Multistage Optimisation Strategy For Solving Production Planning Problems

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Abstract: The production planning problems for a supply chain (SC) for low-volume production in make-to-order environment are considered. A generic framework for describing the strategic planning process for a SC is developed. The propagation of uncertainty and variability is included in approach proposed. Focus is on benchmarking the performance of a SC, understanding the impact of different characteristics to the planning of a SC, and the possibilities of improving the performance of a SC. The multiobjective optimization problem has been formulated and solved by use of global optimization methods (GA) and parallel computing.

Keywords: supply chain planning, low-volume and make-to-order production, multistage optimal stochastic planning, Pareto-optimal solutions, parallel computing.

1. INTRODUCTION

Let us concentrate on the strategic modelling of a SC and assume that a production system involves several independent companies in the production and delivery process. Such an approach has been stemmed from a desire to provide value to the industry by appropriate techniques and solutions to real problems.

It is important to notice that along with the recent pace for competitiveness the trial-and-error solution appears almost useless, and proper analysis tools are a key prerequisite for significant improvements in the manufacturing systems. What is required is an abstract model for SC planning to provide a sound theoretical basis, from which ideas for an increased performance of a SC can be sprouted, and a more effective SC realization can be derived.

Optimisation has become an essential part of the SC planning problem solving. Stochastic programming techniques are most suitable for supply chain systems, because they address the issues of optimal decision-making under uncertainty and variability (Dormer et al. 2005).

A natural response to the complexity of a supply chain is to manage the various SC entities independently, i.e., to allow each entity to use local information and to implement locally optimal management polices. This approach can lead to an inefficient SC. In order to improve the performance, the coordination of activities is needed.

2. UNCERTAINTES AND VARIABILITY IN A MANUFACTORING ENVIRONMENT

There are many sources of variability in a SC: variation of dimensions of parts (products), process times, machine failure/repair time, quality measures, setup times, etc. According to the variability law (Hopp et al. 2001): increasing variability always degrades the performance of a production system. From an analytical point of view, a SC is a network with the following sources of variability: supplier, manufacturing and demand.

Product variety results in recurrent manufacturing process variations that are related to machine set-ups, cycle times, labour, etc. It is essential to limit the number of options of products and to minimize process variation using the coordination of the product and process variety from both design and production perspective.

In the following it is assumed that in a production system products are relatively homogenous and form families of similar products, with a variability of product structure, dimensions, materials and other features of products, volume of orders.

The generic bill of the material (GBOM) (Tseng et al. 2000) concept has been used to describe the product families. GBOM is a structure common to a set of similar products in a family, it represents multiple product elements, variety parameters and their value instances, and various relationships. The GBOM of a product family could be represented in the form of and/or tree.

For each OR leaf to simulate the variability, the random number generator for a discrete distribution with the given probabilities of occurrences of the alternative components or features is used.

The process time, the actual time that is needed to manufacture a part or assemble the product and consequently workstation (WS) workload fluctuates as a result of product variety. To simulate the variability of process time, the random number generator for normal distribution with a given mean and standard deviation is used.

In order to deal with the stochastic nature of demand and supply, the stochastic scenario trees are used (Puigjaner et al. 2008). Each scenario has some probability ω_i of the occurrence of the demand or supply parameter ξ, which can be an objective measure derived by statistical information, forecasting methods or a subjective measure of likelihood. Scenarios can be the result of a discretisation of a continuous probability distribution $f_p(\xi)$.

3. MULTISTAGE OPTIMIZATION STRATEGY

To match the planning model more closely with the real situation, the problem of planning for a SC is recommended to be decomposed and a multi-level task structure to be used.

This has led to the optimisation scheme represented in Fig. 1:

The capacity planning for each enterprise on a lower level. It is considered that each enterprise is autonomous to develop its resources and to make decisions about the efficiency of resource utilization. The capacity of each enterprise is optimised on the basis of production tasks and the average function of profit for an enterprise i is determined on the upper level.

Integrated manufacturing planning for a SC on an upper level in order to realize the best coordinated strategy for the whole SC based on the capacities determined on the lower level.

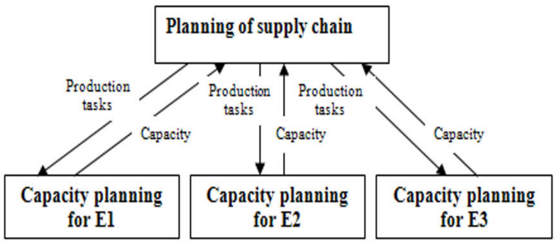

Fig. 1. Schematic representation of the hierarchy of planning tasks for a SC.

Each unit of a SC may have different resource arrangement to focus on one or more criteria of performance. There is a need to align the goal of each enterprise with the common objectives of a whole SC. The hierarchical approach that provides sharing of information and coordination between planning tasks of different levels must be used.

3.1 SC Planning – integrated model

A supply chain structure can be viewed as a network of suppliers, manufacturing plants, transporters, and customers, organized to acquire raw materials, convert these raw materials to finished products, and distribute these products to customers.

Strategic level supply chain planning involves different decisions, with time horizons more than one (a half) year. This work focused on:

a) configuration decisions, consider the number, capacity, and technology of the facilities

b) production decisions, consider the aggregate quantities for purchasing, processing and distribution of products.

In order to understand how different SCs work, a simple, yet representative SC network $\Theta = (N, A)$ is considered (Fig. 2), where N is the set of nodes and A is the set of arcs.

The set N consists of the set of suppliers S, the set of manufacturing enterprises E and the set of customers C, i.e., $N = S \cup E \cup C$. The manufacturing enterprises $E_i;\ i = 1, n$ include different workstations (manufacturing centres) M and assembling facilities F, i.e., $E = M \cup F$. Components are purchased by different suppliers $S_j;\ j = 1, m$, assuming that there are different customers $C_u;\ u = 1, l$.

Let P be the set of products flowing through the supply chain and k number of products ($p^1, p^2, \ldots, p^k$) assembled out of m components.

In order to solve the planning task the processing/purchasing times $a_{i,j}$ of each component of a GBOM on each enterprise and machining centre are required. The planning decisions consist of routing the flow of product $p^k \in P$ from the supplier to the customers. The flow of product p^k from a node i to a node j of the network is denoted by x_{ij}^k ($(ij) \in A$).

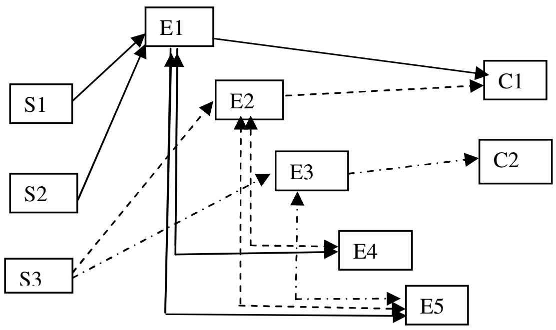

Fig. 2. Schema of a simple supply chain network, where nodes N represent: Si - suppliers; Ej - processing facilities; Cu – customers, and arcs represent the material flows for different products.

Because the real manufacturing systems have a stochastic nature and a significant variability of parameters, it is necessary to have a mechanism for estimating the activities at each node of a SC in response to the variability in demand, the variability in process times, etc.

The random data vector is represented by $\hat{\xi} = (\hat{d}_j, \hat{a}_{i,w})$, while $\xi = (d_j, a_{i,w})$ stands for its particular realization. The resulting formulation leads to a conceptual planning framework that guides the selection of supply chain strategies seeking higher total net profit:

$$\text{Max} \sum_{i=1}^{m}\sum_{j=1}^{k}(r_i \times X_i^k - X_{j,i} \times (M_i + c_j^k * \hat{a}_{i,j}) - \sum_{i=1}^{m}\sum_{w=1}^{v} Inv_w^i \times m_w^i \quad (1)$$

Subject to:

$$\sum_{i \in N} X_{ij}^K - \sum_{l \in N} X_{jl}^k = 0; \forall j \in E; \forall k \in K, \quad (2)$$

(balance of material flow for each product and node)

$$X_\delta^k \leq \hat{d}_{k,\delta} \quad (3)$$

(random demand for all product variants k and customers δ)

$$\sum_{i=1}^{m} \hat{a}_{iw} * X_i \leq m_w \times F_w \quad (4)$$

(for all machines w)

$$\sum_{i=1}^{m} X_{i,u} \times M_i \leq \mu_i \quad (5)$$

(for all materials and components (suppliers) u)

$X_i \geq 0$, for all *i,j*, (6)

where

r_i - revenue from one unit of the sold product p^i;

$\hat{a}_{i,j}$ - stochastic time required for the processing product i on the machine j;

c_j^k - per-unit cost of the processing product k at the facility j

F_w - capacity of the processing unit (machine) w;

M_i, μ_i - cost and resource of the material u;

Inv_w^i - investment costs to implement the machine w in the enterprise i;

X_i - quantity of the products p_i produced during the period analyzed;

m_w -number of processing facilities (machines) for WS w;

$\hat{d}_{k,\delta}$ -stochastic demand for the product k and the custom. δ.

The first, second and third term in formula (1) stand for the sales revenue, material and production costs and investments, respectively. The possibilities of outsourcing of production are considered whenever the enterprise is incapable of satisfying the demand. A prevalent approach for optimization under uncertainty is the multistage stochastic programming, which deals with problems involving a sequence of decisions that react to uncertainties that evolve over time (Puigjanel et al. 2008, Sousa et al. 2008). At each stage, we make decisions based on currently available information, i.e., past observations and decisions prior to the realization of future events. In our model, the two-staged stochastic optimization approach is used (Santoso et al. 2005, Alfieri at al. (2005), Shah et al. 2005). The first stage decisions based on the estimation of an average situation consist of the configuration decisions (capacity: numbers of machines m_w in enterprises). The second-stage consists of product processing from suppliers to customers in an optimal fashion based upon the given configuration and the realized uncertain scenario.

The objective for the first stage is to minimize the expected capacity investment costs $\mathrm{E}[Q(m^i, \xi)]$ for each enterprise *i*. The optimal value $Q(m, \xi)$ of the second stage problem is a function of the first stage decision variable and a realization (or a scenario) $\hat{\xi} = (c_j, a_{i,w}, r_i, \mu_u)$ of the uncertain parameters. $E[Q(m, \xi)]$ is estimated as a "response surface or surrogate " model for solving the second stage problem and using, for example, the regression analysis. The expectations $\mathrm{E}[Q(m^i, \xi)]$ are taken with respect to the probability distribution of $\hat{\xi} = (c_j, a_{i,w}, r_i, \mu_u)$.

There is a potential source of difficulty in solving the proposed problem: an evaluation of the objective function $\mathrm{E}[Q(m^i, \xi)]$ involves computing the expected value of the discrete value function $Q(m^i, \xi)$. This might involve solving a large number of linear programs of the second-stage problem, one for each scenario of the uncertain problem parameter realization. For example, as result, we have for E2 the function for estimating the objective function:

$$\mathrm{E}[Q(m^2, \xi)] = a_0^2 + a_1^2 \times m_1^2 + \ldots + a_n^2 \times m_n^2 =$$

$$987{,}1 - 100 \times m_1^2 - 100 \times m_2^2 - 20 \times m_3^2. \quad (7)$$

In (7) a_i^2 and m_i^2 stand for the regression coefficients and number of machines, respectively (for enterprise 2).

The tool for indentifying effective and robust policies in the face of variability and randomness is statistical simulation combined with multiple sources of variability.

Both, analytical and numerical techniques are used to estimate the influence of different sources of variability, to determine the robustness of planning decisions and to tune a supply chain.

Pooling variability is, for example, one strategy to reduce the effect of the variability and to increase the robustness of decisions. Considering the variability pooling (Hopp et al. 2001), for example, a less variable demand for products from a diversified customers than from any single customer can be expected. An analogous effect is encountered with the subcontracting of production between different enterprises (Fig. 3), etc.

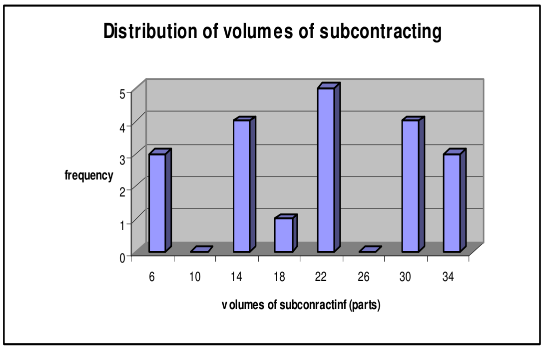

Fig. 3. Distribution of volumes of subcontracting.

In an academic example it is assumed that about 99% of acceptance of the total customer demand was received (with a minimal capacity of enterprises determined from the capacity planning task. According to the Pareto-optimal decisions, implementing three additional machines, 100% of acceptance of the possible product demand can be received.

In order to create an environment for cooperative behaviour, it is recommended to conduct negotiations between enterprises for the development of the SC performance, based on additional performance measures of enterprises. To choose different measures of performance for different enterprises of a SC, it is useful to estimate them. Different performance indicators to measure the efficiencies of performance for any enterprise are given by (Hopp et al. 2001). In the current study the following performance indicators are considered:

- Throughput TH efficiency E_{TH} in terms of whether the output is adequate to satisfy demand, $E_{TH} = \min(TH, D)/D$, where D is an average demand rate for a product.
- Utilization efficiency E_u is the fraction of time the workstations are busy, $E_u = \frac{1}{n}\sum_{i=1}^{n}\frac{TH(i)}{r^*(i)}$, where $r^*(i)$ - ideal rate of workstation i (not including detractors).
- Cycle time efficiency E_{CT} as the ratio of the best possible cycle time to the actual cycle time , $E_{CT} = T_0^*/CT$, where T_0^* is raw process time (not including detractors).

Performance of an ideal manufacturing system requires all efficiency measures equal to 1.0. For real enterprises the efficiency measures are <1.0.

3.2 Enterprise level planning - strategic capacity

Resource planning and investment decisions are the determining factors for a SC profitability. The resource investment process for the machine building industry is characterized by long lead times in investment costs. As a result, these decisions need to be made early, are costly and difficult to change later on. Investing in resource flexibility is one of the strategies of prime importance in today's competitive environment (Bish 2005).

There are several issues to address the strategic capacity planning:

a) How much and when should capacity be added?

b)What type of capacity should be added?

In the following it is assumed that a reasonable set of technology options can be generated and that cost, capacity and variability parameters can be estimated for each option.

To frame the capacity-planning problem at the plant level, we use the sc "modern views of the role of capacity". The traditional view is based on the only question whether there is enough capacity to meet a manufacturing task, and the answer is either yes or no. A modern view is more realistic and consistent, providing that cycle times (CT) and work in process (WIP) levels grow continuously with an increasing capacity utilized.

That means that for capacity planning we must consider other measures of performance in addition to the cost and processing times. WIP, mean CT, and CT variance, for example, which are affected by capacity decisions.

The resulting formulation of a capacity planning task for an enterprise is a bi-criterial nonlinear integer planning task:

find the number of machines $m_1^i, m_2^i, \ldots\ldots, m_n^i$ for each enterprise i and for each workstation that will give

$$\text{Min}\begin{cases} E\left[Q(m^i,\xi)\right] = a_0^i + a_1^i \times m_1^i + a_2^i \times m_2^i + \ldots + a_n^i \times m_n^i \\ \sum_{j=1}^{n} CT(m_j^i) = \sum_{j=1}^{n}\left(\frac{c_a^2 + c_e^2}{2}\right)\times\left(\frac{u^{\sqrt{2(m+1)}-1}}{m_j(1-u(m_j^i))}\right)\times t_e + t_e \end{cases} \qquad (8)$$

subject to constraints:

$$u(m_W^i) = \frac{\sum_{j=1}^{n} X_j \times t_{e,j}}{m_w \times F_w} \le [u_w], \quad \text{for } w=1,n, \qquad (9)$$

$$m_{j_i}^i \ge 0, \text{and } m_j^i \text{ are integer for } j = 1, n,$$

where

t_{ei} - mean effective process time for a machine, including out gages, set-ups, rework and other routine disruptions;

c_e - effective coefficient of variation (CV) for the machining time, considering out gages, set-ups, rework and other routine disruptions;

c_a - coef. of variation of the time between arrival to a WS;

F_w - resource of time for the WS w;

$[u_w]$- recommended values of utilization of a WS.

The posed problem could be solved using the Pareto-optimal approach and estimating the Pareto-optimal curve (Pareto-front) (You et al. 2008).

4. SOLUTION OF THE OPTIMISATION PROBLEM

Let us consider nonlinear integer multiobjective optimisation problem (8)-(9) formulated in section 3.2. The solution procedure is implemented in MATLAB 8 code. The core of the solution algorithm is the Genetic Algorithm and Direct Search Toolbox function **gamultiobj().** The function call can be written as

```
[m,result,eflag,output,population,scores] =
gamultiobj(@(m)obj_fn(m),N,[],[],[],[],lb,ub,options)
```

Both objectives given by formula (8) are coded in objective function **obj_fn**(). Above N stands for number of design variables, lb and ub are vectors defining lower and upper bounds of design variables, respectively. The structure options contain several input parameters (population size, number of generations, etc.). The return values m, result, eflag, output, population, scores stand for the set of design variables corresponding to optimal value of the objective function, the optimal values of the objective functions, exit-flag identifying the reason why the algorithm terminated, final population and the corresponding values of its fitness (objective) functions, respectively. Latter function covers the solution of nonlinear real-valued optimisation problems but does not support use of integer variables. In order to include integer design parameters the MATLAB built-in function **gaplotpareto()** has been completed by author's code. Repetitive runs of the program have been performed. The population size 100 and the number of generations 30 are found to be big enough for convergence. Note, that **gamultiobj()** uses only the tournament selection function and elite individuals automatically (sorts noninferior individuals above inferior ones). However, all functions used have open code and can be rewritten by user (including mutation, crossover, etc.).

5. NUMERICAL RESULTS

The relative importance of the objectives is not generally known for the whole SC until the system's best performance is determined and the trade-off between the objectives is understood. The Pareto front of the objective functions is much more informative and helpful for making the final decision in comparison with approaches where the objective functions are combined into a single objective (weighted summation, compromise programming, etc.). The Pareto front of the objectives (8) for enterprise E1 is given in Figures 4 and 5.

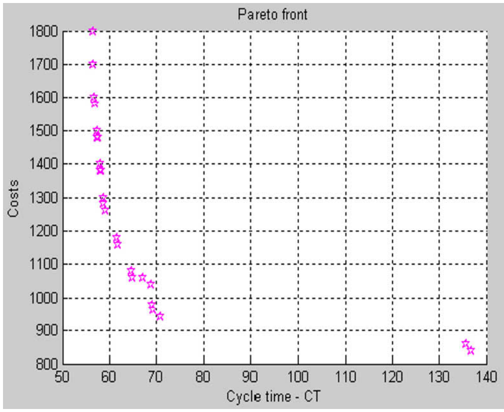

Fig. 4. Example of Pareto-optimal curve for capacity costs and CT for enterprise E1(integer programming).

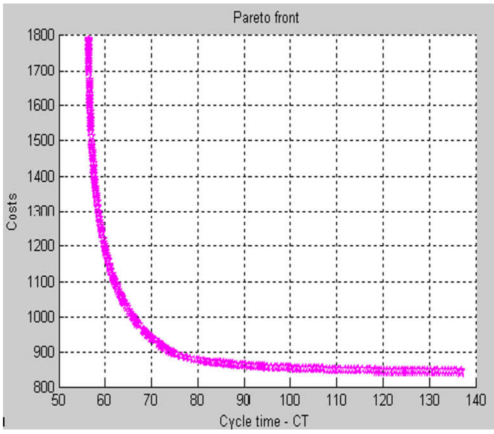

Fig. 5. Example of Pareto-optimal curve for capacity costs and CT for enterprise E1(continous variables).

It can be seen from Figure 5 that the capacity cost reduction has been achieved primarily by increasing the cycle time up to 80 and the influence of the further increase of the cycle time on capacity costs is marginal.

Obviously, the optimisation algorithm proposed gives good opportunity for applying parallel computing in both levels upper (enterprises can be treated in parallel) and lower (GA algorithms in enterprise level). Parallel computing is supported in MATLAB 8, but here are also some restrictions. For example the break and return statements, the global and persistent variables, references to a nested function, etc. cannot be used in the parfor-loops. Also nested parfor-loops are not supported. Note that the latter restriction is not principal, since a call of a function that contains another parfor-loop is supported.

In the current paper the main attention is paid to enterprise level modelling and the parallel computing is applied in realization of multiobjective genetic algorithm. The individuals of the population are evaluated in parallel. The number of parallel processes used is determined by value of the matlabpool (up to 4). Current study is performed on computer with dual-core processor and the value of the matlabpool is set to 2. The computational cost of the objective function and the population size are main factors influencing the efficiency of the parallel processing considered above. Interesting is fact that in the case of relatively small population size (up to 50) and computationally cheap objective function (depend on complexity of computation formulas and size of arrays used) the parallel computation may appear even more time consuming than sequational computing. Latter fact can be explained with the fact that the computational time of the work done for parallelisation exceed the time saved due to use of parallel computing. However, an analysis done show that in the case of computationally expensive objective function the computing time reduction up to 20% can be achieved (not all tasks of the algorithm are performed in parallel). The further improvement of the performance of a SC can be achieved by increasing the number of parallel processes and applying parallel computing in both levels.

6. DICUSSION

First, note that the optimisation problem posed is based on analytical approaches: queuing theory and stochastic process theory. Such an approach gives first estimate of the system's performance (the order of magnitude of the flowtime, bottle-necks, etc.), but has also some limitations:

– the queueing relations assume infinite buffers,

– the queueing relations cannot be used for flowlines that feature rework or bypassing,

– the queueing relations assume machines that process one lot type at a time

In order to overcome these shortcomings the simulation of the SC networks is necessary. Modelling of the SC networks with finite buffers is in progress.

7. CONCLUSIONS

In this study a model and a methodology for planning product manufacturing for a supply chain has been developed. The proposed stochastic approach is based on statistical simulation and on the use of a sample of an average approximation scheme. Such an approach is acceptable in the case of higher variability and multiple resources of uncertainty in the supply chain planning in the make-to-order environment. The bi-criterion optimisation framework was implemented to obtain the trade-offs between responsiveness and economics of the capacity planning model. The solution algorithm for nonlinear multiobjective optimisation problem has been treated and implemented in MATLAB 8 code including integer programming and parallel computing. The further improvement of the performance of a SC has been discussed.

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