$$\chi^2 = (d-Am)^T N^{-1} (d-Am)$$

$$\nabla \chi^2 = -2A^T N^{-1}(d-Am)$$

$$r \equiv d-Am$$

best fit: $\nabla \chi^2 = 0 \Rightarrow A^T N^{-1} r = 0$

$$\nabla^2 \chi^2 = 2A^T N^{-1} A$$

$$\nabla^3 \chi^2 = 0$$

$$\chi^2 = (d - A(m))^T N^{-1} (d - A(m))$$

$$-2 \frac{\partial A}{\partial m}^T N^{-1}(d - A(m)) \quad A_m \equiv \frac{\partial A}{\partial m}$$

$$\frac{\partial^2 \chi^2}{\partial m_i \partial m_j} = -2 \frac{\partial^2 A}{\partial m^2} N^{-1}(d - A(m)) + 2 \frac{\partial A}{\partial m}^T N^{-1} \frac{\partial A}{\partial m}$$

$$= -2 A_{mm}^T N^{-1} r + 2 A_m^T N^{-1} A_m$$

$$\nabla^2 \chi^2 \approx A_m^T N^{-1} A_m^T$$

guess: m_0 : $(A_m^T N^{-1} A_m)^{-1} \Rightarrow A_m^T N^{-1} A_m \delta m = A_m^T N^{-1} r$

Levenberg - Marquardt

Newton's: $A_m^T N^{-1} A_m \, dm = A^T N^{-1} r$

one param: $\delta m_i = \dfrac{\nabla \chi^2_{m_i}}{(1+\lambda) \nabla^2 \chi^2_{m_i}}$

$$\left[A_m^T N^{-1} A_m + \lambda \, \mathrm{diag}(\,) \right] dm = A^T N^{-1} r$$