

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

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1 MASON'S GAIN FORMULA

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

6 NYQUIST PLOT

7 PHASE MARGIN

8 GAIN MARGIN

9 COMPENSATORS

9.1 Phase Lead

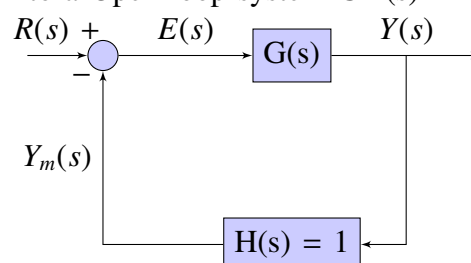
10 OSCILLATOR

10.1. A unity feedback control system is characterised by the open-loop transfer function

$$G(S) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} \quad (10.1.1)$$

the value of the k for which the system oscillates at 2 rad/s

Solution: Modelling Closed loop system $G(s)$ into a Open loop system $G_m(s)$

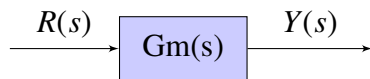


$$E(s) = R(s) - H(s)Y(s) \quad (10.1.2)$$

$$G(s) = \frac{Y(s)}{E(s)} \quad (10.1.3)$$

$$G(s) = \frac{Y(s)}{R(s) - H(s)Y(s)} \quad (10.1.4)$$

$$Gm(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} \quad (10.1.5)$$



Characteristic equation :

$$1 + G(s)H(s) = 0 \quad (10.1.6)$$

For a unity feedback system , $H(s) = 1$

$$1 + G(s) = 0 \quad (10.1.7)$$

$$1 + \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} = 0 \quad (10.1.8)$$

$$s^3 + ks^2 + 4s + 3 = 0 \quad (10.1.9)$$

For the system to oscillate poles should lie on imaginary axis. Constructing the routh array for the characteristic equation(10.1.9).

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & k & 3 \\ s^1 & \frac{3-4k}{k} & 0 \\ s^0 & 3 & 0 \end{array} \quad (10.1.10)$$

For the system to have poles on imaginary axis, any one of the entire row in a Routh's matrix should be all zeros.

$$\frac{3-4k}{k} = 0 \text{ or } k = \frac{3}{4} \quad (10.1.11)$$

substituting value of k in (10.1.9).

$$s^3 + \frac{3}{4}s^2 + 4s + 3 = 0 \quad (10.1.12)$$

$$s = \frac{-3}{4}, +2j, -2j \quad (10.1.13)$$

This show that at $k = 3/4$, system oscillates at frequency 2 rad/s.

10.2. Finding the system response $gm(t)$ in time domain

Solution: From equation (10.1.5)

$$Gm(s) = \frac{2(s+1)}{s^3 + \frac{3}{4}s^2 + 4s + 3} \quad (10.2.1)$$

Partial Fractions

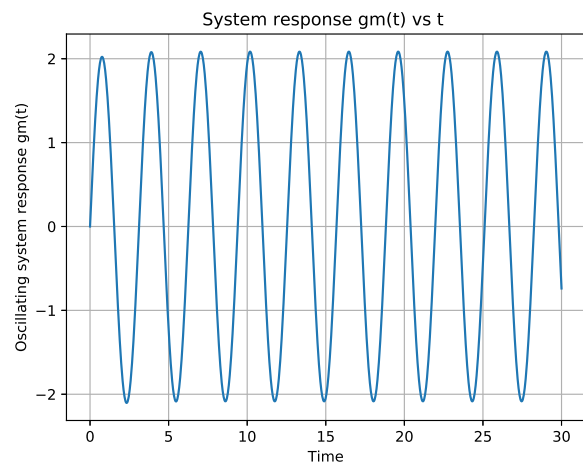
$$Gm(s) = \frac{8}{73(s + \frac{3}{4})} + \frac{-8s + 152}{73(s^2 + 4)} \quad (10.2.2)$$

Apply inverse Laplace transform

$$gm(t) = \frac{8}{73}e^{\frac{-3t}{4}}u(t) + \left(\frac{-8}{73}\right)\sin(2t) + \left(\frac{-152}{73}\right)\cos(2t) \quad (10.2.3)$$

10.3. Plotting $gm(t)$ in time domain.

codes : <https://github.com/varunsankarmoparthy/EE2227-CONTROLSYSTEMS/blob/master/codes/sysresponseplo>



This shows that system oscillates at 2 rad/sec.