Document for : **MATLAB Central**, **File exchange**.

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2D Auto correlation function

In this document, we explain the method of calculating the auto-correlation function, it is used in statistical studies,signal and image processing, estimation filtering ...etc.

the Wiener-Khinchine theorem **[1]** (also known as the **Wiener–Khintchine theorem** and sometimes as the **Wiener–Khinchin–Einstein theorem** or the **Khinchin–Kolmogorov theorem**) states that the power spectral density of wide-sense-stationary-random process is the Fourier transform of the corresponding auto-correlation function,
Norbert Wiener first published this theorem in 1930, and Aleksandr Khinchin did so independently in 1934. Albert Einstein had probably anticipated the idea in a brief two-page memo in 1914 .

Let's consider a 2d signal X , its auto-correlation function $R_x(u,v)$ is related to the power spectral density $S_x(a,b)$ via the theorem [1] :

$$S_x(a,b)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_x(a,b)e^{-2\pi i(au+bv)}du\,dv=F[R_x(a,b)] \quad (1)$$

On the other hand , the power spectral density is the square modulus of the Fourier transform of the 2d signal X :

$$S_x=|F(X)|^2=F(X).F(\bar{X}) \quad (2)$$

with F(X), the Fourier transform, and $F(\bar{X})$ the conjugate transform .

$$F(X)=Y(a,b)=\sum_{u=1}^{N}\sum_{v=1}^{M} X(u,v).\exp(\frac{-2\pi i ua}{N}+\frac{-2\pi i vb}{M}) \quad (3)$$

$$F(\bar{X})=Y(\bar{a},b)=\sum_{u=1}^{N}\sum_{v=1}^{M} X(u,v).\exp(\frac{2\pi i ua}{N}+\frac{2\pi i vb}{M}) \quad (4)$$

From the equations (1) and (2) the Auto-correlation function of the Signal X is given by :

$$R(X)=F^{-1}[F(X).F(\bar{X})] \quad (5)$$

Application in MATLAB:

Let's take two examples, First let's try to visualize the auto-correlation function of random gray scale image , we choose the 'circui.tif" :

```
I=imread('circuit.tif');
I=im2double(I); % convert from uint8 to double
```

Now we implement the equation (5) :

```
B=abs(fftshift(ifft2(fft2(I).*conj(fft2(I)))));
```

Note that B and I have the same size 280x272, and for better representation we normalize the auto-correlation function :

```
[n p]=size(I);
B=B/(n*m);
figure,imshow(I);
figure, surf(B), shading interp
```

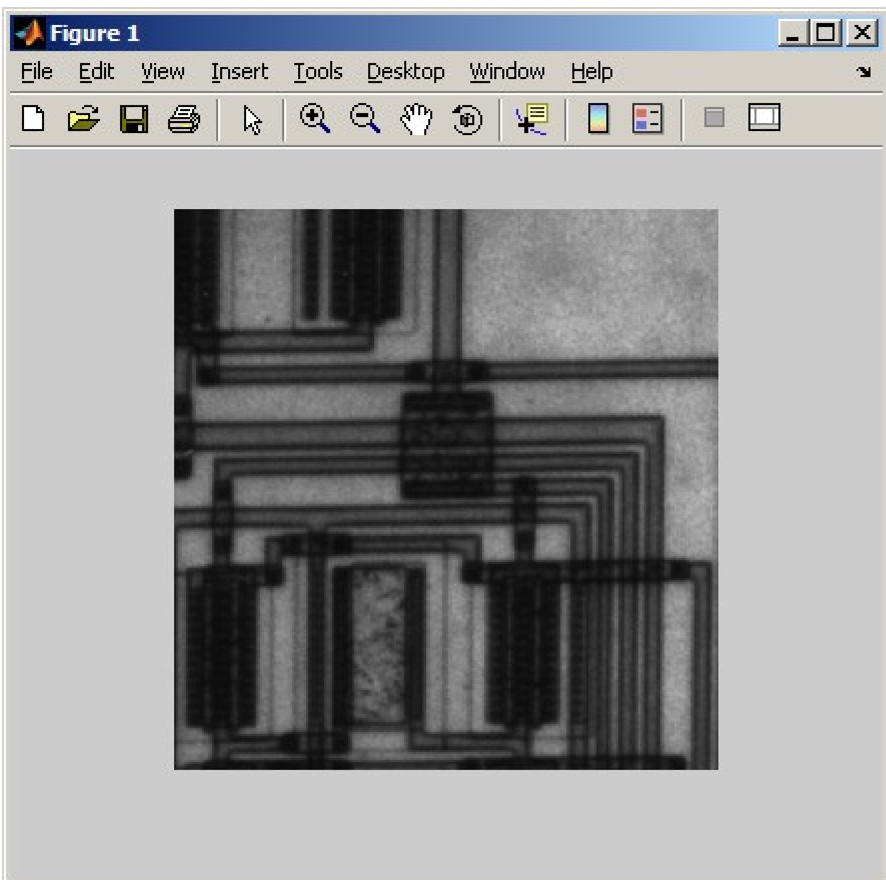


Figure 1 : Image 'ciruit.tif'

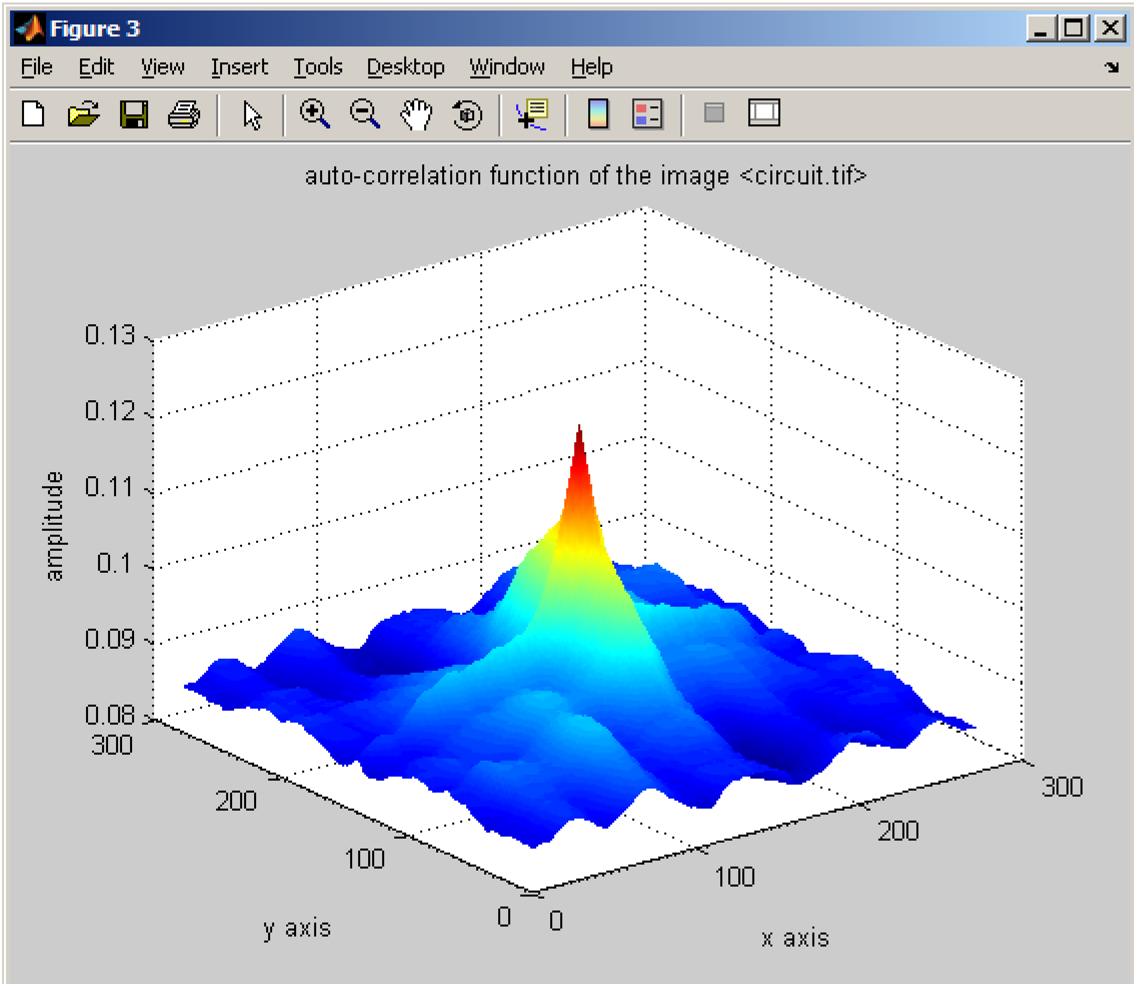


Figure 2 : Auto-correlation of the image 'circuit.tif'

Now , we take special case, a two dimensional Gaussian process, lets' consider a 2d Gaussian signal of size(460,480) with mean=0.0036 and variance = 1.0046 :

```
I2=randn(460,480);

 mean(I2(:))

ans =

   0.0036

>> var(I2(:))

ans =

   1.0046
```

The signal fluctuates around the value zeros as we can see in the figure n° 3:

surf(I2);

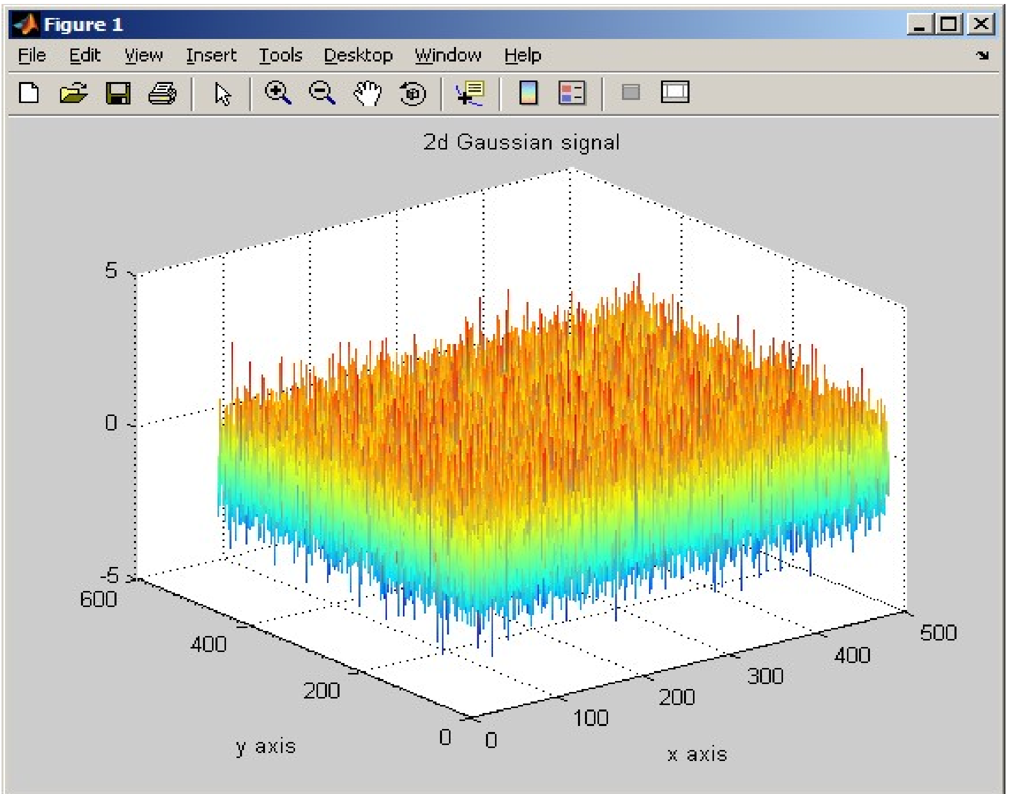


Figure 3 : 2D Gaussian signal

The signal can be seen as an image that contains noise if we set the azimuth to 0 and elevation to 90:
view(0,90) :

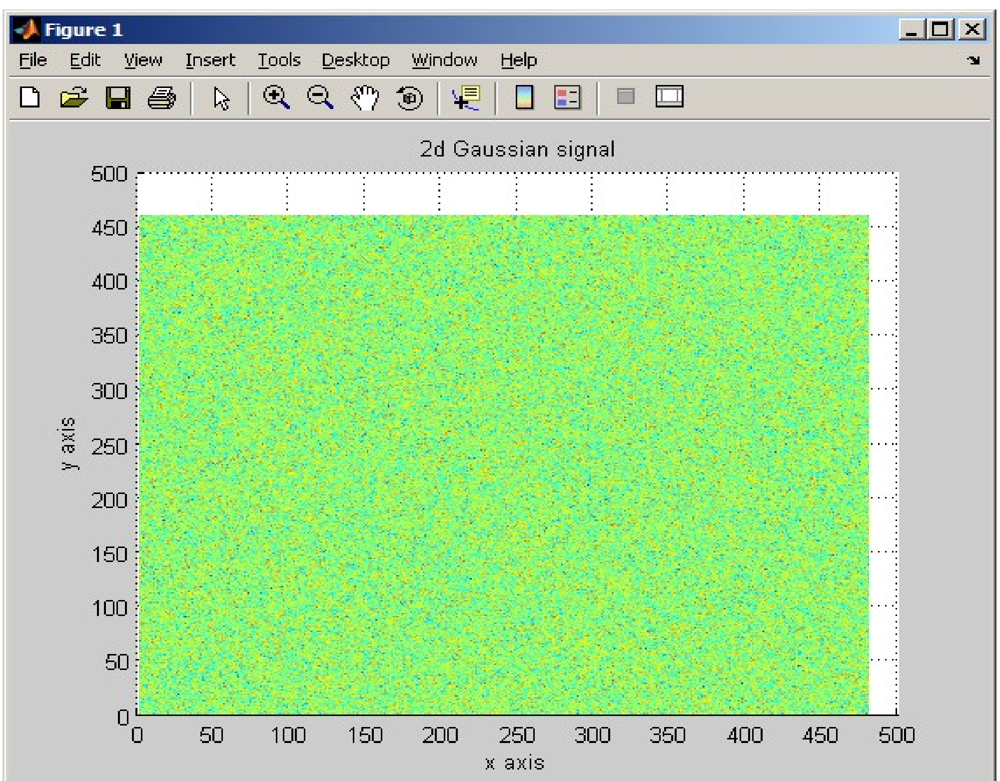
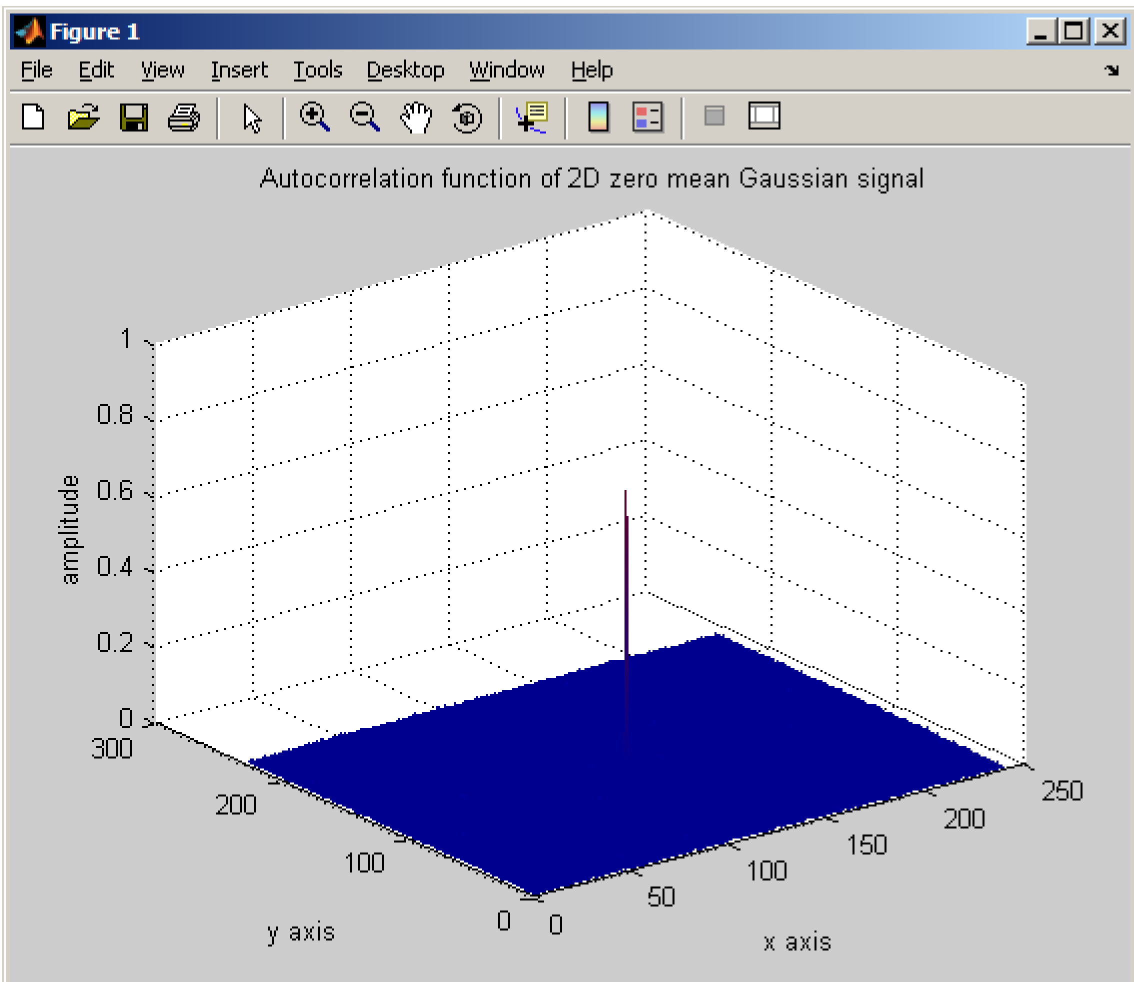


Figure 4 : Image view of the Gaussian signal

Then , if we calculate the auto-correlation of this signal, we find that is has a special form : it is symmetric , with no correlation except in the center which has a 2d Dirac impulsion .
Note : for better visualization we divide the size of signal to have X [230,240] :

```
I2=randn(230,240);
B2=abs(fftshift(ifft2(fft2(I2).*conj(fft2(I2))))./(230*240);
surf(B2)
shading interp
xlabel(' x axis')
ylabel(' y axis')
 zlabel(' amplitude')
title(' Autocorrelation function of 2D zero mean Gaussian signal')
```



The auto-correlation of the X signal is : $r(n,m)=\sigma^2.\delta(n,m)$ with :

σ : the constant power spectral density.
$\delta(n,m)$: 2d Dirac impulsion .

References :

[1] : **http://en.wikipedia.org/wiki/Wiener%E2%80%93Khinchin_theorem#cite_note-0**

[2] : Dennis Ward Ricker (2003). *Echo Signal Processing*. Springer. ISBN 1-4020-7395-X.

[3] : http://mathworld.wolfram.com/Wiener-KhinchinTheorem.html