Control Systems

G V V Sharma[*]

Contents

Abstract—**This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

1 Mason's Gain Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 State-Space Model

5.1 Controllability and Observabiity

5.2 Second Order System

6 Nyquist Plot

7 Phase Margin

8 Gain Margin

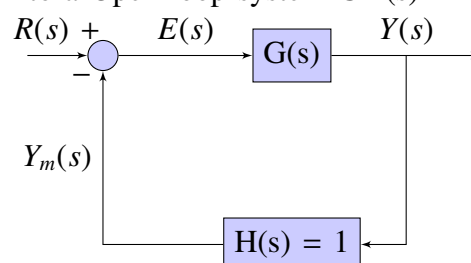## 9 Compensators

9.1 Phase Lead

10 Oscillator

10.1. A unity feedback control system is characterised by the open-loop transfer function

$$G(S) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} \quad (10.1.1)$$

the value of the k for which the system oscillates at 2 rad/s

Solution: Modelling Closed loop system G(s) into a Open loop system Gm(s)
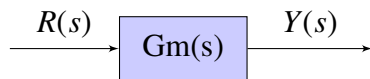
$$E(s) = R(S) - H(s)Y(s) \qquad (10.1.2)$$

$$G(S) = \frac{Y(s)}{E(s)} \qquad (10.1.3)$$

$$G(S) = \frac{Y(s)}{R(s) - H(s)Y(s)} \qquad (10.1.4)$$

$$Gm(S) = \frac{Y(S)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} \qquad (10.1.5)$$

$R(s) \rightarrow \boxed{Gm(s)} \xrightarrow{Y(s)}$

Characteristic equation :

$$1 + G(s)H(s) = 0 \qquad (10.1.6)$$

For a unity feedback system , H(s) = 1

$$1 + G(s) = 0 \qquad (10.1.7)$$

$$1 + \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} = 0 \qquad (10.1.8)$$

$$s^3 + ks^2 + 4s + 3 = 0 \qquad (10.1.9)$$

For the system to oscillate poles should lie on imaginary axis. Constructing the routh array for the characteristic equation(10.1.9).

$$\begin{pmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ k & 3 \\ \frac{3-4k}{k} & 0 \\ 3 & 0 \end{pmatrix} \qquad (10.1.10)$$

For the system to have poles on imaginary axis, any one of the entire row in a Routh's matrix should be all zeros.

$$\frac{3 - 4k}{k} = 0 \ or \ k = \frac{3}{4} \qquad (10.1.11)$$

substituting value of k in (10.1.9).

$$s^3 + \frac{3}{4}s^2 + 4s + 3 = 0 \qquad (10.1.12)$$

$$s = \frac{-3}{4}, +2j, -2j \qquad (10.1.13)$$

This show that at k = 3/4, system oscillates at frequency 2 rad/s.

10.2. Finding the system response gm(t) in time domain
Solution: From equation (10.1.5)

$$Gm(S) = \frac{2(s+1)}{s^3 + \frac{3}{4}s^2 + 4s + 3} \qquad (10.2.1)$$
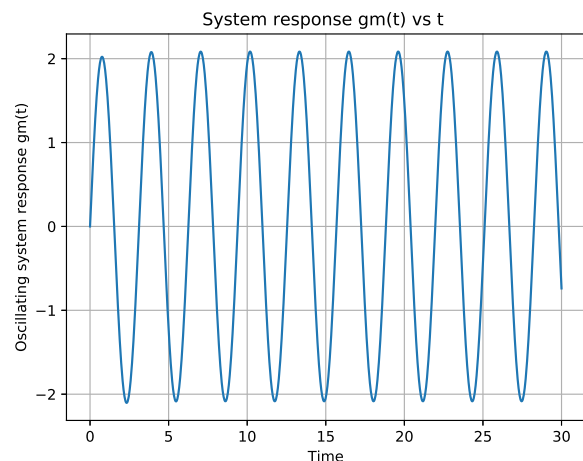
Partial Fractions

$$Gm(S) = \frac{8}{73(s + \frac{3}{4})} + \frac{-8s + 152}{73(s^2 + 4)} \qquad (10.2.2)$$

Apply inverse Laplace transform

$$gm(t) = \frac{8}{73}e^{\frac{-3t}{4}}u(t) + (\frac{-8}{73})\sin(2t) + (\frac{-152}{73})\cos(2t) \qquad (10.2.3)$$

10.3. Plotting gm(t) in time domain.

https://github.com/varunsankarmoparthi/
EE2227−CONTROLSYSTEMS/blob/
master/codes/sysresponseplot.py

This shows that system oscillates at 2 rad/sec.

10.4. Verifying Gm(s) using Rouths array

https://github.com/varunsankarmoparthi/
EE2227−CONTROLSYSTEMS/blob/
master/codes/routharrayverification.py

This shows that system is oscillating and stable.