MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

(a) By issuing bonds, governments are able to raise money to finance projects without reducing expenses on other projects, whereas printing more money leads to inflation or even hyperinflation, raising the price level significantly.

(b) If the Federal Reserve plans to raise the federal funds rate, then the inflationary pressure will be reduced, and so that the yield curve might be flatten in the long-term.

(c) Since the beginning of the COVID-19 pandemic, the US Fed has employed quantitative easing by increasing the purchase of Treasury securities and mortgage-backed securities (MBS) in order to increase the supply of domestic money and decrease the interest rates.

2.

The selected 11 bonds are: CAN 0.50 Mar 22, CAN 0.25 Aug 22, CAN 1.75 Mar 23, CAN 0.25 Aug 23, CAN 2.25 Mar 24, CAN 1.50 Sep 24, CAN 1.25 Mar 25, CAN 0.50 Sep 25, CAN 0.25 Mar 26, CAN 1.00 Sep 26, CAN 1.25 Mar 27. The maturity dates of 11 selected bonds are from March 1, 2022 to March 1 to 2027, a five year period from now. Since they have semi-annual coupon payments, I choose those bonds with maturity dates close to 0.5, 1, 1.5, . . . , 4.5, 5 years from now, which allows me to calculate the yield curves using the bootstrapping technique. Also, most of those bonds have a duration of 5 years and coupon rates ranging from 0.25% to 2.25%.

3.

The Principal Component Analysis (PCA) is a technique to reduce the dimensionality that aims to find a low-dimensional representation of the data by projecting the data onto a subspace which maximizes the projected variance, or equivalently, minimizes the reconstruction error. The eigenvectors and eigenvalues always come in pairs, in which the eigenvectors determine the direction and the eigenvalues determine the magnitude that tells us how much variance (spread out) of data in this direction. Thus, the optimal subspace is given by the top eigenvectors with large eigenvalues of the empirical covariance matrix.

Empirical Questions - 75 points

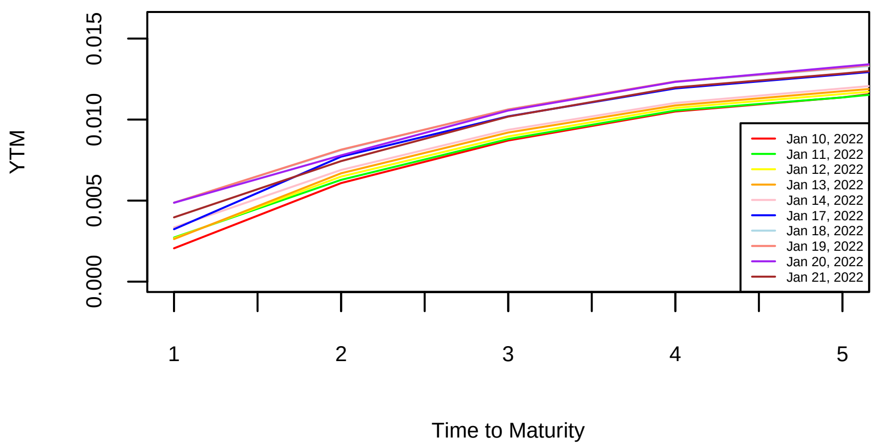
4.

(a) Since the maturity date of 11 bonds are not exactly 6 months apart and 0.5, 1, 1.5,. . . ,4.5, 5 years from now, a linear interpolation technique can be applied in this case. Then the bonds' yield in 0.5 year, 1 year, 1.5 years. . . can be calculated using the function approx() in R, which can help us do the linear interpolation.

Table 1: YTM of 11 Selected Bonds

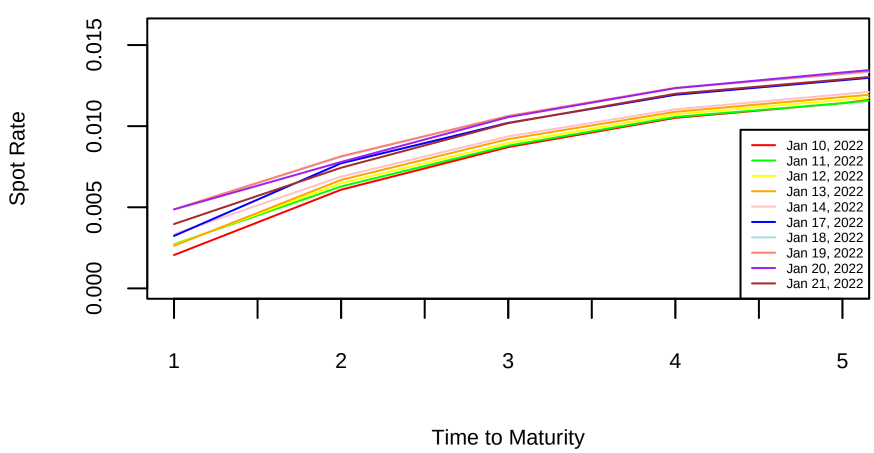
	2022-01-10	2022-01-11	2022-01-12	2022-01-13	2022-01-14	2022-01-17	2022-01-18	2022-01-19	2022-01-20	2022-01-21
CAN 0.50 Mar 22	0.0020563	0.0027185	0.0026734	0.0026264	0.0033433	0.0032352	0.0048715	0.0048701	0.0048686	0.0039661
CAN 0.25 Aug 22	0.0060815	0.0062811	0.0064827	0.0066865	0.0068923	0.0077105	0.0081148	0.0081442	0.0077928	0.0074379
CAN 1.75 Mar 23	0.0087103	0.0087980	0.0089618	0.0092016	0.0093670	0.0102023	0.0106302	0.0106213	0.0105580	0.0101847
CAN 0.25 Aug 23	0.0104999	0.0105586	0.0107207	0.0108730	0.0110258	0.0119225	0.0123278	0.0123493	0.0123296	0.0119851
CAN 2.25 Mar 24	0.0113701	0.0113669	0.0115795	0.0117510	0.0119230	0.0127887	0.0131619	0.0131961	0.0132652	0.0128387
CAN 1.50 Sep 24	0.0126074	0.0123784	0.0124454	0.0126016	0.0128227	0.0137075	0.0140283	0.0142294	0.0141778	0.0137774
CAN 1.25 Mar 25	0.0135365	0.0134100	0.0135813	0.0136626	0.0139770	0.0147921	0.0152062	0.0152752	0.0152921	0.0147844
CAN 0.50 Sep 25	0.0142934	0.0141379	0.0140558	0.0140728	0.0143521	0.0151127	0.0156075	0.0158902	0.0158050	0.0154288
CAN 0.25 Mar 26	0.0146557	0.0145048	0.0145968	0.0145614	0.0148301	0.0156068	0.0161749	0.0163030	0.0162492	0.0158089
CAN 1.00 Sep 26	0.0151443	0.0150467	0.0151291	0.0159410	0.0164085	0.0161285	0.0166489	0.0167818	0.0167015	0.0161973
CAN 1.25 Mar 27	0.0170830	0.0170028	0.0170699	0.0179818	0.0184528	0.0179997	0.0184718	0.0186043	0.0185239	0.0180496

Figure 1: YTM Curves Corresponding to Each Day



(b)

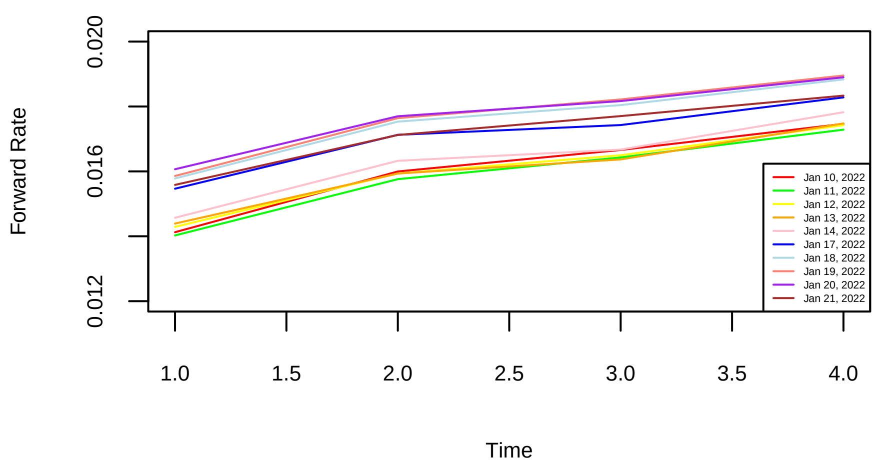
Figure 2: Spot Curves Corresponding to Each Day



First of all, listing the bonds by the maturity dates, so that CAN 0.50 Mar 22 is the first one and CAN 1.25 Mar 27 is the last one. Consider an equation, $P_i = \sum_j cf_i^j(1+\frac{r_j}{n})^{-nt_i^j}$, where P_i is the dirty price of the i^{th} bond, cf_i^j is the j^{th}cash flow of the i^{th} bond , n is 2 since the bond compounds 2 times a year, t_i^j is the time between the calulated date and the date of j^{th}cash flow of the i^{th} bond, and finally, r_j is the spot rate of j^{th} bond, which is unknown. By solving r_j, one can obtain the spot rate of j^{th} bond. Also, using the boostrapping technique, r_1 is needed to solve for r_2, and r_1 and r_2 are needed to solve for r_3, so on so forth. For example, CAN 0.50 Mar 22, the first bond, which has the maturity less then 6 months, so it can be treated as a zero coupon bond. By finding the root of $P = cf_1^1(1+\frac{r_1}{2})^{-2t_1^1}$, r_1 can be calculated. Next, the second bond, CAN 0.25 Aug 22, has a maturity between 6 months and 1 year, we can extend the former equation to $P = cf_2^1(1+\frac{r_1}{2})^{-2t_2^1} - cf_2^2(1+\frac{r_2}{2})^{-2t_2^2}$, and solve for the unknown spot rate, r_2. Repeating the same thing for all remaining bonds and all remaining days, we can obtain the spot curve with terms ranging from 1-5 years from my chosen bonds in part 2.

(c)

Figure 3: Forward Curves Corresponding to Each Day



Spot rate of each bond are required to calculated for the forward rate. The formula I used is $\frac{f_{1,i}}{2} = \left(\frac{(1+\frac{r_{3+2i}}{2})^{2i}}{(1+\frac{r_3}{2})^2}\right)^{\frac{1}{2i-2}} - 1$, where $i = 2, 3, 4, 5$. In this formula, $f_{1,i}$ is the 1-year forward curve with terms ranging from 2-5 years from my chosen data. Since the time to maturity of bonds 3, 5, 7, 9, and 11 is approximately 1, 2, 3, 4, and 5 years, we use r_{3+2i} as the 2,..,5-year forward rate and r_3 to be the 1-year spot rate. Also, since the bond compounds 2 times a year, so we divide each rate by 2 and times the power term by 2.

5.

Table 2: Covariance Matrix for the Time Series of Daily Log-Returns of Yield

	log_returns_ytm1	log_returns_ytm2	log_returns_ytm3	log_returns_ytm4	log_returns_ytm5
log_returns_ytm1	0.0011319	0.0008985	0.0007941	0.0007237	0.0000998
log_returns_ytm2	0.0008985	0.0007467	0.0006693	0.0006044	0.0000398
log_returns_ytm3	0.0007941	0.0006693	0.0006345	0.0005846	0.0000828
log_returns_ytm4	0.0007237	0.0006044	0.0005846	0.0005702	0.0000496
log_returns_ytm5	0.0000998	0.0000398	0.0000828	0.0000496	0.0006387

Table 3: Covariance Matrix for the Time Series of Daily Log-Returns of Forward Rates

	log_returns_f12	log_returns_f13	log_returns_f14	log_returns_f15
log_returns_f12	0.0005774	0.0005356	0.0004785	0.0003735
log_returns_f13	0.0005356	0.0005518	0.0005109	0.0004125
log_returns_f14	0.0004785	0.0005109	0.0005176	0.0003996
log_returns_f15	0.0003735	0.0004125	0.0003996	0.0003505

6.

Table 4: Eigenvalues of Covariance Matrix (YTM)

x
0.0029733
0.0006329
0.0000910
0.0000219
0.0000029

Table 5: Eigenvectors of Covariance Matrix (YTM)

x_1	x_2	x_3	x_4	x_5
-0.6062856	-0.0108189	0.6156028	0.4403023	-0.2438600
-0.4962449	-0.0763313	0.1670596	-0.5712704	0.6274199
-0.4547909	0.0036111	-0.3901238	-0.4481357	-0.6634229
-0.4192797	-0.0390733	-0.6640039	0.5279023	0.3210852
-0.0594014	0.9962513	-0.0051433	-0.0166595	0.0604215

Table 6: Eigenvalues of Covariance Matrix (Forward)

x
0.0018740
0.0000902
0.0000249
0.0000081

Table 7: Eigenvectors of Covariance Matrix (Forward)

x_1	x_2	x_3	x_4
-0.5293275	0.7519690	0.0433578	-0.3904807
-0.5395572	0.0412977	0.1494937	0.8275410
-0.5106573	-0.4105899	-0.7336709	-0.1799225
-0.4097798	-0.5140555	0.6614369	-0.3610105

The first eigenvalue of ytm covariance matrix is 2.973265e-03, corresponding to the eigenvector (-0.60628562,-0.49624489,-0.45479093,-0.41927966,-0.05940138), indicating that the variance of this time series is 2.973265e-03 in the negative direction.

The first eigenvalue of forward covariance matrix is 1.876378e-03, corresponding to the eigenvector (-0.5295236,-0.5395006,-0.5105605,-0.4097217), indicating that the variance of this time series is 1.876378e-03 in the negative direction.

References and GitHub Link to Code

References:

- Federal Reserve issues FOMC statement. Board of Governors of the Federal Reserve System. (n.d.). Retrieved February 13, 2022, from https://www.federalreserve.gov/newsevents/pressreleases/monetary20210127a.htm
- Team, T. I. (2022, February 8). Quantitative easing (QE). Investopedia. Retrieved February 13, 2022, from https://www.investopedia.com/terms/q/quantitative-easing.asp

GitHub Link : git@github.com:ElsaGrx/APM466-A1.git