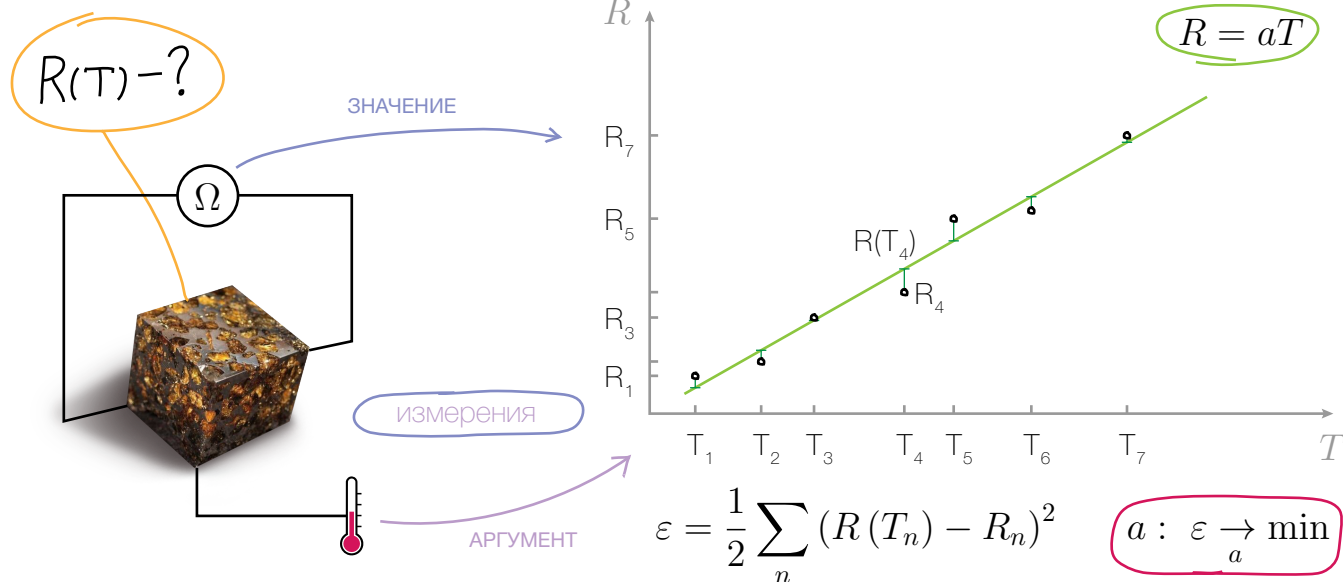
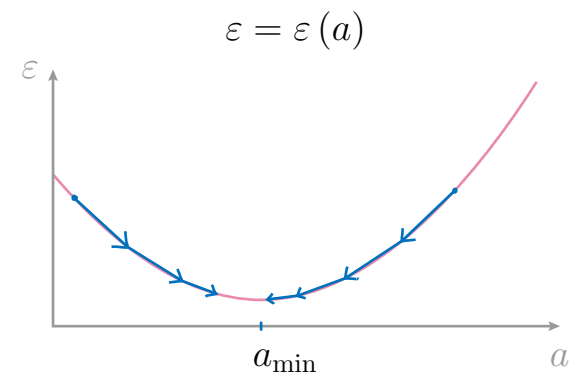


# Аппроксимация неизвестных функций

## Физические свойства



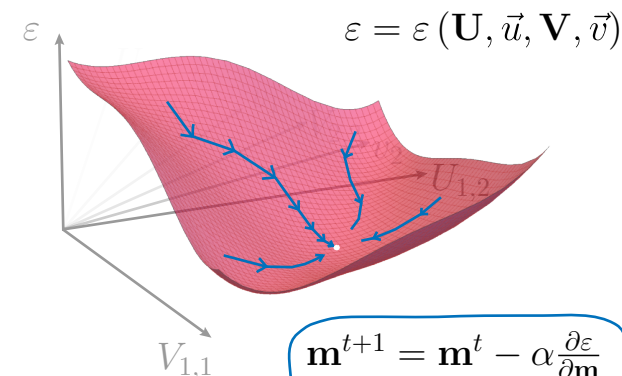
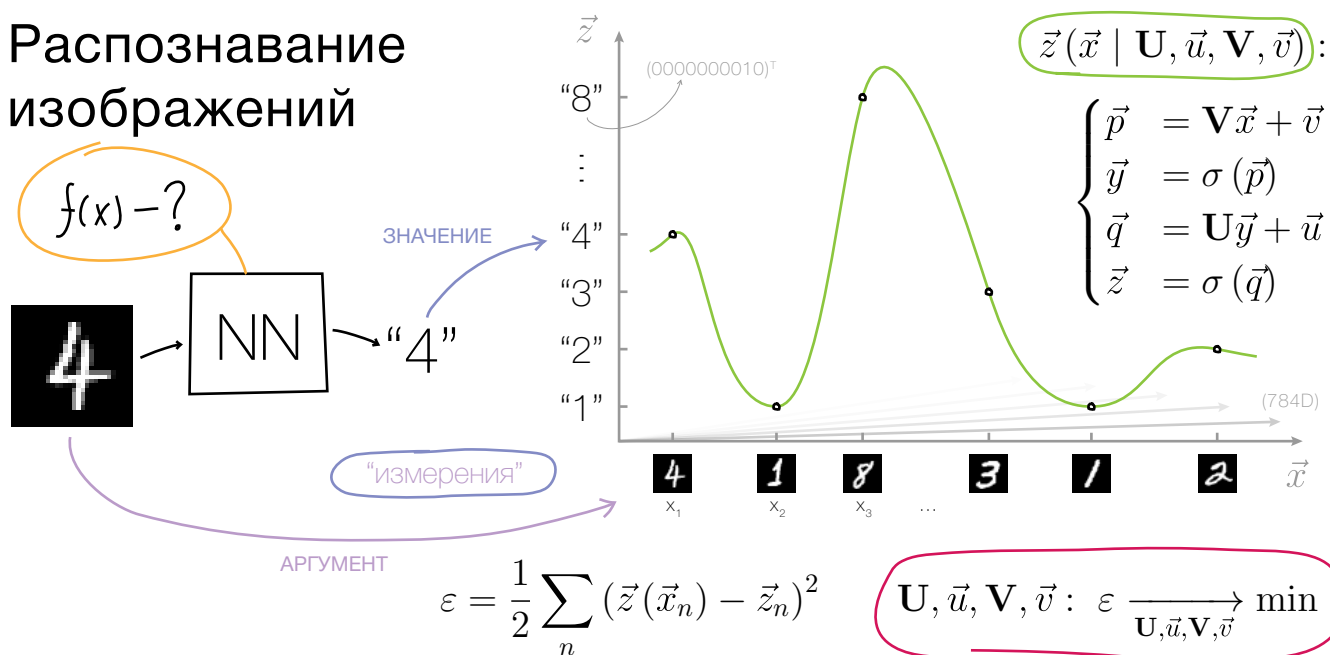
## Градиентный спуск



$a^{t+1} = a^t - \alpha \frac{d\varepsilon}{da}(a^t)$

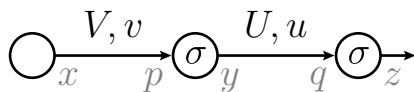
$a \xrightarrow{t \rightarrow +\infty} a_{\min}$

## Распознавание изображений



# Обучение нейросети

Упрощённый одномерный случай



$$\begin{aligned} p &= Vx + v \\ y &= \sigma(p) \\ q &= Uy + u \\ z &= \sigma(q) \\ \varepsilon &\stackrel{(\approx)}{=} \frac{1}{2} (z - l)^2 \end{aligned}$$

$$\frac{\partial \varepsilon}{\partial \mathbf{m}} \left\{ \begin{aligned} \frac{\partial \varepsilon}{\partial V} &= \frac{\partial \varepsilon}{\partial p} \frac{\partial p}{\partial V} = \frac{\partial \varepsilon}{\partial p} x \\ \frac{\partial \varepsilon}{\partial v} &= \frac{\partial \varepsilon}{\partial p} \frac{\partial p}{\partial v} = \frac{\partial \varepsilon}{\partial p} = \frac{\partial \varepsilon}{\partial y} \frac{\partial y}{\partial p} = \frac{\partial \varepsilon}{\partial y} \sigma'(p) \\ \frac{\partial \varepsilon}{\partial y} &= \frac{\partial \varepsilon}{\partial q} \frac{\partial q}{\partial y} = \frac{\partial \varepsilon}{\partial q} U \\ \frac{\partial \varepsilon}{\partial U} &= \frac{\partial \varepsilon}{\partial q} \frac{\partial q}{\partial U} = \frac{\partial \varepsilon}{\partial q} y \\ \frac{\partial \varepsilon}{\partial u} &= \frac{\partial \varepsilon}{\partial q} \frac{\partial q}{\partial u} = \frac{\partial \varepsilon}{\partial q} = \frac{\partial \varepsilon}{\partial z} \frac{\partial z}{\partial q} = \frac{\partial \varepsilon}{\partial z} \sigma'(q) \\ \frac{\partial \varepsilon}{\partial z} &= (z - l) \end{aligned} \right.$$

**Back Propagation**

$x/\vec{x}$  — изображение (АРГУМЕНТ)

$l/\vec{l}$  ( $= z_n/\vec{z}_n$ ) — метка (ЗНАЧЕНИЕ)

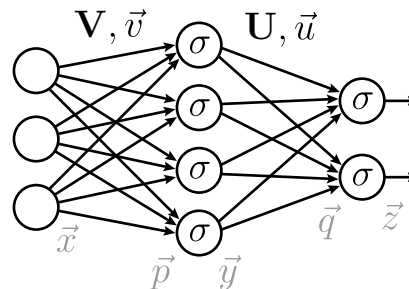
Функция активации

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Производная

$$\sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$

Полный вывод



$$\begin{aligned} \vec{p} &= \mathbf{V}\vec{x} + \vec{v} \\ \vec{y} &= \sigma(\vec{p}) \\ \vec{q} &= \mathbf{U}\vec{y} + \vec{u} \\ \vec{z} &= \sigma(\vec{q}) \\ \varepsilon &\stackrel{(\approx)}{=} \frac{1}{2} (\vec{z} - \vec{l})^2 \end{aligned}$$

$$\frac{\partial \varepsilon}{\partial \mathbf{m}} \left\{ \begin{aligned} \frac{\partial \varepsilon}{\partial \mathbf{V}} &= \frac{\partial \varepsilon}{\partial V_{ij}} = \sum_k \frac{\partial \varepsilon}{\partial p_k} \frac{\partial p_k}{\partial V_{ij}} = \frac{\partial \varepsilon}{\partial p_i} \frac{\partial p_i}{\partial V_{ij}} = \frac{\partial \varepsilon}{\partial p_i} x_j = \frac{\partial \varepsilon}{\partial \vec{p}} \otimes \vec{x} \\ &\text{столбец} \times \text{строка} = \text{матрица (тензорное произведение)} \\ \frac{\partial \varepsilon}{\partial \vec{v}} &= \frac{\partial \varepsilon}{\partial v_i} = \frac{\partial \varepsilon}{\partial p_i} \frac{\partial p_i}{\partial v_i} = \frac{\partial \varepsilon}{\partial p_i} = \frac{\partial \varepsilon}{\partial y_i} \frac{\partial y_i}{\partial p_i} = \frac{\partial \varepsilon}{\partial y_i} \sigma'(p_i) = \frac{\partial \varepsilon}{\partial \vec{y}} \odot \sigma'(\vec{p}) \\ &\text{поэлементное умножение (произведение Адамара)} \\ \frac{\partial \varepsilon}{\partial \vec{y}} &= \frac{\partial \varepsilon}{\partial y_i} = \sum_j \frac{\partial \varepsilon}{\partial q_j} \frac{\partial q_j}{\partial y_i} = \sum_j \frac{\partial \varepsilon}{\partial q_j} U_{ji} = \sum_j U_{ij}^T \frac{\partial \varepsilon}{\partial q_j} = \mathbf{U}^T \frac{\partial \varepsilon}{\partial \vec{q}} \\ &\text{матрица} \times \text{столбец} = \text{столбец} \\ \frac{\partial \varepsilon}{\partial \mathbf{U}} &= \dots = \frac{\partial \varepsilon}{\partial \vec{q}} \otimes \vec{y} \\ \frac{\partial \varepsilon}{\partial \vec{u}} &= \dots = \frac{\partial \varepsilon}{\partial q_i} \frac{\partial q_i}{\partial u_i} = \frac{\partial \varepsilon}{\partial q_i} = \frac{\partial \varepsilon}{\partial \vec{z}} \odot \sigma'(\vec{q}) \\ \frac{\partial \varepsilon}{\partial \vec{z}} &= \vec{z} - \vec{l} \end{aligned} \right.$$

**Back Propagation**