Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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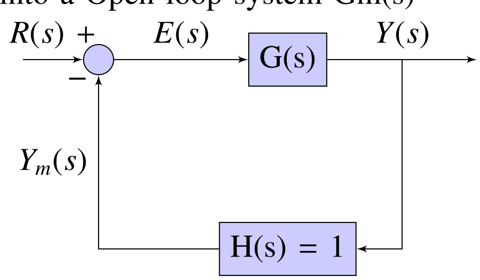
10.1. A unity feedback control system is characterised by the open-loop transfer function

$$G(S) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} \tag{10.1.1}$$

the value of the k for which the system oscillates at 2 rad/s

Solution: Modelling Closed loop system G(s) into a Open loop system Gm(s)

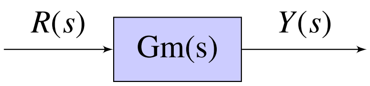


$$E(s) = R(S) - H(s)Y(s) \tag{10.1.2}$$

$$G(S) = \frac{Y(s)}{E(s)} \tag{10.1.3}$$

$$G(S) = \frac{Y(s)}{R(s) - H(s)Y(s)} \tag{10.1.4}$$

$$Gm(S) = \frac{Y(S)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} \tag{10.1.5}$$



Characteristic equation :

$$1 + G(s)H(s) = 0 \tag{10.1.6}$$

For a unity feedback system , H(s) = 1

$$1 + G(s) = 0 \tag{10.1.7}$$

$$1 + \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} = 0 \tag{10.1.8}$$

$$s^3 + ks^2 + 4s + 3 = 0 \tag{10.1.9}$$

For the system to oscillate poles should lie on imaginary axis. Constructing the routh array for the characteristic equation(10.1.9).

$$\begin{pmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ k & 3 \\ \frac{3-4k}{k} & 0 \\ 3 & 0 \end{pmatrix} \tag{10.1.10}$$

For the system to have poles on imaginary axis, any one of the entire row in a Routh's matrix should be all zeros.

$$\frac{3-4k}{k} = 0 \ or \ k = \frac{3}{4} \tag{10.1.11}$$

substituting value of k in (10.1.9).

$$s^3 + \frac{3}{4}s^2 + 4s + 3 = 0 \tag{10.1.12}$$

$$s = \frac{-3}{4}, +2j, -2j \tag{10.1.13}$$

This show that at k = 3/4, system oscillates at frequency 2 rad/s.

10.2. Finding the system response gm(t) in time domain

Solution: From equation (10.1.5)

$$Gm(S) = \frac{2(s+1)}{s^3 + \frac{3}{4}s^2 + 4s + 3} \tag{10.2.1}$$

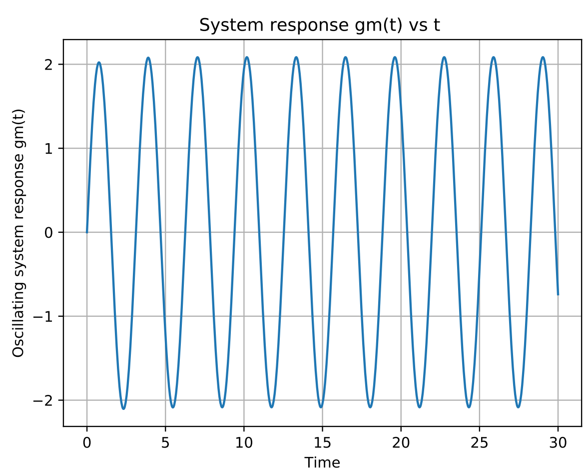
Partial Fractions

$$Gm(S) = \frac{8}{73(s + \frac{3}{4})} + \frac{-8s + 152}{73(s^2 + 4)} \tag{10.2.2}$$

Apply inverse Laplace transform

$$gm(t) = \frac{8}{73}e^{\frac{-3t}{4}}u(t) + (\frac{-8}{73})\sin(2t) + (\frac{-152}{73})\cos(2t) \tag{10.2.3}$$

10.3. Plotting gm(t) in time domain.
https://github.com/varunsankarmoparthi/EE2227-CONTROLSYSTEMS/blob/master/codes/sysresponseplo



This shows that system oscillates at 2 rad/sec.

10.4. Verifying Gm(s) using Rouths array
https://github.com/varunsankarmoparthi/EE2227-CONTROLSYSTEMS/blob/master/codes/routharrayverifi
This shows that system is oscillating and stable.