MATLAB PROJECT 2

Please include this page in your Group file, as a front page. Type in the group number and the names of all members WHO PARTICIPATED in this project.

GROUP # <u>5</u>

FIRST AND LAST NAMES (UFID numbers are NOT required):

- 1. Corey Wolfe
- 2. Thomas Pena Reina
- 3. Manuel Vera Miranda
- 4. Elton Li
- 5. Ekaterina Krysova
- 6. Dany Rashwan

By including your names above, each of you had confirmed that you did the work and agree with the work submitted.

Part I. Elementary Row Operations

Exercise 1

```
type ele1
function E1=ele1(n,r,i,j)
E1=eye(n);
E1(j,:)=E1(j,:)+r*E1(i,:);
end
type ele2
function E2=ele2(n,i,j)
E2=eye(n);
temp=E2(j,:);
E2(j,:)=E2(i,:);
E2(i,:)=temp;
end
type ele3
function E3=ele3(n,j,k)
E3=eye(n);
E3(j,:)=k*E3(j,:);
end
format
format compact
A=[0\ 1\ 3\ 1;\ 2\ 4\ 6\ -2;\ 3\ 1\ 4\ 2]
A = 3 \times 4
            1 3 1
4 6 -2
1 4 2
E2=ele2(3,1,2);
%switches first and second rows
```

```
A1=E2*A
```

E3=ele3(3,1,1/2);

%multiplies row 1 by 1/2

A2=E3*A1

E1=ele1(3,-3,1,3);

%multiplies row 1 by -3 and adds it to the 3rd row

A3=E1*A2

E1=ele1(3,5,2,3);

%multiplies row 2 by 5 and adds it to the 3rd row

A4=E1*A3

E3=ele3(3,3,1/10);

%multplies the 3rd row by 1/10

A5=E3*A4

E1=ele1(3,-3,3,2);

%multiplies the 3rd row by -3 and adds it to the 2nd row

A6=E1*A5

E1=ele1(3,-3,3,1);

```
%multiplies the 3rd row by -3 and adds it to the 1st row A7=E1*A6
```

```
E1=ele1(3,-2,2,1);
%multiples the 2nd row by -2 and adds it to the 1st row.
A8=E1*A7
```

```
%A8 is AN, and is in final reduced echelon form.
Ared=rref(A);
%Ared is matlabs reduced form matrix.
if (Ared == A8)
    fprintf 'Your reduced form matches MATLABs reduced form!'
end
```

Your reduced form matches MATLABs reduced form!

Part II. Basic Operations

Exercise 2

```
type inverses
```

```
% function used to find the inverse of a matrix
function F=inverses(A)
% matrix A is (n x n), for it to be inveretible
% A needs to have the n pivot points
rankA = rank(A);
n = size(A,1);
% determines if A is invertible
if (rankA ~= n)%returns empty matrix F and message
    F = []
    fprintf("matrix A is not invertible");
    return
else
    F = rref([A eye(n)]);
   %eliminates the first block of the matrix, results in inverse
    F = (F(:,(n+1:(2*n))))
    fprintf("P is the inverse of A using MATLAB function");
```

```
P = inv(A) %MATLAB build in inverse
    % function closetozeroroundoff used with p=5
    DiffMat = closetozeroroundoff(abs(F-P),5);
    % compares matrices F and P
    if isequal(DiffMat,zeros(n))
        fprintf("\nMatrices F and P match");
        fprintf("check code");
    end
end
type closetozeroroundoff
function B = closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
B=A;
end
%(a)
A=[4 \ 0 \ -7 \ -7; -6 \ 1 \ 11 \ 9; 7 \ -5 \ 10 \ 19; -1 \ 2 \ 3 \ -1]
A = 4 \times 4
                  -7
     4
            0
                         -7
           1
     -6
                  11
                         9
      7
            -5
                  10
                         19
             2
                         -1
F= inverses(A);
F = 4 \times 4
   -19 -14
                  0
                        7
                  -2 196
   -549 -401
   267 195
                  1 -95
   -278 -203 -1
                       99
P is the inverse of A using MATLAB function
P = 4 \times 4
  -19.0000 -14.0000 -0.0000
                                    7.0000
 -549.0000 -401.0000 -2.0000 196.0000
  267.0000 195.0000
                          1.0000 -95.0000
 -278.0000 -203.0000 -1.0000 99.0000
Matrices F and P match
%(b)
A=[1 -3 2 -4; -3 9 -1 5; 2 -6 4 -3; -4 12 2 7]
A = 4 \times 4
      1
           -3
                  2
                        -4
                        5
     -3
           9
                  -1
```

-3

4

2

2

-4

-6

12

```
F = inverses(A);
F =
     []
matrix A is not invertible
%(c)
A=magic(3);
F = inverses(A);
F = 3 \times 3
   0.1472 -0.1444 0.0639
-0.0611 0.0222 0.1056
-0.0194 0.1889 -0.1028
P is the inverse of A using MATLAB function
P = 3 \times 3
     0.1472 -0.1444 0.0639
            0.0222 0.1056
0.1889 -0.1028
    -0.0611
   -0.0194
Matrices F and P match
%(d)
A=hilb(5);
F = inverses(A);
F = 5 \times 5
                  -300
4800
                                 1050 -1400
-18900 26880
79380 -117600
179200
           25
                                                             630
                              -18900
         -300
                                                           -12600
                                79380
         1050
                   -18900
                                                          56700
        -1400 26880 -117600
-12600 56700
                               -117600
                                                          -88200
                                                           44100
                                             -88200
P is the inverse of A using MATLAB function
P = 5 \times 5
10<sup>5</sup> ×
   0.0105 -0.1890 0.7938 -1.1760 0.5670
   -0.0140 0.2688 -1.1760 1.7920 -0.8820
     0.0063 -0.1260 0.5670 -0.8820 0.4410
Matrices F and P match
%(e)
A=magic(6);
F = inverses(A);
F =
```

Part III. Solving Equations

matrix A is not invertible

Exercise 3

Part 1. Solving a system Ax = b

```
type solvesys.m
```

```
function [C,N]=solvesys(A,b)
    % This function takes as input an n*n matrix A and an n*1 vector b
    % If matrix A is invertible, the outputs are matrix C, whose
    % columns x1, x2, x3 are the three solutions to Ax=b obtained
    % using the three methods \, inv, and rref; and N is the vector whose
    % 3 entries are the 2-norms of the vectors of the differences
    % between each two distinct solutions.
    [~,n]=size(A);
     format long
     if rank(A) < n %checks if the matrix is not invertible
         disp('A is not invertible'); %returns the empty matrices C and N
         C = [];
         N = \lceil \rceil;
         if rank([A b]) ~= rank(A) %checks if the system is inconsistent
             disp('The system is inconsistent');
         else
             disp('The system is consistent, but the solution is not unique');
         end
         return %ends the function if A is not invertible
     end
     x1 = A \ ; %first method for solution
     x2 = inv(A)*b; %second method for solution
     refMat = rref([A b]); %matrix obtained from reducing the extended matrix
     x3 = refMat(:, end); %third method; last column of reduced matrix
     C = [x1 \ x2 \ x3]; %solution matrix
     disp('Solutions obtained by different methods are the columns of ');
     disp(C);
     n1=norm(x1-x2);
     n2=norm(x2-x3);
     n3=norm(x3-x1);
     disp('Norms of differences between solutions are the entries of');
     N=[n1;n2;n3] %matrix of differences between solutions
end
%(a)
A=magic(4); I=eye(size(A,1)); b=I(:,end)
b = 4 \times 1
      0
       0
       0
       1
solvesys(A, b);
```

A is not invertible
The system is inconsistent

```
%(b)
A=magic(4),b=A(:,end)
```

```
A = 4 \times 4
   16
        2
             3
                 13
   5 11 10
                  8
        7
             6
                 12
   4 14 15
                  1
b = 4 \times 1
   13
    8
   12
    1
```

solvesys(A,b);

A is not invertible

The system is consistent, but the solution is not unique

%(c) A=magic(5); b=fix(10*rand(size(A,1),1))

b = 5×1 0 2 5 9

solvesys(A, b);

Solutions obtained by different methods are the columns of Column 1

- -0.033653846153846
- 0.029807692307692
- 0.376923076923077
- 0.099038461538461
- -0.087500000000000

Column 2

- -0.033653846153846
- 0.029807692307692
- 0.376923076923077
- 0.099038461538461
- -0.0875000000000000

Column 3

- -0.033653846153846
- 0.029810298102981
- 0.376923076923077
- 0.099041533546326
- -0.0875000000000000

```
Norms of differences between solutions are the entries of
N = 3 \times 1
10<sup>-5</sup> ×
    0.00000000005732
    0.402832488834191
    0.402832488834704
%(d)
A=eye(6); b=fix(10*rand(size(A,1),1))
b = 6 \times 1
       1
       9
       9
       4
       8
       1
solvesys(A, b);
Solutions obtained by different methods are the columns of
      1
            1
      9
            9
                  9
      9
            9
                  9
      4
            4
                  4
      8
            8
                  8
                  1
Norms of differences between solutions are the entries of
N = 3 \times 1
       0
       0
       0
```

In part (d), since matrix A was the 6x6 identity matrix, the solution to the system is vector b.

```
%(e)
A=magic(7), b=fix(10*rand(size(A,1),1))
A = 7 \times 7
     30
            39
                   48
                           1
                                               28
                                 10
                                        19
     38
            47
                    7
                           9
                                        27
                                               29
                                 18
     46
             6
                    8
                          17
                                 26
                                        35
                                               37
      5
            14
                   16
                          25
                                 34
                                        36
                                               45
     13
            15
                   24
                          33
                                 42
                                       44
                                               4
     21
            23
                   32
                          41
                                 43
                                        3
                                               12
     22
            31
                   40
                          49
                                 2
                                        11
                                               20
b = 7 \times 1
      4
      9
      7
      9
      0
      8
```

solvesys(A, b);

```
Solutions obtained by different methods are the columns of
   -0.026359753203607
   0.091580446131941
   -0.074532510678690
    0.106530612244898
   -0.119383009017560
   0.179620313241576
    0.088258186995729
  Column 2
   -0.026359753203607
   0.091580446131941
   -0.074532510678690
   0.106530612244898
   -0.119383009017560
   0.179620313241576
   0.088258186995729
  Column 3
   -0.026359143327842
   0.091575091575092
   -0.074534161490683
   0.106529209621993
   -0.119383825417202
    0.179620034542314
    0.088258471237195
Norms of differences between solutions are the entries of
N = 3 \times 1
10^{-5} \times
    0.00000000010292
    0.587883777771398
    0.587883777778084
%(f)
A=hilb(7), b
```

```
A = 7 \times 7
   1.000000000000000 0.50000000000000
                                              0.333333333333333 ...
   0.500000000000000
                        0.3333333333333333
                                              0.250000000000000
   0.3333333333333333
                        0.250000000000000
                                              0.200000000000000
   0.2500000000000000
                       0.200000000000000
                                              0.166666666666667
   0.200000000000000
                      0.166666666666667
                                              0.142857142857143
   0.166666666666667
                        0.142857142857143
                                              0.125000000000000
   0.142857142857143
                        0.125000000000000
                                              0.1111111111111111
b = 7 \times 1
     4
     9
     7
     9
     6
     0
     8
```

solvesys(A, b);

```
Solutions obtained by different methods are the columns of
   1.0e+08 *
  Column 1
   0.001739080006816
  -0.074064480272672
   0.745516802628954
  -2.991962410221816
  5.619398418736615
  -4.949073376185741
  1.650208565312696
  Column 2
  0.001739080008332
  -0.074064480336399
  0.745516803265350
  -2.991962412762030
  5.619398423488756
  -4.949073380358478
  1.650208566700776
  Column 3
  0.001739080000000
  -0.074064480000000
  0.745516800000000
  -2.991962410000000
   5.619398420000000
  -4.949073370000000
   1.650208560000000
Norms of differences between solutions are the entries of
N = 3 \times 1
   0.698448925446246
   1.351941001755965
   0.866713758011968
```

Of the above exercises, it can be seen that the difference between solutions is small enough in most cases except in part (f), where it is greater than 2 between x1 and x2. The only one where it is truly 0 is in part (d), because of what was stated above.

Part 2. Condition numbers

```
c1 = cond(magic(7))

c1 = 7.111323446624705

c2 = cond(hilb(7))

c2 = 4.753673563768867e+08
```

c1 is significantly smaller than c2. This indicates that having a large condition number increases the error when solving a system of equations, and is the reason why the difference in the norms in part (f) is so much greater than in part (e).

```
A = hilb(7)
```

```
A = 7 \times 7
   1.0000000000000000
                         0.5000000000000000
                                              0.333333333333333 ...
   0.5000000000000000
                         0.3333333333333333
                                              0.2500000000000000
   0.333333333333333
                         0.2500000000000000
                                              0.200000000000000
   0.250000000000000
                         0.200000000000000
                                              0.166666666666667
   0.2000000000000000
                         0.166666666666667
                                              0.142857142857143
                         0.142857142857143
   0.1666666666666667
                                              0.125000000000000
   0.142857142857143
                         0.125000000000000
                                              0.1111111111111111
```

b=ones(7,1)

x=A\b

b1=b+0.01

y=A\b1

y = 7×1 10⁴ x 0.000707000004265 -0.033936000170225 0.381780001638653 -1.696800006364036 3.499650011654935 -3.359664010061042 1.213212003300447

norm(x-y)

ans = 5.242180560287886e+02

c3=rcond(A)

c3 = 1.015027595488996e-09

A = magic(7)

$A = 7 \times 7$						
30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

b=ones(7,1)

x=A\b

x = 7×1 0.005714285714286 0.005714285714286 0.005714285714286 0.005714285714286 0.005714285714286 0.005714285714286

0.005714285714286

b1=b+0.01

y=A\b1

```
y = 7×1

0.005771428571429

0.005771428571429

0.005771428571429

0.005771428571429

0.005771428571429

0.005771428571429

morm(x-y)

ans = 1.511857892036916e-04

c3=rcond(A)
```

Comparing the sensibility perturbations, we can see that the difference of the norms of x and y in hilb(7) is much greater (>10^6) than in magic(7). And comparing the reciprocal condition numbers c3, we can see that it was significantly smaller in hilb(7) than in magic(7).

In other words, when a slight change was made in b, the solution with matrix hilb(7) was much more sensitive to this change than the matrix magic(7) because the condition number of hilb(7) is much greater than in magic(7).

Generally, we can see that a matrix with a large condition number is more sensitive to perturbations than a matrix with a smaller condition number.

Part IV. Area, Graphics, and Volume in Matlab

Exercise 4

```
format
type areavol
```

```
%Creates the function 'areavol'
function D = areavol(A)

% Variable to determine if parallelogram or parallelipiped
isParallelogram = 0;

% Gets the number of columns to detect different vectors

% If it's 2 then it will be parallelogram, otherwise it's a parallelipiped
if isequal(size(A,2),2)
    isParallelogram = 1;
end

% Checks if linearly dependent
r = rank(A); % gets the rank
[rows, ~] = size(A); % gets the number of rows

% rows > rank, so these vectors are not independent.
if rows > r
    if isequal(isParallelogram, 1) % Cannot be built - parallelogram
```

%(a)

A=randi(10,2)

$$A = 2 \times 2$$
10 8
7 8

D=areavol(A)

The area of the parallelogram is D = 24.0000

%(b)

A=fix(10*rand(3))

D=areavol(A)

The volume of the parallelpiped is D = 336

%(c)

A=magic(3)

D=areavol(A)

The volume of the parallelpiped is D = 360

%(d)

B=randi([-10,10],2,1); A = [B,3*B]

 $A = 2 \times 2$

D=areavol(A)

Parallelogram cannot be built.

D = 0

%(e)

X=randi([-10,10],3,1);Y=randi([-10,10],3,1);A=[X,Y,X-Y]

 $A = 3 \times 3$

D=areavol(A)

Parallelipiped cannot be built.

D = 0

Exercise 5

R1 = [1, 0; 0, -1]

 $R1 = 2 \times 2$

R2 = [-1, 0; 0, 1]

 $R2 = 2 \times 2$

0 1

VS = [1, 0; 2, 1]

 $VS = 2 \times 2$

type transf

function C=transf(A,E)

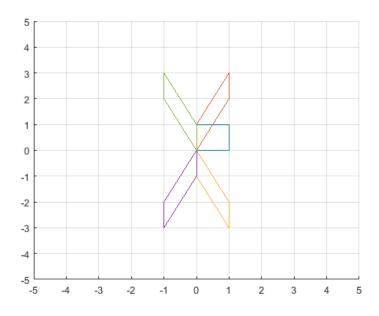
 $\ensuremath{\text{\%}}$ Create a function which transforms an image and plots it

C=A*E;

```
% linear transformation; Call the obtained matrix C,C represents an image
% represented by E under the transformation whose standard matrix is A
x=C(1,:);y=C(2,:);
% Call the first row of C x and call the second row y
plot(x,y)
% Plot x against y
v=[-5 \ 5 \ -5 \ 5];
% Create vector v which will be holding the boundaries of the plot
% Add axes and grid to the plot
end
E=[0 1 1 0 0;0 0 1 1 0];
A=eye(2);
hold on
Warning: MATLAB has disabled some advanced graphics rendering features by switching to
software OpenGL. For more information, click here.
grid on
E=transf(A,E)
E = 2 \times 5
            1
                   1
                           0
                                  0
      0
      0
                   1
A=VS;
E=transf(A,E)
E = 2 \times 5
             1 1 0
      0
                                  0
      0
             2
A=R1;
E=transf(A,E)
E = 2 \times 5
      0
            1
                  1
      0
          -2 -3 -1
A=R2;
E=transf(A,E)
E = 2 \times 5
           -1 -1
      0
           -2 -3 -1
      0
```

% Multiply matrices A and E. Here A is the standard matrix of a

```
A=R1;
E=transf(A,E)
```



```
E = 2 \times 5
0 -1 -1 0 0
0 2 3 1 0
```

Part V. Cofactors, Determinants, and Inverse Matrices

Exercise 6

```
type closetozeroroundoff.m
```

```
function B=closetozeroroundoff(A,p) A(abs(A)<10^{-}p)=0; B=A; end
```

type cofactor.m

```
function C=cofactor(a)
format compact
[m,n] = size(a);
C = zeros(m,n);
for i = 1:m
    for j = 1:n
        a_i_j = zeros(m-1, n-1);
```

```
a_i_j(1:(i-1), 1:(j-1)) = a(1:(i-1), 1:(j-1));
        a i j(i:(m-1), 1:(j-1)) = a((i+1):m, 1:(j-1));
        a_{i_j}(1:(i-1), j:(n-1)) = a(1:(i-1), (j+1):n);
        a_{i_j(i:(m-1), j:(n-1))} = a((i+1):m, (j+1):n);
        C(i,j) = (-1)^{(i+j)} * det(a_i_j);
    end
end
disp( 'the matrix of cofactors is')
disp(C)
end
 type determine.m
function D = determine(a,C)
D = [];
n = size(a,1);
if(rank(a) < n)
    fprintf('the determinant of the matrix is');
    D = 0
    return
else
   E = zeros(n,2);
   for i = 1:n
        E(i,1) = C(i,:) * transpose(a(i,:));
        E(i,2) = transpose(C(:,i)) * a(:,i);
    end
    for i=1:n
        for j=1:2
            if closetozeroroundoff((E(1,1)-E(i,j)), 7) \sim= 0
                disp('Something went wrong!');
                return
            end
        end
    end
    d=det(a);
    if closetozeroroundoff((d-E(1,1)),7) == 0
        disp('The determinant is ')
        D=E(1,1)
    else
        disp('Check the code!')
    end
end
end
 type inverse.m
function B = inverse(a,C,D)
\mathsf{B} = [];
if(rank(a) < size(a, 1))
    fprintf('a is not invertible');
    return
```

else

B = (1/D) * transpose(C);

```
end
F = inv(a);
if (closetozeroroundoff(abs(B-F),7) == zeros(size(B)))
   fprintf('the inverse is calculated correctly and it is the matrix');
else
   fprintf('What is wrong?!')
end
end
 %(a)
 a=diag([1,2,3,4])
a = 4 \times 4
                       0
     1
           0
                 0
            2
     0
                  0
                         0
     0
            0
                  3
                         0
     0
 C=cofactor(a);
the matrix of cofactors is
   24
        0 0 0
    0
         12
    0
         0
               8
                    0
    0
         0
               0
                    6
 D=determine(a,C);
The determinant is
D = 24
 B=inverse(a,C,D)
the inverse is calculated correctly and it is the matrix
B = 4 \times 4
                0
    1.0000
                              0
                                         0
                          0
             0.5000
         0
                                         0
               0
          0
                         0.3333
                                         0
          0
                     0
                                   0.2500
                               0
 %(b)
 a=ones(4)
a = 4 \times 4
    1
            1
                  1
                        1
```

C=cofactor(a);

```
the matrix of cofactors is
    0
          0
               0
                     0
    0
          0
                0
          0
                0
                      0
    0
    0
          0
                0
                      0
```

D=determine(a,C);

the determinant of the matrix is D = 0

B=inverse(a,C,D)

a is not invertible
B =
[]

%(c) a=magic(5)

$a = 5 \times 5$ 17 24 1 8 15 23 5 7 14 16 6 13 20 22 4 10 12 19 3 21 11 18 25 9

C=cofactor(a);

the matrix of cofactors is 1.0e+05 * -1.5340 0.2373 -0.2503 2.1873 0.1397 0.1560 -1.8915 2.5935 -0.3315 0.2535 -1.7940 -0.2340 0.1560 0.5460 2.1060 0.0585 0.6435 0.1560 2.2035 -2.2815 0.1723 0.0747 1.8460 0.5622 -1.8753

D=determine(a,C);

The determinant is D = 5.0700e+06

B=inverse(a,C,D)

the inverse is calculated correctly and it is the matrix $B = 5 \times 5$ -0.0049 0.0512 -0.0354 0.0012 0.0

```
0.0034
       -0.0373
0.0431
                 -0.0046
                        0.0127
                                  0.0015
                 0.0031
-0.0303
        0.0031
                         0.0031
                                  0.0364
0.0047
       -0.0065
                 0.0108
                        0.0435 -0.0370
0.0028
        0.0050
                 0.0415
                        -0.0450
                                  0.0111
```

```
%(d)
 a=magic(6)
a = 6 \times 6
    35
                       26
                             19
                                    24
          1
                 6
     3
          32
                7
                       21
                             23
                                    25
                 2
    31
           9
                       22
                             27
                                    20
     8
          28
               33
                       17
                            10
                                    15
    30
          5
                 34
                      12
                            14
                                    16
          36
                 29
     4
                      13
                             18
                                    11
 C=cofactor(a);
the matrix of cofactors is
  1.0e+06 *
   2.5894
           2.5894
                   -1.2947
                            -2.5894 -2.5894
                                               1.2947
  -0.0000
          0.0000
                   -0.0000
                             0
                                    -0.0000
                                              -0.0000
  -2.5894
          -2.5894
                   1.2947
                            2.5894
                                    2.5894
                                             -1.2947
  -2.5894
          -2.5894
                   1.2947
                           2.5894
                                    2.5894
                                              -1.2947
   0.0000
          0.0000
                  -0.0000
                           0.0000 -0.0000
                                              -0.0000
   2.5894
            2.5894
                   -1.2947
                           -2.5894
                                     -2.5894
                                              1.2947
 D=determine(a,C);
the determinant of the matrix is
D = 0
 B=inverse(a,C,D)
a is not invertibleB =
    []
 %(e)
 a=hilb(4)
a = 4 \times 4
    1.0000
             0.5000 0.3333
                                    0.2500
    0.5000
             0.3333
                        0.2500
                                    0.2000
    0.3333
              0.2500
                         0.2000
                                    0.1667
    0.2500
              0.2000
                         0.1667
                                    0.1429
 C=cofactor(a);
the matrix of cofactors is
   0.0000 -0.0000 0.0000
                            -0.0000
  -0.0000
          0.0002
                  -0.0004
                            0.0003
   0.0000
          -0.0004
                    0.0011
                             -0.0007
  -0.0000
           0.0003
                   -0.0007
                             0.0005
```

D=determine(a,C);

The determinant is

B=inverse(a,C,D)