

1. 利用下面的函数表, 作出差商表, 写出相应的牛顿插值多项式以及插值误差表达式并计算 $f(1.5)$ 的近似值

x	1.0	2.0	3.0	4.0
$f(x)$	2.0	4.0	5.0	3.0

$$\checkmark \quad x_0 = 1, \quad x_1 = 2, \quad x_2 = 3, \quad x_3 = 4$$

$$\text{则 } f[x_0, x_1] = \frac{2-4}{1-2} = 2$$

$$f[x_1, x_2] = \frac{5-4}{3-2} = 1$$

$$f[x_2, x_3] = \frac{3-5}{4-3} = -2$$

$$f[x_0, x_1, x_2] = \frac{2-1}{1-3} = -\frac{1}{2}$$

$$f[x_1, x_2, x_3] = \frac{1-(-2)}{2-4} = -\frac{3}{2}$$

$$f[x_0, x_1, x_2, x_3] = \frac{-\frac{1}{2}-(-\frac{3}{2})}{1-4} = -\frac{1}{3}$$

故差商表如下所示

i	x_i	$f(x_i)$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i]$
0	1	2			
1	2	4	2		
2	3	5	1	$-\frac{1}{2}$	
3	4	3	-2	$-\frac{3}{2}$	$-\frac{1}{3}$

$$\therefore \text{插值多项式为 } N_3(x) = 2 + 2(x-1) - \frac{1}{2}(x-1)(x-2) - \frac{1}{3}(x-1)(x-2)(x-3)$$

$$f(1.5) \approx N_3(1.5) = 3$$

$$\text{插值误差表达式为: } R_3(x) = \frac{f^{(4)}(\xi)}{24}(x-1)(x-2)(x-3)(x-4)$$

设利用 $\{x_i\}_{i=0}^2$ 构造的 Newton 插值多项式为 $N_2(x)$ 利用 $\{x_i\}_{i=1}^3$ 构造的 Newton 插值多项式为 $\tilde{N}_2(x)$

$$\text{则 } N_2(x) = 2 + 2(x-1) - \frac{1}{2}(x-1)(x-2)$$

$$\tilde{N}_2(x) = 4 + (x-2) - \frac{3}{2}(x-2)(x-3)$$

$$\therefore R_3(x) = \frac{x-x_0}{x_0-x_3} (N_2(x) - \tilde{N}_2(x)) = \frac{x-1}{-3} (x^2-5x+6) = -\frac{1}{3}(x^3-6x^2+11x-6)$$

2. 利用数据 $f(0)=2.0$, $f(1)=1.0$, $f(3)=0.25$, $f'(3)=0.6$, 构造出 3 次插值多项式, 给出其插值误差表达式, 并计算 $f(2)$ 的近似值.

作差商表

0	$f(0)=2$			
1	$f(1)=1$	$f[0,1]=-1$		
3	$f(3)=\frac{1}{4}$	$f[1,3]=-\frac{3}{8}$	$f[0,1,3]=\frac{5}{24}$	
3	$f'(3)=\frac{3}{5}$	$f[3,3]=\frac{3}{5}$	$f[1,3,3]=\frac{39}{80}$	$f[0,1,3,3]=\frac{67}{720}$

$$\therefore N_3(x) = 2 - x + \frac{5}{24}x(x-1) + \frac{67}{720}x(x-1)(x-3)$$

$$f(2) \approx N_3(2) = \frac{83}{360} \approx 0.230556$$

$$\text{插值余项: } R(x) = \frac{f^{(4)}(\xi)}{24}x(x-1)(x-3)^2$$

3. 设 $f(x) = 20x^3 + 3x + 2020$ 求 $f[1,2]$ 和 $f[1,2,3,4]$

$$f[1,2] = \frac{f(1)-f(2)}{1-2} = \frac{2043-2186}{1-2} = 143$$

$$f[2,3] = \frac{f(2)-f(3)}{2-3} = \frac{2186-2569}{2-3} = 383$$

$$f[3,4] = \frac{f(3)-f(4)}{3-4} = \frac{2569-3312}{3-4} = 743$$

$$f[1,2,3] = \frac{f[1,2]-f[2,3]}{1-3} = \frac{143-383}{1-3} = 120$$

$$f[2,3,4] = \frac{f[2,3] - f[3,4]}{2-4} = \frac{383-743}{2-4} = 180$$

$$f[1,2,3,4] = \frac{f[1,2,3] - f[2,3,4]}{1-4} = \frac{120-180}{1-4} = 20$$