# Homework 5

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Course: *ECE 6143 Machine Learning* – Professor: *Yury Dvorkin*
Due date: *December 2, 2021*

### Problem 1

**Solution.** First, we need to denote some events:

| Denote | Events |
|---|---|
| $A_1$ | The car is behind the door 1 |
| $A_2$ | The car is behind the door 2 |
| $A_3$ | The car is behind the door 3 |
| $B_1$ | I choose the door 1 first |
| $B_2$ | I choose the door 2 first |
| $B_3$ | I choose the door 3 first |
| $C_1$ | The host open the door 1 |
| $C_2$ | The host open the door 2 |
| $C_3$ | The host open the door 3 |

Suppose I chose door 1, and the host opened the door 3, so the question is to compare

$$P(A_1 \mid B_1, C_3)$$
$$P(A_2 \mid B_1, C_3)$$

We can get that $P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$
Because the host knows that the car is behind which door, so:

$$P(C_3 \mid A_1) = \frac{1}{2}$$
$$P(C_3 \mid A_2) = 1$$
$$P(C_3 \mid A_3) = 0$$
$$P(C_3) = \frac{\frac{1}{2} + 1 + 0}{3} = \frac{1}{2}$$

$$P(A_1 \mid B_1, C_3) = \frac{P(B_1, C_3 \mid A_1) * P(A_1)}{P(B_1, C_3)}$$
$$\because P(B_1, C_3 \mid A_1) = P(B_1 \mid A_1) * P(C_3 \mid A_1) = \frac{1}{3} * \frac{1}{2} = \frac{1}{6}$$
$$P(B_1, C_3) = P(B_1) * P(C_3) = \frac{1}{3} * \frac{1}{2} = \frac{1}{6}$$
$$\therefore P(A_1 \mid B_1, C_3) = \frac{\frac{1}{6} * \frac{1}{3}}{\frac{1}{6}} = \frac{1}{3}$$

Similarly:

$$P(A_2 \mid B_1, C_3) = \frac{P(B_1, C_3 \mid A_2) * P(A_2)}{P(B_1, C_3)}$$
$$\because P(B_1, C_3 \mid A_2) = P(B_1 \mid A_2) * P(C_3 \mid A_2) = \frac{1}{3} * 1 = \frac{1}{3}$$
$$P(B_1, C_3) = P(B_1) * P(C_3) = \frac{1}{3} * \frac{1}{2} = \frac{1}{6}$$
$$\therefore P(A_2 \mid B_1, C_3) = \frac{\frac{1}{3} * \frac{1}{3}}{\frac{1}{6}} = \frac{2}{3}$$

Conclusion: Change the door.

## Problem 2

**Solution.** The distribution:

$$p\left(x_1, \ldots, xt\right) = \prod_{i=1}^{5} p\left(x_i \mid \text{ parents }_i\right) = p\left(x_1\right) p\left(x_2 \mid x_1\right) p\left(x_3\right) p\left(x_4 \mid x_1, x_3\right) p\left(x_5 \mid x_2, x_4\right)$$

By using the Bayes ball algorithm:
1. **False**: Because we do not know $x_1$, so $x_4$ would affect each other. If the student is hard working, maybe the lead him have good performance in testing.
2. **False**: Because we know $x_5$, so $x_2$ and $x_4$ would affect each other. If the student have good grade, but he is good at taking tests, maybe it leads him understand the material better.
3. **True**: Because we already know $x_1$ and $x_3$, so that $x_2$ and $x_4$ would not affect each other.
4. **False**: We know $x_4$, so that $x_1$ and $x_3$ would affect each other, and $x_1$ would have impact in $x_5$, so that $x_3$ and $x_5$ are not conditionally independent.
5. **True**: $x_5$ and $x_3$ are conditionally independent because we already know $x_1, x_2, x_4$, therefore $x_5$ and $x_3$ have been determined and can not affect each other.
6. **False**: Using D-separation method, we get route 1-2-5-4-3
7. **True**: Using D-separation method, we get that 1-4-3 blocked, 1-2-5 blocked
8. **True**: Using D-separation method, we get that 2-5-4-3 blocked, 2-1-4-3 blocked
9. **False**: Using D-separation method, we get route 2-5-4-3
10. **False**: Using D-separation method, we get route 3-4-1-2

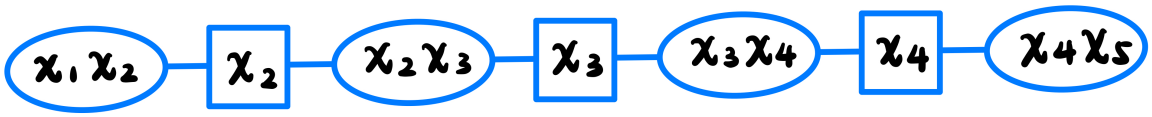
## Problem 3

**Solution.** See the **Figure 1** in the **final page**.

## Problem 4

**Solution.** This is the Constructed Junction Tree:
(The source code is in the attachment.)

Result:




| | $x_2 = 0$ | $x_2 = 1$ | |
|---|---|---|---|
| $x_1 = 0$ | 0.0405 | 0.4451 | 0.4856 |
| $x_1 = 1$ | 0.3237 | 0.1908 | 0.5145 |
| | 0.3642 | 0.6359 | |

| | $x_2 = 0$ | $x_2 = 1$ | |
|---|---|---|---|
| $x_1 = 0$ | 0.2601 | 0.1040 | 0.3641 |
| $x_1 = 1$ | 0.0578 | 0.5780 | 0.6358 |
| | 0.3179 | 0.6820 | |

| | $x_2 = 0$ | $x_2 = 1$ | |
|---|---|---|---|
| $x_1 = 0$ | 0.1192 | 0.1987 | 0.3179 |
| $x_1 = 1$ | 0.6395 | 0.0426 | 0.6821 |
| | 0.7587 | 0.2413 | |

| | $x_2 = 0$ | $x_2 = 1$ | |
|---|---|---|---|
| $x_1 = 0$ | 0.5690 | 0.1897 | 0.7587 |
| $x_1 = 1$ | 0.0603 | 0.1810 | 0.2413 |
| | 0.6293 | 0.3707 | |

## Problem 5

**Solution.**
(The source code is in the attachment.)

Day 1: **Happy** Day 2: **Angry** Day 3: **Angry** Day 4: **Angry** Day 5: **Angry**

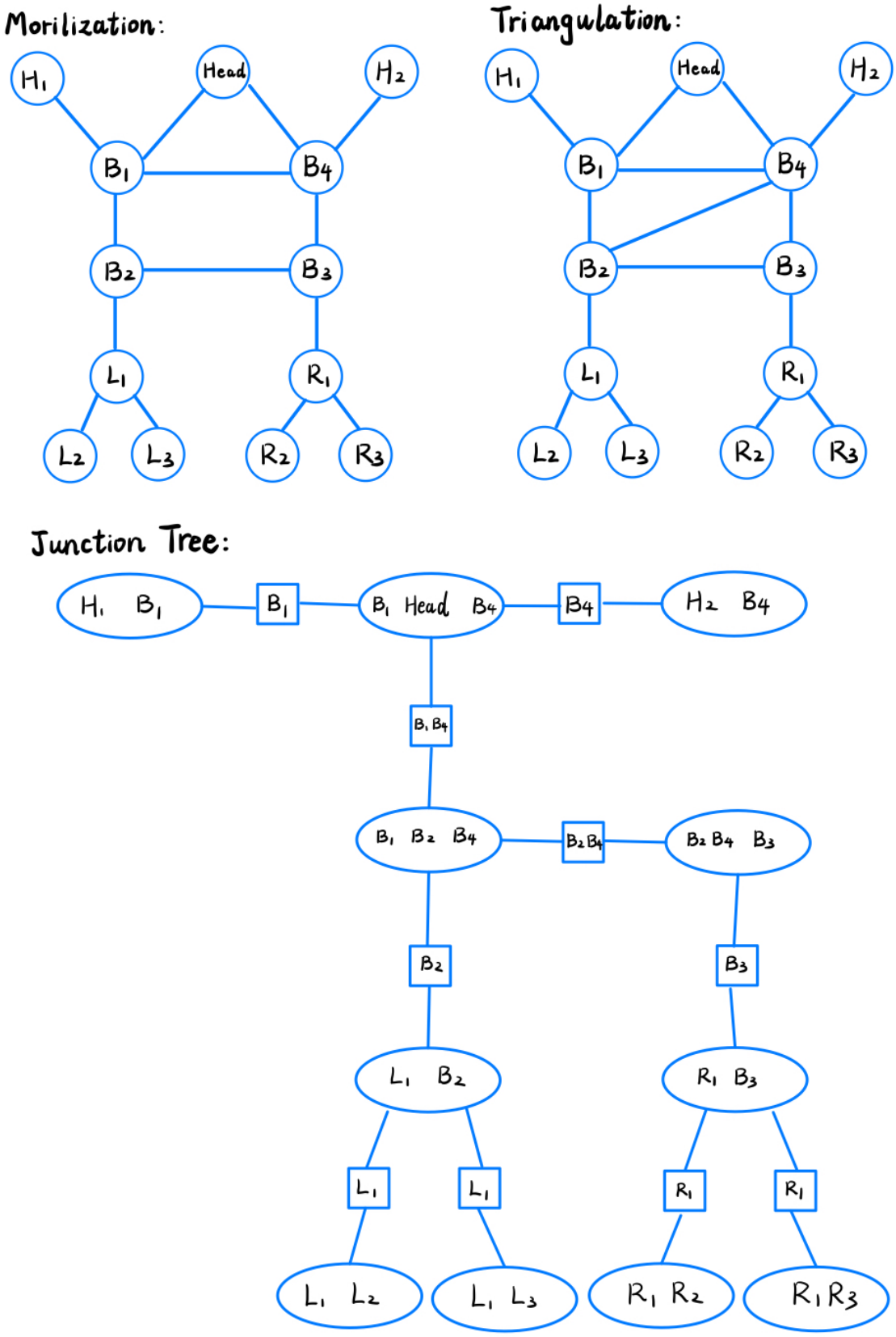



Figure 1: Problem 3