Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```


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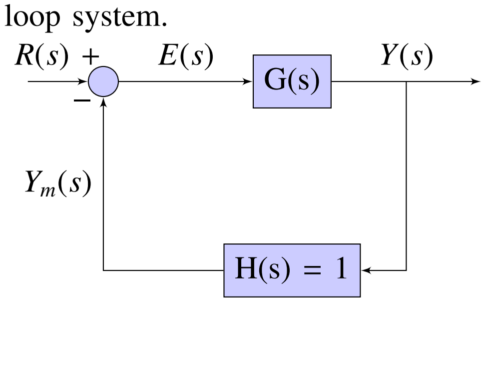
10.0.1. A unity feedback control system is characterised by the open-loop transfer function

$$G(S) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} \tag{10.0.1.1}$$

Find the value of the k for which the system oscillates at 2 rad/s. Verify your result through a program.

Solution: Fig. **??** models the equivalent closed loop system.



The characteristic equation is

$$1 + G(s) = 0 \tag{10.0.1.2}$$

$$\implies 1 + \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} = 0 \tag{10.0.1.3}$$

$$\text{or, } s^3 + ks^2 + 4s + 3 = 0 \tag{10.0.1.4}$$

Constructing the routh array for (10.0.1.4)

$$\begin{vmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & 4 \\ k & 3 \\ \frac{3-4k}{k} & 0 \\ 3 & 0 \end{vmatrix} \tag{10.0.1.5}$$

For the system to oscillate, poles should lie on the imaginary axis.

$$\implies \frac{3-4k}{k} = 0, \text{ or, } k = \frac{3}{4} \tag{10.0.1.6}$$

Substituting in (10.0.1.4),

$$s^3 + \frac{3}{4}s^2 + 4s + 3 = 0 \tag{10.0.1.7}$$

$$\implies s = \frac{-3}{4}, \pm 2\jmath \tag{10.0.1.8}$$

The following code verifies the result.

```
codes/ee18btech11030/ee18btech11030.py
```

10.0.2. Sketch the impulse response of the closed loop system.

Solution: The closed loop response

$$G_m(s) = \frac{G(s)}{1+G(s)} = \frac{2(s+1)}{s^3 + \frac{3}{4}s^2 + 4s + 3} \tag{10.0.2.1}$$

$$= \frac{8}{73(s+\frac{3}{4})} + \frac{-8s+152}{73(s^2+4)} \tag{10.0.2.2}$$

$$\implies g_m(t) = \frac{8}{73}e^{-\frac{3t}{4}}u(t) - \left(\frac{8}{73}\right)\sin(2t) + \left(\frac{152}{73}\right)\cos(2t) \tag{10.0.2.3}$$

The following code

```
codes/ee18btech11030/ee18btech11030_1.py
```

plot Fig. 10.0.2 . This shows that system oscillates at 2 rad/sec.

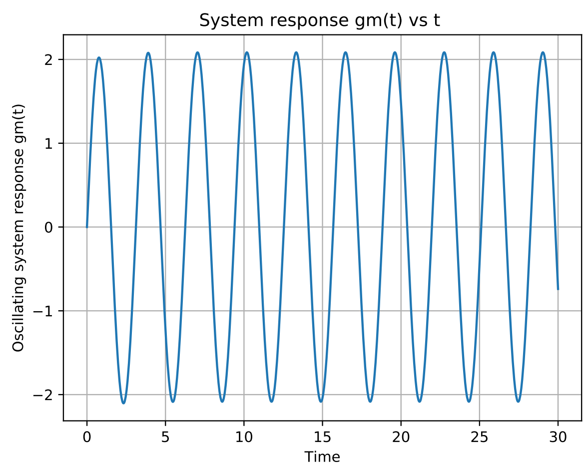


Fig. 10.0.2