all average total costs of the former are larger than those of the latter. The average demands of Types 1 and 2 with high seasonality are a little higher than those with moderate seasonality, by 1.948% and 0.07891%, respectively; but all total costs of the former are much larger than those of the latter, by 20.93%, 20.84%, and 20.70%, respectively. Therefore, increases in seasonality lead to substantial increases in total costs.
(4) When seasonality is fixed and average demand corresponding to each season is constant, the total cost for each method increases with the unit inventory cost, but the range is relatively small.
(5) When unit inventory cost is fixed the cost savings of the integrated model over *Rule* I and *Rule* II increase with seasonality as shown in Tables 7 and 8. Since the setup-cost savings of the globally optimal family schedule over traditional family schedules are constant in each month as demonstrated earlier, the cost savings obtained from the more reasonable planning of production through the integrated model increase with seasonality.

Table 7
Average total cost and percentage difference between *Rule* I and the integrated model.

	No seasonality		Moderate seasonality		High seasonality	
	Cost difference ($1,000,000)	Percentage (%)	Cost difference ($1,000,000)	Percentage (%)	Cost difference ($1,000,000)	Percentage (%)
Low	0.52	1.15	0.59	1.21	0.69	1.17
Medium	0.52	1.14	0.58	1.18	0.68	1.15
High	0.53	1.15	0.59	1.19	0.68	1.14

Table 8
Average total cost and percentage difference between *Rule* II and the integrated model.

	No seasonality		Moderate seasonality		High seasonality	
	Cost difference ($1,000,000)	Percentage (%)	Cost difference ($1,000,000)	Percentage (%)	Cost difference ($1,000,000)	Percentage (%)
Low	0.30	0.67	0.38	0.78	0.49	0.83
Medium	0.29	0.64	0.37	0.76	0.48	0.81
High	0.30	0.65	0.38	0.77	0.48	0.80

(6) When unit inventory cost is fixed the percentage of total cost savings of the integrated model over *Rule* I reaches its maximum value at moderate seasonality, while that of the integrated model over *Rule* II increases with seasonality, as shown in Tables 7 and 8.

(7) The cost savings and percentage of cost savings of the integrated model over *Rules* I and II do not fluctuate substantially with the varying of unit inventory costs, as shown in Tables 7 and 8. That is, varying unit inventory costs from 5% to 20% does not significantly change the production plans and schedules.

6.3. Managerial implications

Several managerial implications can be drawn from this study:

(1) Managers can use the proposed integrated model to plan and schedule production more effectively and economically, than they could with traditional hierarchical models, if setup times for the products within the same family can be ignored. This is often the case in cellular manufacturing production systems. The obtained production plans and family processing sequence cannot only provide lowest cost, but also ensure feasibility for their implementation and consistency when disaggregating the aggregate plans.

(2) When other parameters are fixed, substantial cost savings can be realized by lowering the seasonality of products, while the fluctuation of unit inventory costs within a small range does not change the total cost significantly. That is, managers can increase the profitability of products substantially by reducing seasonalities, instead of reducing the unit inventory costs.

(3) The savings in cost that managers can realize through the implementation of the production plans obtained with the integrated model increase with seasonality, while the cost savings of the integrated model over hierarchical models are almost constant when the unit inventory costs vary within 5% and 20% of unit cost.

(4) Managers can make tradeoffs between the solution complexity and the total cost of hierarchical models, monolithic models and the integrated model. The two adjustment rules proposed in this paper for traditional hierarchical models can be easily implemented in practice and need less computational effort. On the contrary, monolithic models can provide optimal total cost, but need higher computational effort. The proposed integrated model provides another alternative to decision-makers.

7. Conclusions and future research directions

Our integrated APP and family disaggregation model considers sequence-dependent family setup times as part of the production capacity. These setup times are incorporated into the original capacity constraints of the APP model to ensure feasibility of production plans. It is also seen that the family setup costs and all costs considered in traditional APP models are components of the objective function to ensure optimality.

A series of problem sets with different unit inventory costs and seasonalities are employed to compare the performance of the integrated model and traditional HPP models. Results show that the total cost of the integrated model is consistently less than that of the HPP models. For all problem sets with the same seasonality, the total cost is increasing with the unit inventory cost. However the variation of unit inventory cost only brings little fluctuation in total costs. The cost savings of the integrated model over HPP models brought about by varying unit inventory costs are constant. For the problem sets with the same unit inventory cost, the total cost increases drastically with increasing seasonality, as did the cost savings realized through the integrated model.

Finally, the integrated model is applied to a mold manufacturing plant. Our results show that the integrated model can save as much as $672,570, compared with the adjustment *Rule* I that the plant is using. Meanwhile, the production plans are also reasonable and feasible. The proposed integrated model with higher solution complexity and lower total cost, and adjustment *Rule* II with lower solution complexity and higher total cost can be used to meet differing managerial requirements.

The integrated approach proposed in this paper can be used in other similar hierarchical systems with some interactions between different levels, such as the hierarchical system of machine layout and production planning problem.

Our work can be extended in two ways. First, relax the assumption that all families are produced in each period. Without this assumption the integrated model will be harder to solve.

Therefore, metaheuristics like , Simulated Annealing and Tabu search, could be used to solve the problem to obtain an acceptable solution in a reasonable amount of time. Second, incorporate pricing decisions into the integrated model with varying production costs in different periods. Deng and Yano (2006) suggest that manufacturers should be more aggressive in suppressing demand prior to peak periods in the presence of capacity constraints while continuing to produce at capacity and building inventory so as to increase supply for the most profitable periods. That is, the seasonal variations in demand should be enlarged under the condition of constant production cost, but in fact the production costs during the peak season are higher since costly overtime, even subcontracting, is used to meet the demand. Therefore, joint production and pricing decisions with varying production costs in different periods are worth further study.

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