$$A^T A m = A^T d$$

SVD: $A = USV^T$ (?)

$$V S \underbrace{\cancel{U^T U}}_{I} S V^T m = V S U^T d$$

$$V S^2 V^T m = V S U^T d$$

mult by V^T $\Downarrow$

$$S^2 V^T m = S U^T d$$

$$\Rightarrow \quad V^T m = S^{-1} U^T d$$

$$\boxed{m = \underbrace{V S^{-1} U^T}_{\text{Pseudo-inverse}} d}$$

$$\left[\quad\right] {\downarrow} n_d = \left[\quad\right]\left[\quad\right]\left[\quad\right]$$

$n_p \rightarrow$

$\hat{A}$ = pinv.

$$A m = d$$

$$m = \hat{A} d$$

Let $N = LL^T$

$$A^T (LL^T)^{-1} A m = A^T (LL^T)^{-1} d$$

$$A^T L^{T-1} L^{-1} A m = A^T L^{T-1} L^{-1} d$$

$$\Rightarrow (L^{-1}A)^T (L^{-1}A) m = (L^{-1}A)^T (L^{-1} d)$$

Let $\tilde{A} \equiv L^{-1} A$

$\tilde{d} \equiv L^{-1} d$

$$A^T N^{-1} A m = A^T N^{-1} d$$

$$\Rightarrow \tilde{A}^T \tilde{A} m = \tilde{A}^T \tilde{d}$$

Solved by $m = \tilde{A}_{pinv} \tilde{d}$

I want to know errors on m

Say we know truth: m_t

want $(m-m_t)^2 \Rightarrow \langle (m-m_t)(m-m_t)^T \rangle$

$\langle d = Am_t \rangle \Rightarrow d = Am_t + n \qquad \langle n^2 \rangle = N$

$d_t = Am_t$

$A^T N^{-1} A m = A^T N^{-1} d$

$A^T N^{-1} A m_t = A^T N^{-1} d_t \Rightarrow m - m_t \Rightarrow A^T N^{-1} A (m - m_t)$

$\shortparallel Am_t$

$= A^T N^{-1} (d - d_t)$

$= A^T N^{-1} n$

$$\langle (m-m_t)(m-m_t)^T \rangle \Rightarrow m-m_t = (A^T N^{-1} A)^{-1} A^T N^{-1} n$$

$$= \langle (A^T N^{-1} A)^{-1} A^T N^{-1} n n^T N^{-1} A (A^T N^{-1} A)^{-1 \cancel{T}} \rangle$$

$$\langle n n^T \rangle_{ij} = \langle n_i n_j \rangle = \sigma_{ij}^2 \delta_{ij} \Rightarrow \langle n n^T \rangle = N$$

$$\langle (A^T N^{-1} A)^{-1} A^T N^{-1} N N^{-1} A (A^T N^{-1} A)^{-1} \rangle$$

$$\langle (A^T N^{-1} A)^{-1} A^T N^{-1} A (A^T N^{-1} A)^{-1} \rangle$$

$$= (A^T N^{-1} A)^{-1}$$

prediction uncertainty:

$$\langle (d - d_{true})(d - d_{true})^T \rangle \qquad \langle d = Am \rangle \qquad d_t = Am_t$$

$$d - d_t \sim A(m - m_t)$$

$$\langle A(m - m_t)(m - m_t)^T A^T \rangle$$

$$= A(A^T N^{-1} A)^{-1} A^T$$